Pramāṇa, Vol. 7, No. 5, 1976, pp. 313-315. © Printed in India.

Bare quark masses from the transverse cut offs in $e^+ e^-$ interactions

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MS received 12 June 1976

Abstract. From the transverse cuts off of the electron positron annihilation in the SPEAR II regime, we obtain positivity domains of bare quark masses.

Keywords. Electron positron; hamiltonian density.

In a recent paper, (Gronau *et al* 1975) made an interesting observation that the coefficient α of $\cos^2\theta$ term of the single particle inclusive distribution $e^+ e^- \to h + x$ exhibits a unique dependence on $\langle p_T{}^2\rangle$, the transverse momentum of the jet. Careful determination of α will determine $\langle p_T{}^2\rangle$ once the momentum of the particle is known.

On the other hand, in view of the phenomenology of the deep inelastic scattering and the present situation in the electron positron annihilation physics, bare quark mass has acquired some meaning through the hadronic hamiltonian density or the photon sectral representation. Such bare quark mass appears in the hadronic hamiltonian density (Testa 1975, Leutwyler 1974)

$$\theta_{00} = \bar{q}\,(-\,i)\,q + m_0\bar{q}\lambda_0 q + m_8\bar{q}\lambda_8 q \tag{1}$$

as

$$m_u = m_d = \sqrt{\frac{2}{3}}\,m_0 + \frac{m_8}{\sqrt{3}} \quad \text{and} \quad m_s = \frac{\sqrt{2}}{3}\,m_0 - \frac{2m_8}{\sqrt{3}}. \tag{2}$$

In the exact SU (3) limit

$$m_u = m_d = m_s \quad \text{and} \quad m_8 = 0. \tag{3}$$

In the physics of electron positron annihilation, a corresponding quantity appears & in the spectral representation of the free field singularity structure (Frishman 1972).

$$\rho\,(g) = \frac{2}{3s}\left(m^2 + \frac{s}{2}\right)\frac{\pi}{2}\left(1 - \frac{4m^2}{s}\right)^{1/2} \mathrm{Tr}\,\frac{\bar{Q}\bar{Q}}{(4\pi)^2} \tag{4}$$

where $s = Q^2$ is the centre of mass energy, $\bar{Q}$ is the quark charge and m is the bare mass of the quarks.

The aim of the present note is to determine the positivity domain of the bare quark mass as a function of $\langle p_T{}^2\rangle$ of the jet. To this end, we use eq. (1) of Gronau *et al viz.*,

$$\frac{d\sigma}{d\Omega_{\text{jet}}} \infty\ 1 + \alpha_j \cos^2\theta_j + \alpha_j P^2 \sin^2\theta_j \cos^2\theta_j \tag{5}$$

where $-1 < \alpha_j < 1$, and other terms have usual significance.

The coefficient function α multiplying the $\cos^2\theta$ term is obtained by integrating eq. (5) with respect to the solid angle of the jet, giving

$$Q^2 \frac{d\sigma}{\frac{d^3p}{E}} \sim 1 + \alpha(Q^2, p)\cos^2\theta + \alpha(Q^2, p) P^2 \sin^2\theta \cos^2\psi \tag{6}$$

with

$$\alpha(Q^2, p) = \alpha_j \frac{1 - \frac{3}{2}\frac{\langle p_T^2\rangle}{p^2}}{1 + \frac{1}{2}\alpha_j \frac{\langle p_T^2\rangle}{p^2}}. \tag{7}$$

In order to relate the bare quark mass to $\langle p_T^2\rangle$, of the jet, we make use of the leading order parton/light cone expansion for $e^+e^- \to h + x$. The result (Choudhury 1976) is

$$Q^2 \frac{d\sigma}{\frac{d^3p}{E}} = \frac{\pi\alpha^2}{2} F_2^{ep}(x)\left[(1+\cos^2\theta)\left(1 - \frac{2m^2}{s}\right) + 4(1-\cos^2\theta)\frac{m^2}{s}\right] \tag{8}$$

which leads to

$$Q^2 \frac{d\sigma}{\frac{d^3p}{E}} \simeq 1 + \frac{1 - \frac{6m^2}{s}}{1 + \frac{2m^2}{s}}\cos^2\theta + \frac{1 - \frac{6m^2}{s}}{1 + \frac{2m^2}{s}} P^2 \sin^2\theta\cos^2\psi. \tag{9}$$

Thus under the high energy limit,

$$\frac{S}{8}\frac{\frac{\langle p_T^2\rangle}{p^2}}{\left(1 - \frac{1}{2}\frac{\langle p_T^2\rangle}{p^2}\right)} < m^2 < \frac{S}{4}\frac{\left(1 - \frac{\langle p_T^2\rangle}{p^2}\right)}{\left(1 - \frac{1}{2}\frac{\langle p_T^2\rangle}{p^2}\right)} \tag{10}$$

which is our main result.

Stability of the positivity domain of the bare quark mass with respect to the second and third order nonleading terms has been investigated earlier (Choudhury 1976). For example

$$0 < m^2 < 3\cdot 28\ \text{GeV}^2 \left[\frac{1 + \frac{1}{4}\epsilon}{1 - \frac{1}{4}\epsilon}\right]^{\frac{1}{2}}: \quad \text{(2nd order term)}$$

$$0 < m^2 < 3\cdot 87\ \text{GeV}^2 \left[\frac{1 + \frac{1}{4}\epsilon}{1 - \frac{1}{4}\epsilon}\right]^{\frac{1}{2}}: \quad \text{(3rd order term)}$$

showing relative stability of the positivity domains. In the above, ϵ is ratio of the charges of the spin 0 to spin $\frac{1}{2}$ constituents.

Experimentally (Morehouse 1975) the mean momentum observed for events with > 3 prongs are

$$s = 23\cdot 04\ \text{GeV}^2; \qquad \langle p^2\rangle = 0\cdot 25\ \text{GeV}^2$$

$$s = 38\cdot 44\ \text{GeV}^2; \qquad \langle p^2\rangle = 0\cdot 30\ \text{GeV}^2$$

$$s = 54\cdot 76\ \text{GeV}^2; \qquad \langle p^2\rangle = 0\cdot 374\ \text{GeV}^2.$$

Table 1. p_T^2 as a function of bare quark (mass)^2 in GeV^2

p_T^2 (GeV)^2	$\sqrt{S} = 4\cdot 8$ GeV	$\sqrt{S} = 6\cdot 2$ GeV	$\sqrt{S} = 7\cdot 4$ GeV
0	$0 < m^2 < 5\cdot 76$	$0 < m^2 < 9\cdot 61$	$0 < m^2 < 13\cdot 59$
0·01	$0\cdot 11 < m^2 < 5\cdot 64$	$0\cdot 15 < m^2 < 6\cdot 51$	$0\cdot 18 < m^2 < 13\cdot 41$
0·02	$0\cdot 23 < m^2 < 5\cdot 52$	$0\cdot 32 < m^2 < 9\cdot 2$	$0\cdot 37 < m^2 < 13\cdot 21$
0·03	$0\cdot 35 < m^2 < 5\cdot 39$	$0\cdot 50 < m^2 < 9\cdot 0$	$0\cdot 57 < m^2 < 12\cdot 75$
0·04	$0\cdot 47 < m^2 < 5\cdot 25$	$0\cdot 68 < m^2 < 8\cdot 9$	$0\cdot 77 < m^2 < 12\cdot 82$
0·05	$0\cdot 59 < m^2 < 5\cdot 12$	$0\cdot 86 < m^2 < 8\cdot 7$	$0\cdot 97 < m^2 < 12\cdot 60$
0·06	$0\cdot 71 < m^2 < 4\cdot 9$	$1\cdot 05 < m^2 < 8\cdot 55$	$1\cdot 19 < m^2 < 12\cdot 40$
0·07	$0\cdot 83 < m^2 < 4\cdot 8$	$1\cdot 25 < m^2 < 8\cdot 34$	$1\cdot 41 < m^2 < 12\cdot 18$
0·08	$0\cdot 96 < m^2 < 4\cdot 66$	$1\cdot 46 < m^2 < 8\cdot 14$	$1\cdot 63 < m^2 < 11\cdot 96$
0·09	$1\cdot 00 < m^2 < 4\cdot 49$	$1\cdot 67 < m^2 < 7\cdot 93$	$1\cdot 86 < m^2 < 10\cdot 73$
0·10	$1\cdot 20 < m^2 < 4\cdot 32$	$1\cdot 90 < m^2 < 7\cdot 7$	$2\cdot 10 < m^2 < 11\cdot 19$

In table 1, we record the positivity domains of the bare quark (mass)^2 as a function (P_T^2) in the SPEAR II regime. We now recall that Testa obtained $m_u = m_d = 11$ MeV and $m_s = 274$ MeV. Null plane dynamics of Leutwyler put $m_u = m_d = 54$ MeV and 125 MeV $< m_s <$ 150 MeV. SPEAR II regime seems to rule out such a light bare mass, if it makes sense to consider the light cone algebra almost exact even up to a few nonleading terms, as we have done in the present note.

To conclude, our analysis is complementary to the work of Gronau *et al* in the sense that if the transverse cut off is inferred from the parameter α, so does the bare quark mass from the transverse cut off.

Acknowledgement

Part of this work was done, when the author was at the University of Liverpool. He thanks C Michael for comments on the manuscript.

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