Pramāṇa, Vol. 7, No. 5, 1976, pp. 307-312. © Printed in India.

SU(4), broken Zweig rule and the null plane

D K CHOUDHURY
Tata Institute of Fundamental Research, Bombay 400005

MS received 12 June 1976

Abstract. Using Glauber's theory of eikonal approximation, an algebraic representation of the broken Zweig rule in the constituent quark model is obtained. Null plane language is then elaborated. It is pointed out that the exotic current associated with the disconnected quark diagrams, which break the Zweig rule, can have canonical dimension.

Keywords. Glauber's theory; Zweig rule; SU(4).

The recent discovery of ψ (3100) and ψ' (3700) (Augustin *et al* 1974, Aubert *et al* 1974, Bacci *et al* 1974, Abrams *et al* 1974) has focussed considerable attention on the dynamics and symmetrics of the broken Zweig rule. Mechanisms for the broken Zweig rule have recently been investigated within the context of quark gluon field theory (Appelquist and Politzer 1975, Fritzsch and Minkowski 1975), and the dual models of hadrons (Freund and Nambu 1975). While in the former the smallness of the non-planar/disconnected quark diagram is ascribed to the smallness of the quark gluon coupling constant due to the asymptotic freedom, in the latter the smallness is ascribed to the smallness of the $J^P = 1^-$ closed loop Pomeron daughter coupling to the hadron. Of course, the validity of asymptotic freedom or Ore-Powell calculation (Ore and Powell 1949) to the decays of hadrons is not rigorous. However, the experimental result is too tempting to warrant a strictly puritan view on the validity of such a technique. With such a *raison d'etre* in mind, we apply Glauber's theory of eikonal approximation (Glauber 1959) to the decay processes and demonstrate an algebraic representation of the broken Zweig rule in a constituent quark picture (Dalitz 1966). Null plane language is then elaborated upon.

A single quark transition operator is repr sented by

$$M^{(1)}(q) = f_q\, \sigma^{(1)}.q \sum_{a=1}^{15} \lambda^a.\pi^a \tag{1}$$

in SU(8) space which gives the amplitude for the connected quark diagram (figure 1). Here λ^a are the SU (4) generators and σ and q are spin and momentum operators of the constituent quark and the emitted meson respectively.

In order to incorporate the non-planar or disconnected quark diagrams (figure 2), one needs to invoke multiple scattering effects of the meson on the second quark or antiquark (figure 3). Here, the meson emitted from the first quark gets scattered from the second quark (or antiquark). To obtain an algebraic representation to such diagrams, we decompose the momentum

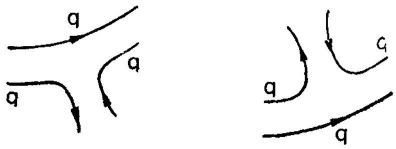

Figure 1. Connected quark diagram for single quark amplitude.

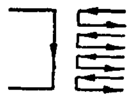

Figure 2. Disconnected quark diagrams of the broken Zweig rule

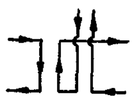

Figure 3. Non-planar quark diagrams as double scattering effect.

$$q = p + K$$

where p = momentum canonically conjugate to the impact parameter b, as done by Choudhury some time back (Choudhury 1968) (hereafter referred to as I). Then eq. (1) in the impact parameter representation becomes

$$a^{(1)}(b) = \frac{1}{2\pi}\int d^2 p\, e^{-ip.b}\, M^{(1)}(q). \tag{2}$$

The amplitude for the meson emitted from the quark at $-\frac{1}{2}S$ and scattered by the antiquark at $+\frac{1}{2}S$ becomes

$$\underset{1\to 2}{M}(p) = \langle f | \frac{1}{2\pi}\int d^2 b\, e^{i(b-\frac{1}{2}s).p}\, a^{(1)}(b)\, e^{ix(b-s)} \,|\, i\rangle \tag{3}$$

where $| i\rangle$ and $\langle f |$ are the initial and final mesonic states, $x(b - s)$ is as defined by Glauber (1959) which can be expressed in terms of the pion quark amplitude $\pi + Q^{(2)} \to \pi + Q^{(2)}$, defined by $f^{(2)}(p')$, *viz.*,

$$e^{ix(-s)} = 1 - \frac{1}{2\pi i q}\int e^{ip'.s} f^{(2)}(p')\, d^2 p'. \tag{4}$$

This then finally yields the total decay amplitude as

$$F_{fi} = \sum_{i\neq j}^{2} \langle f | e^{-ip(s/2)} M^{(i)}(q) \,|\, i\rangle$$

$$-\frac{1}{2\pi i q}\int \langle f | e^{i(p'-(p/2)).s} M^{(i)}(q) f^{(i)}(p')\, d^2 p' \,|\, i\rangle. \tag{5}$$

As in I, one can neglect the spin flip part of the pion quark amplitude proportional to $\sigma \cdot \hat{n}$, (where $\hat{n}$ is the unit vector perpendicular to the initial and final meson momentum), which follows from the invariance of the scale dimension of the axial current within the algebraic approach of $SU_w(8)$.

The pion quark amplitude has the SU(4) decomposition

$$4 \times 15 = 4 + \overline{20'} + 36 \tag{6}$$

to be compared with the SU(3) decomposition

$$3 \times 8 = 3 + \bar{6} + 15. \tag{7}$$

Hence the enlargement of internal symmetry from SU(3) to SU(4) will not change the number of invariant amplitudes, so that $\pi_\alpha + Q \to \pi_\beta + Q$ amplitude is represented by

$$f(p') = X\,\delta_{\alpha\beta} + iy f_{\alpha\beta\gamma}\,\lambda_\gamma + Z d_{\alpha\beta\gamma}\,\lambda_\gamma. \tag{8}$$

Here $f_{\alpha\beta\gamma}$ and $d_{\alpha\beta\gamma}$ are the SU(4) λ-matrices defined as

$$[\lambda_k, \lambda_l] = 2i f_{klm}\,\lambda_m$$

$$\{\lambda_k, \lambda_l\} = \delta_{kl} + 2\, a_{klm}\,\lambda_m \tag{9}$$

and x, y, z are the linear combinations of the three invariant amplitudes occurring in the decomposition, eq. (6). Assuming that the meson quark amplitude is purely imaginary, one can use eq. (3·12) of (I). Thus under the assumption of Gaussian shapes of hadronic form factors $F(p) = e^{-\beta(p^2/4)}$, and the diffractive slope (a) of the pion quark amplitude, the effective meson meson coupling $g_{I\gamma}$ becomes

$$g_{I\gamma} = f_a \langle f | \sum_{i=1}^{2} \lambda_\alpha^{(i)} \cdot \pi_\alpha \,|\, i \rangle \left[1 - \frac{1}{(\beta + a/2)\, 8\pi}\, \tilde{g}_{I\gamma} \right] \tag{10}$$

where $\tilde{g}_{I\gamma}$ is the effective coupling for the non-planar quark diagrams given by figure 3, to be

$$\tilde{g}_{I\gamma} = \frac{\langle f | \sum\limits_{i \neq j=1}^{2} \sum\limits_{\alpha,\beta=1}^{15} \lambda_\alpha^{(i)} \cdot \pi_\alpha \,(X\delta_{\alpha\beta} + iy\, f_{\alpha\beta\gamma}\, \lambda_{\alpha\beta\gamma}\, \lambda_k^{(j)} + Z d_{\alpha\beta\gamma}\, \lambda_\gamma^{(j)} \,|\, i \rangle}{\langle f | \sum\limits_{i=1}^{2} \lambda_\alpha^{(i)} \cdot \pi_\alpha \,|\, i \rangle}. \tag{11}$$

The SU(3) decomposition of eq. (6) is given by Amati *et al* (1964)

$$3 \oplus 1 \oplus \{\bar{8} \oplus 3 \oplus \bar{6} \oplus \bar{3}\} \oplus \{6 \oplus 3 \oplus 15 \oplus 1 \oplus 8 \oplus \bar{3}\}. \tag{12}$$

As noted by Deloff (1967) the invariant amplitudes of (7) do not differ by order of magnitudes; so we may assume it to be true even for the SU(3) decomposition, eq. (12). This then allows us to infer that second term of eq. (10) is $\sim 5\%$ of the first term for $\sqrt{b}/2 \approx 4$ GeV^{-1} and $\sqrt{a}/2 \approx 2\cdot 8$ GeV^{-1}. Thus the larger size of the particle and/or the larger size of the diffractive slope of the pion quark amplitude reduces the contribution of figure 3, within our formalism. Hence relative stability of ψ and ψ' compared with ϕ is therefore attributed to broken SU(4) parametrized through β and α. It now follows that all the processes,

Table 1. Mesonic SU(4) operators with non-planar diagrams. Superscripts denote quark indices $i \neq j = 1, 2$.

State	Operators
$\pi^{\pm}$	$(\lambda_1\pm i\lambda_2)^{(i)} + \frac{y}{x}\{\pm\lambda_3^{(i)}(\lambda_1\pm i\lambda_2)^{(i)} \mp\lambda_3^{(j)}(\lambda_1\pm i\lambda_2)^{(i)} + \frac{1}{2}(\mp(\lambda_4\pm i\lambda_5)^{(j)}(\lambda_6\mp i\lambda_7)^{(i)} \pm(\lambda_4\pm i\lambda_5)^{(i)}(\lambda_6\mp i\lambda_7)^{(j)} \pm(\lambda_9\pm i\lambda_{10})^{(i)}(\lambda_{11}\mp i\lambda_{12})^{(j)} \mp(\lambda_9\pm i\lambda_{10})^{(j)}(\lambda_{11}\mp i\lambda_{12})^{(i)}\} + \frac{z}{x}\{\frac{1}{\sqrt{3}}(\lambda_8^{(i)}(\lambda_1\mp i\lambda_2)^{(j)} + \lambda_8^{(j)}(\lambda_1\pm i\lambda_2)^{(i)} + \frac{1}{2}(\lambda_4\pm i\lambda_5)^{(i)}(\lambda_6\mp i\lambda_7)^{(j)} + (\lambda_4\pm i\lambda_5)^{(j)}(\lambda_6\mp i\lambda_7)^{(i)} + (\lambda_9\pm i\lambda_{10})^{(i)}(\lambda_{11}\mp i\lambda_{12})^{(j)} + (\lambda_9\pm i\lambda_{10})^{(j)}(\lambda_{11}\mp i\lambda_{12})^{(i)}) + \frac{1}{\sqrt{6}}(\lambda_{15}^{(i)}(\lambda_1\pm i\lambda_2)^{(j)} + \lambda_{15}^{(j)}(\lambda_1\pm i\lambda_2)^{(i)})\}$
$K^{\pm}$	$(\lambda_4\pm i\lambda_5)^{(i)} + \frac{y}{2x}\{\pm(\lambda_1\pm i\lambda_2)^{(i)}(\lambda_6\pm i\lambda_7)^{(j)} \mp(\lambda_1\mp i\lambda_2)^{(j)}(\lambda_6\pm i\lambda_7)^{(i)} \pm\lambda_3^{(i)}(\lambda_4\pm i\lambda_5)^{(j)} \mp\lambda_3^{(j)}(\lambda_4\pm i\lambda_5)^{(i)} \pm\sqrt{3}\,\lambda_8^{(i)}(\lambda_4\pm i\lambda_5)^{(j)} \mp\sqrt{3}\,\lambda_8^{(j)}(\lambda_4\pm i\lambda_5)^{(i)} \mp(\lambda_9\pm i\lambda_{10})^{(j)}(\lambda_{13}\mp i\lambda_{14})^{(i)} \pm(\lambda_9\pm i\lambda_{10})^{(i)}(\lambda_{13}\mp i\lambda_{14})^{(j)}\} + \frac{z}{2x}\ (\lambda_1\pm i\lambda_2)^{(i)}(\lambda_6\pm i\lambda_7)^{(j)} + (\lambda_1\pm i\lambda_2)^{(j)}(\lambda_6\pm i\lambda_7)^{(i)} + \lambda_3^{(i)}(\lambda_4\pm i\lambda_5)^{(j)} + \lambda_3^{(j)}(\lambda_4\pm i\lambda_5)^{(i)} - \frac{1}{\sqrt{3}}\lambda_8^{(i)}(\lambda_4\pm i\lambda_5)^{(j)} - \frac{1}{\sqrt{3}}\lambda_8^{(j)}(\lambda_4\pm i\lambda_5)^{(i)} + (\lambda_9\pm i\lambda_{10})^{(i)}(\lambda_{13}\mp i\lambda_{14})^{(j)} + \frac{2}{\sqrt{6}}\lambda_{15}^{(i)}(\lambda_4\pm i\lambda_5)^{(j)} + \frac{2}{\sqrt{6}}\lambda_{15}^{(j)}(\lambda_4\pm i\lambda_5)^{(i)}\}$

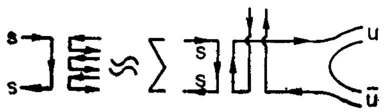

Figure 4. $\phi \rightarrow 3\pi, \psi \rightarrow$ hadrons, in the present picture. Σ indicate all possible intermediate hadronic states in the sequential decays.

which are allowed only by the diagram, figure 2, will proceed *via* figure 3 through sequential two-body processes. As in illustration, $\phi \rightarrow 3\pi$ decay proceeds *via* the factorizable amplitude $(\phi \rightarrow \rho\pi) \cdot (\rho \rightarrow \pi\pi)$ as given in figure 4. Similar sequential diagrams for $\psi \rightarrow 3\pi$ or $\psi' \rightarrow \psi + 2\pi$ can easily be drawn. Because of the non-planar diagram, figure 3, the SU (4) mesonic operators are modified from their usual simple forms. In table 1 we explicitly give the representations for pion and kaon, showing that the non-planar diagrams operate through the terms proportional to y/x and z/x. As a result, a pion, for example, can spend part of the time as a mixture of K, $\bar{K}$, D, $\bar{D}$, n, n_c, while kaon as that of π, K, $\bar{K}$ D, $\bar{D}$, n, n_c, $F^{\pm}$, before emerging from the hadronic structure (Galliard *et al* 1974).

Let us now use the null plane language (Fritzsch and Gell Mann 1972) for eq. (10). The pionic transition amplitude, eq. (1) is related by PCAC to the divergence of the axial current, defined on the null plane to be

$$\tilde{F}_\pi^{(i)} = \sqrt{2} \int d^4(x^+)\, q_+^+(x)\, \sigma_z \frac{\lambda i}{2} q_+(x) \tag{13}$$

with the usual notation (Weyers 1973). Since canonical scale invariance allows us to neglect the spinflip past of the pion quark amplitude, Dirac part of eq. (13) remains unaltered, while the SU(4) part gets the replacement

$$15 \rightarrow 15 \oplus 15 \oplus (4 \oplus \bar{20}' \oplus 36). \tag{14}$$

Hence, the non-planar diagrams invariably invoke the presence of exotic currents, as far as its internal symmetry representation is concerned. Theoretical argument for such an operator had already been advocated some time back (Osborn 1974). In the present note, we have demonstrated that such an operator can be simulated as a multiple scattering effect and can be related to the non-planar diagram of figure 3, which in turn breaks the Zweig rule. However, the scale dimension of the axial current is unmodified by the non-planar diagrams. This is to be contrasted with the recent work of Carlitz and Weyers (Carlitz and Weyers 1975), where exotic currents are invariably linked with the currents of higher dimensions.

Note added in proof

After this work was completed, many new developments have taken place in the subject of Zweig rule; to mention one, the dual scheme of Rosen Zweig (1975). In comparison with the above scheme, our scheme is of course a much more modest and *ad hoc* scheme. For other schemes on Zweig rule, see Pasupathy (1975 and 1976).

Acknowledgement

Part of this work was done when the author was at the University of Liverpool. He thanks C Michael for discussions and for comments on the manuscript. He also thanks J Pasupathy for bringing to his notice the most recent work on Zweig rule.

References

Abrams G S *et al* 1974 *Phys. Rev. Lett.* **33** 1453
Amati D, Bacry H, Nuyts J and Prentki J 1964 *Nuovo Cimento* **34** 1732
Appelquist T and Politzer H D 1975 *Phys. Rev. Lett.* **34** 43
Aubert J J *et al* 1974 *Phys. Rev. Lett.* **33** 1404
Augustin J E *et al* 1974 *Phys. Rev. Lett.* **33** 1406
Bacci C *et al* 1974 *Phys. Rev. Lett.* **33** 1408
Carlitz H and Weyers J 1975 *Phys. Lett.* **56B** 154
Choudhury D K 1968 *Phys. Rev.* **174** 2101
Dalitz R H 1966 *Quark Sub-Structure for Mesonic and Baryonic States* Tokyo Summer School
Deloff A 1967 *Nucl. Phys.* **B2** 597
Freund P G O and Nambu Y 1975 *Phys. Rev. Lett.* **34** 1645
Fritzsch H and Gell Mann M 1972 Talk presented at XVI-Int. Conf. High Energy Phys., Chicago
Fritzsch H and Minkowski P 1975 *Nuovo Cimento A* (to be published)
Gaillard M K, Lee B W and Rosner J L 1974 Fermilab-Pub. 74/86 THY
Glauber R J 1959 in *Lectures in Theoretical Physics* (Interscience Publ., New York) Vol. 1
Ore A and Powell J L 1949 *Phys. Rev.* **75** 1696
Osborn H 1974 *Nucl. Phys.* **B80** 90
Pasupathy J 1975 *Phys. Lett.* **58B** 71
Pasupathy J 1976 *Phys. Rev.* **D12** 2929
Rosen Zweig and Chew O F 1975 *Phys. Lett.* **3** 93
Weyers J 1973 *CERN Report* Ref. TH 1743