Pramāṇa, Vol. 7, No. 5, 1976, pp. 291–306. © Printed in India.

A unified U_3 gauge theory of weak and electromagnetic interactions with six quark-flavours and lepton-types

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MS received 17 April 1976

Abstract. Motivated by a desire to attempt a unified description of some recently observed phenomena such as the unusual events in the deep underground experiments at Kolar, the dilepton events initiated by high energy laboratory neutrino beams, the possible production of a new heavy lepton in $e^+ e^-$ annihilation along with the conventional weak interaction processes, a U_3 gauge theory of weak and electromagnetic interactions is proposed. The theory makes use of six flavours of quarks (Charm, Taste and Grace plus the three old SU_3 flavours), each in three colours, and correspondingly six lepton-types. The introduction of the corresponding fermions, and their assignment to representations of the proposed U_3 group, is dictated by the stringent requirements of attaining an anomaly free renormalizable gauge theory and of ensuring that the neutral currents turn out strangeness-preserving. A spontaneous gauge symmetry breaking mechanism is employed to generate the mechanical masses of the vector gauge-bosons of the theory (other than the photon) and of the quarks and the leptons. Diagonalisation of the quark mass matrix so generated enables a natural introduction of mixing angles including the Cabibbo-angle.

Keywords. U_3(W)-gauge theory; weak and electromagnetic interactions; Kolar events; new leptons; new flavours; spontaneous symmetry breaking; Cabibbo-angle.

1. Introduction

With the availability of high energy neutrino beams in the laboratory and the setting up of deep underground experiments exploiting cosmic ray neutrinos in recent years, a whole series of new weak interaction phenomena are beginning to show up. On the theoretical side a large number of attempts have been made to construct models of weak and electromagnetic interactions following the elegant proposal of Weinberg (1967) and Salam (1968) for constructing $SU_2 \otimes U_1$ gauge theories that are spontaneously (vacuum) broken to provide a 'soft' generation of mechanical masses of the various particles involved. The theories so constructed are renormalizable—a property that is widely considered as most desirable, at least for theories of weak and electromagnetic interactions.

In recent underground experiments in the Kolar gold mines, Krishnaswami *et al* (1975) reported having observed some very unusual events. Assuming that these events truly represent a new weak interaction phenomenon, and assuming that they are initiated by the conventional neutrinos (electronic or muonic) in cosmic rays, the question naturally arises whether they may be given a possible description in terms of a unified gauge theory of weak and electromagnetic interactions. The $SU_2 \otimes U_1$ gauge models proposed so far in the literature do not seem to provide a natural place for such events consistent with the above assumptions.

In the present work we propose a U_3 ($\equiv SU_3 \otimes U_1$) gauge theory of weak and electromagnetic interactions that might perhaps enable us to interpret the Kolar events in a unified framework along with all the new and the old weak interaction phenomena. To distinguish the use of the U_3 group made here from that made in other contexts, we shall refer to our proposal as a U_3(W) [$\equiv SU_3$(W) $\otimes$ U_1(W)] gauge theory. We shall assume that the Kolar events represent a certain spectacular decay mode of a new heavy *charged* lepton. Recent experiments (Benvenuti *et al* 1975 *a*; Faissner *et al* 1976) using high energy-muonic-neutrinos produced in the laboratory have so far only indicated that the supposed Kolar particle, if produced by muonic neutrinos, is most unlikely to be electrically neutral. It is not known whether it is produced (if at all) by the muonic neutrinos or by the electronic-neutrinos (in the latter case laboratory experiments will find it hard to produce it, whereas there is no such limitation on cosmic ray neutrinos, a sizable fraction of which is of the electronic type). Future experiments will have to decide the issue. In the meanwhile we assume the existence of an electron type heavy lepton E (with charge $Q = -1$) and of a muon type heavy lepton M (with charge $Q = -1$). We then assume that $\mathcal{L}_1 \equiv (\nu_e, e_L, E_L)$ and $\mathcal{L}_2 \equiv (\nu_\mu, \mu_L, M_L)$ (subscript L standing for the left-handed chiral projection) form two triplets belonging to the $\underline{3}$-representation of SU_3(W).

In constructing the U_3(W) gauge theory we have to comply with very severe theoretical as well as phenomenological constraints. The theoretical constraint is supplied by demanding that no axial-vector anomalies be present in the U_3(W) gauge theory. With this group the only simple way of getting rid of all such anomalies is to ensure their cancellation between the lepton sector and the quark sector. Phenomenologically we have the constraint that the quarks enter the theory in such a way that *no* strangeness-changing neutral currents appear. Furthermore the quarks must be introduced in a manner consistent with the conventional hadron spectroscopy. To take care of all these constraints in the enlarged framework of the SU_3(W) $\otimes$ U_1(W) gauge theory we first assume that apart from the conventional three flavours carried by the " light " quarks (u, d, s) we have three additional flavours—the first of which is the now well known *Charm* ($\mathcal{C}$) flavour and the remaining two are new flavours named *Taste* ($\mathcal{T}$) and *Grace* ($\mathcal{G}$) carried respectively, by three " heavy " quarks c, t and g. Each of the quarks, c, t and g carries the same electric charge $Q = +2/3$ and is singlet under the conventional strong interaction SU_3. Each of the six flavours of quarks is further assumed to appear in three *colours* (red, blue and white) to take care of the requirements of hadron spectroscopy. For each colour then we form two triplets $q_1 \equiv (d'_L, u'_L, t'_L)$ and $q_2 \equiv (s'_L, c'_L, g'_L)$ belonging to the *same* (to avoid strangeness-changing neutral currents) representation-$\underline{3}^*$, of the group SU_3(W). Here the prime indicates suitable mixtures of the quarks to be discussed later. The representation-$\underline{3}^*$ is chosen, since the lepton triplets have been placed in the representation-$\underline{3}$ and we aim for a cancellation of all anomalies between the quarks and the leptons. To ensure this fully we have to introduce, in fact, six triplets of (L-projections of) leptons [each belonging to the $\underline{3}$-representation of $SU_3(W)$] corresponding to the six ($\underline{3}^*$) triplets of quarks (including colour). Thus we have *six lepton-types* in the theory. We have further to make sure that the sum of the electric charges of all the quarks and all the leptons together vanishes to completely get rid of all possible anomalies in the complete theory. The right-handed *R*-components of the various fermions are assumed to be singlets of $SU_3(W)$. While most of the new leptons may be too heavy to be playing any phenomenological role at the present energies, all of them are needed for the above theoretical reason. In any case some of the new heavy leptons will be seen to be possibly playing a role already in the recently discovered new phenomena.

The next technical step is to implement the spontaneous breaking of the $U_3(W)$ gauge symmetry. For this we employ three suitable complex scalar triplets of Higgs fields. It is ensured in this way that eight vector gauge-bosons acquire mechanical masses leaving a ninth vector boson mass-less to be identified with the photon. The symmetry breaking also enables a generation of the fermion mechanical masses. In the case of the quarks the diagonalisation of the so generated mass matrix is used to identify the primed combinations of the quark fields that enter the weak interactions. This provides a natural way of incorporating the *Cabibbo-angle*.

It could very well turn out that the U_3 (W) gauge theory of unified weak and electromagnetic interactions proposed here has relevance to high energy physics (see section 6). In that case a rich crop of phenomena are implied by the theory, some of which are perhaps already showing up. A proper evaluation, however, will require a great deal of experimental work establishing the new quark flavours as well as definitive details of the new weak interaction processes.

The quark flavours and lepton-types employed in the present work are described in section 2, along with the specifications of the $U_3(W)$ representations chosen for the fermions. In section 3, we implement the $U_3(W)$ gauge symmetry breaking by the vacuum and obtain the mechanical masses of the gauge-bosons. In section 4 the fermion masses are introduced using the above symmetry breaking mechanism, thereby bringing in also mixing angles such as the Cabibbo-angle. In section 5 the interactions of the gauge-bosons with the leptons and quarks are given, setting up thereby the main elements for phenomenological applications. The final section 6, is devoted to a brief general discussion of some of the qualitative phenomenological implications of the theory at present. Those readers who are not particularly concerned with the technical theoretical details may easily skip sections 3 and 4.

2. The six quark-flavours and lepton-types

The $U_3(W) \equiv SU_3(W) \otimes U_1(W)$ gauge theory of weak and electro-magnetic interactions we propose here, makes use of six *flavours* of quarks. To the three flavours of the conventional SU_3 carried by the ("light") quarks (u, d, s) with electric charges[II] ($+2/3$, $-1/3$, $-1/3$), we add three more flavours, one of which is the well known *Charm* ($\mathcal{C}$), and the other two to be called *Taste* ($\mathcal{T}$) and

[II] If need be we may cast the theory in terms of integrally charged (Han-Nambu) quarks rather than the fractionally charged (Gell-Mann-Zweig) quarks. However the notation and presentation is much neater in terms of the latter and so we shall employ these in this paper.

Grace ($\mathcal{G}$), carried respectively by three ("heavy") quarks c, t and g. Each of these quarks, c, t, and g carries the same electric charge $Q = +2/3$ and is a singlet under the conventional SU_3. The electric charge is thus represented by

$$Q = I_3 + \frac{Y}{2} + \frac{2}{3}(\mathcal{C} + \mathcal{T} + \mathcal{G}), \tag{2.1}$$

where the only non-zero values of $\mathcal{C}$, $\mathcal{T}$ and $\mathcal{G}$ are: $\mathcal{C} = 1$ for c, $\mathcal{T} = 1$ for t and $\mathcal{G} = 1$ for g. The baryon number of each of the six quarks is $B = 1/3$. Each quark flavour is assumed to come in *three colours* (red, blue and white). It should be *noted* that whenever throughout this paper a quark term is written in the Lagrangian, it is really a sum over all the three colours, though we do not indicate this explicitly to simplify the writing.

Let us denote the eight hermitian generators of SU_3 (W) by G_a, $a = 1, 2, \ldots 8$, and the hermitian generator of $U_1(W)$ by G_0:

$$(G_a, G_b) = i f_{abc} G_c, \; (G_a, G_0) = 0. \tag{2.2}$$

We shall assume that $q_1 \equiv (d'_L, u_L, t'_L)$ and $q_2 \equiv (s'_L, c'_L, g'_L)$ belong to the $\underline{3}^*$-representation of $SU_3(W)$, where

$$u_L \equiv \tfrac{1}{2}(1 + \gamma_5)\, u, \qquad u_R = \tfrac{1}{2}(1 - \gamma_5)\, u, \text{ etc.} \tag{2.3}$$

and

$$\begin{pmatrix} d' \\ s' \end{pmatrix} \equiv S_2 \begin{pmatrix} d \\ s \end{pmatrix}, \quad \begin{pmatrix} g' \\ t' \\ c' \end{pmatrix} \equiv S_3 \begin{pmatrix} g \\ t \\ c \end{pmatrix}. \tag{2.4}$$

Here S_2 and S_3 are 2×2 and 3×3 real orthogonal matrices, respectively. Thus we allow possible mixing of the two negatively charged quarks among themselves, as well as of the three positively charged heavy quarks among themselves. The precise forms of these matrices will be introduced later (section 4) when the gauge-symmetry is broken spontaneously and the thereby generated quark mechanical mass matrices are diagonalized so that the quarks u, d, s, g, t, c become the mechanical-mass eigenstates. This will introduce the quark mixings angles (including the Cabibbo-angle in S_2).

In the $\underline{3}^*$-representations the generators G_a are given by $G_a = \frac{1}{2}(-\tilde{\lambda}_a)$, where λ_a, $a = 1, \ldots, 8$, are the conventional SU_3 hermitian matrices of Gell-Mann (1962) ($\frac{1}{2}\lambda_a$ being the hermitian generators of the $\underline{3}$-representation). We shall also use the ninth Gell-Mann matrix $\lambda_0 \equiv \sqrt{2/3}\,\mathbf{1}$ so that

$$[\lambda_a, \lambda_b] = 2 i f_{abc} \lambda_c, \; (a, b, c = 1, 2, \ldots, 8), \tag{2.5}$$

$$\mathrm{Tr}\,(\lambda_i \lambda_j) = 2\delta_{ij}, \; (i, j = 0, 1, 2, \ldots, 8). \tag{2.6}$$

In terms of the $U_3(W)$ generators the charge operator is given by

$$Q = \left(G_3 + \frac{1}{\sqrt{3}} G_8\right) + G_0, \tag{2.7}$$

so that $G_0 = 1/3$ for each of the above quark triplets q_1 and q_2.

The right-handed, R-components, of all the quarks are taken as singlets of $SU_3(W)$ with $G_0 = Q = 2/3$ for u_R, c'_R, t'_R, g'_R and $G_0 = Q = -1/3$ for d'_R and s'_R.

In introducing leptons into the $U_3(W)$ gauge theory we shall have to meet the requirement that all axial-vector anomalies in the quark-sector are to be cancelled by those in the lepton-sector.[||] Thus corresponding to the six triplets of quarks (including colour), each belonging to the $\underline{3}^*$-representation of $SU_3(W)$, we introduce six triplets of leptons, each belonging to the $\underline{3}$-representation of $SU_3(W)$, and further make sure that the total electric charge of all the quarks and all the leptons together adds upto zero. All anomalies will thus disappear. We are in this way led to six *lepton-types.*

It should be noted that the above requirement can as well be met by taking six quark triplets to belong to the $\underline{3}$-representation and six lepton triplets to belong to the $\underline{3}^*$-representation (of course, using a different set of quarks and leptons). However, we prefer the above choice on phenomenological considerations (see the discussion point (7) of Kolar events in section 6) requiring the leptons E and M to be charged ($Q = -1$) and not neutral.

The six lepton triplets will be denoted by $\mathcal{L}_1 \equiv (\nu_e, e_L, E_L)$, $\mathcal{L}_2 \equiv (\nu_\mu, \mu_L, M_L)$, $\mathcal{L}_3 \equiv (n_{1L}, l_{1L}, L_{1L})$, $\mathcal{L}_4 \equiv (n_{2L}, l_{2L}, L_{2L})$, $\mathcal{L}_5 \equiv (p_{1L}, v_{1L}, V_{1L})$ and $\mathcal{L}_6 \equiv (p_{2L}, v_{2L}, V_{2L})$. The subscript L again stands for the left-handed projection, and we shall assume that ν_e and ν_μ are left-handed (though there is no difficulty in relaxing this if need be). The new leptons E and M have charge $Q = -1$, the former being of the electron-type and the latter of the muon-type. Among the four additional lepton-types, the leptons l_1, l_2, L_1 and L_2 have $Q = -1$; n_1, n_2, v_1, v_2, V_1, V_2 have $Q = 0$; and p_1 and p_2 have $Q = +1$. Correspondingly, we have $G_0 = -2/3$ for $\mathcal{L}_1$, $\mathcal{L}_2$, $\mathcal{L}_3$ and $\mathcal{L}_4$ and $G_0 = 1/3$ for $\mathcal{L}_5$ and $\mathcal{L}_6$. The right-handed projections of the leptons are all taken to be singlets of $SU_3(W)$ and thus have $G_0 = Q$.

We should emphasize that the introduction of the large number of new leptons here has been dictated by the requirement of constructing an anomaly-free renormalizable gauge theory. The leptons of the triplets $\mathcal{L}_4$, $\mathcal{L}_5$, $\mathcal{L}_6$ can all be made very heavy and need not play important phenomenological roles at the present experimental energies. The other three triplets will be seen to be possibly playing crucial roles already (see section 6).

One more point may be noted here. The rather large number of leptons had to be introduced since the two quark triplets q_1 and q_2 of any colour have both been taken to belong to the same triplet $\underline{3}^*$-representation and so cannot cancel anomalies among themselves. Their assignment to the same representation is essential for ensuring that no strangeness-changing neutral currents appear in the theory. Thus rather stringent theoretical as well as phenomenological requirements are at the back of the above choice of the lepton-types in parallel with the quark flavours and colour, making the total number of quarks (including colour) equal to the total number of leptons in the theory. There is present thus a deep seated *quark-lepton analogy* in the theory.

[||] For a general discussion on ways of constructing anomaly free theories, see Georgi and Glashow (1972).

3. Masses of the vector gauge-bosons

To illustrate the standard method[||] of setting up the gauge theory, the vacuum (spontaneous) breaking of the gauge symmetry and the generation thereby of the mechanical masses of the vector gauge bosons in the present model, we start with the example of just one left-handed fermion triplet ($\underline{3}$ of SU_3 (W) with $G_0 = -2/3$: $\psi_L \equiv \frac{1}{2}(1+\gamma_5)(\psi_v, \psi_e, \psi_E)$. This will enable an uncluttered presentation, while the other fermions can be added on later in an obvious manner.

We shall introduce three triplets of complex scalar (Higgs) fields $\phi^{(i)}$; $\phi^{(i)} \equiv (\phi_1^{(i)}, \phi_2^{(i)}, \phi_3^{(i)})$, i = 1, 2, 3, each belonging to the $\underline{3}$-representation of SU_3(W). The charges of the $\phi^{(1)}$ triplet are taken as (0, — 1, — 1) corresponding to $G_0 = -2/3$, while the charges of the triplets $\phi^{(2)}$ and $\phi^{(3)}$ are taken as (+ 1, 0, 0) so that $G_0 = +1/3$ for both of these triplets. The U_3(W) gauge symmetry breaking will be implemented, consistent with charge conservation, by arranging the (degenerate) vacuum state to be such that $\langle\phi_1^{(1)}\rangle_0 = \langle\phi_2^{(2)}\rangle_0 = \langle\phi_3^{(3)}\rangle_0 \neq 0$, while all the remaining components have zero vacuum expectation values.

Before we break the gauge symmetry we have to set up the Lagrangian density invariant under the following $SU_3(W) \otimes U_1(W)$ gauge transformations (in all expressions throughout we shall understand the triplet symbol to mean a column matrix; also repeated indices a, b,... are summed over 1 to 8):

$$\psi_L(x) \to [S(x)\, S_0(x) \psi_L(x)], \tag{3.1}$$

$$\phi^{(i)}(x) \to [S(x) S_0^{(i)}(x) \phi^{(i)}(x)], \tag{3.2}$$

where

$$S(x) = \exp\left[i \frac{\lambda_a}{2} w_a(x)\right], \tag{3.3}$$

$$S_0(x) = S_0^{(1)}(x) = \exp\left[-i \frac{2}{3} w_0(x)\right], \tag{3.4}$$

$$S_0^{(2)}(x) = S_0^{(3)}(x) = \exp\left[i \frac{1}{3} w_0(x)\right], \tag{3.6}$$

and $w_a(x)$, $a = 1, 2, \ldots 8$, and $w_0(x)$ are the nine arbitrary real gauge functions.

It is extremely convenient for computations to introduce corresponding to the reducible nine component complex column matrix $\phi^{(1)} \oplus \phi^{(2)} \oplus \phi^{(3)}$ an associated (3×3) complex matrix Φ whose columns are precisely $\phi^{(1)}, \phi^{(2)}, \phi^{(3)}$; *i.e.*, the elements of the matrix are

$$(\Phi)_{ij} \equiv \phi_i^{(j)}; \quad i, j = 1, 2, 3. \tag{3.6}$$

The gauge transformations (3.2) then can be written all together using a product of (3×3)-matrices:

$$\Phi \to S\Phi T_0, \quad T_0 \equiv \text{diag}\left(e^{-\frac{2i}{3} w_0}, e^{\frac{i}{3} w_0}, e^{\frac{i}{3} w_0}\right). \tag{3.7}$$

[||]A large number of reviews exist in the literature. The author has greatly profited from the reviews of Dass (1973), Bég and Sirlin (1974), Zumino (1972) and Lee (1972). References to the original literature may be traced from these reviews.

To arrive at the gauge invariant Lagrangian we have now to intoduce nine real vector gauge-boson fields $U_{\mu a}(x)$ and $B_\mu(x)$ with the gauge transformation properties:

$$U_\mu \equiv (U_{\mu a}\lambda_a) \to SU_\mu S^{-1} + \frac{2}{if} S(\partial_\mu S^{-1}), \tag{3.8}$$

$$B_\mu \to B_\mu - \frac{1}{f'} \partial_\mu w_0(x), \tag{3.9}$$

where f and f' are real coupling parameters. Defining as usual

$$\mathcal{G}_{\mu\nu a} \equiv \partial_\mu U_{\nu a} - \partial_\nu U_{\mu a} - f f_{abc} U_{\mu b} U_{\nu c} \tag{3.10}$$

$$F_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu, \tag{3.11}$$

and introducing the gauge-covariant derivatives:

$$(D_\mu \psi_L) \equiv \left\{ \partial_\mu + \frac{i}{2} f U_\mu + i f' \left(-\frac{2}{3} \right) B_\mu \right\} \psi_L, \tag{3.12}$$

$$(D_\mu \Phi) \equiv \partial_\mu \Phi + \frac{if}{2}(U_\mu \Phi + B_\mu \Phi t_0), \tag{3.13}$$

where t_0 is a (3×3)-diagonal matrix

$$t_0 \equiv \mathrm{diag}\left(-\frac{2}{3}, \frac{1}{3}, \frac{1}{3} \right) \times \frac{2f'}{f} = -\frac{\sqrt{3}}{2} \sigma \left(\lambda_3 + \frac{1}{\sqrt{3}} \lambda_8 \right); \quad \sigma \equiv \frac{2f'}{\sqrt{3} f}, \tag{3.14}$$

we may now write a $U_3(W)$ gauge invariant Lagrangian density

$$\begin{aligned} L_1 = & -\tfrac{1}{4} \mathcal{G}_{\mu\nu a} \mathcal{G}_{\mu\nu a} - \tfrac{1}{4} F_{\mu\nu} F_{\mu\nu} - \bar{\psi}_L \gamma_\mu (D_\mu \psi_L) \\ & - \mathrm{Tr}\,\{(D_\mu \Phi)^\dagger (D_\mu \Phi)\} - V(\phi). \end{aligned} \tag{3.15}$$

Here $V(\phi)$ stands for a suitable $U_3(W)$ invariant potential function involving invariants built out of the fields $\phi^{(i)}$. Besides the quadratic invariants like $(\phi^{(i)\dagger} \phi^{(i)})$ and the quartic invariants like $(\phi^{(i)\dagger} \phi^{(i)}) (\phi^{(j)\dagger} \phi^{(j)})$, it will also involve the cubic invariant $\epsilon_{ijk} \phi_i^{(1)} \phi_j^{(2)} \phi_k^{(3)}$—since the latter is not only $SU_3(W)$ invariant but $U_1(W)$ invariant as well an account of the sum of the G_0 values of the three scalar triplets being zero.[||]

The parameters entering $V(\phi)$ can be chosen such that we have a vacuum breaking of the gauge symmetry with $\langle \phi_1^{(1)} \rangle_0 = \langle \phi_2^{(2)} \rangle_0 = \langle \phi_3^{(3)} \rangle_0 \neq 0$ (other vacuum expectation values $= 0$). This breaks the $U_3(W)$ gauge symmetry in such a way that Q is still conserved. This means that in the manifold of the 18 real fields comprising the three complex scalar triplets, 8 real scalar fields play the role of massless Goldstone fields that combine with the vector gauge boson fields in such a way as to disappear from the theory, while 8 orthonormal combinations of the gauge bosons become massive leaving only one massless (orthogonal) gauge boson that can be identified with the photon. There will thus remain 10 real scalar fields, all becoming massive, and their masses can be made as large as one pleases by suitably arranging the parameters in $V(\phi)$. A particularly simple choice of $V(\phi)$ is given for illustration in appendix A.

[||]Note thus the subtle contrast with the considerations of Bardakci and Halpern (1972).

These features are best brought out if we make the substitution

$$\Phi = \exp\left[i \frac{\lambda_a}{2} \theta_a (x) \right] \Psi \equiv S(\theta) \Psi, \tag{3.16}$$

where $\theta_a (x)$ are 8 real scalar fields and we impose on Ψ correspondingly 8 conditions

$$\mathrm{Tr}\, [\lambda_a (\Psi - \Psi^\dagger)] = 0, \tag{3.17}$$

so that Ψ involves only 10 independent real fields $\rho_a (x)$, $\rho_0 (x)$ and $\sigma_0 (x)$:

$$\Psi \equiv \tfrac{1}{2} \lambda_a \rho_a + \tfrac{1}{2} \lambda_0 (\rho_0 + i\sigma_0) \equiv \tfrac{1}{2} \rho + \tfrac{1}{2} \lambda_0 (\rho_0 + i\sigma_0). \tag{3.18}$$

The columns of Ψ will be denoted by $\psi^{(i)}$, $i = 1, 2, 3$, and correspond to the $\phi^{(i)}$. Correspondingly, we also make the substitutions:

$$W_\mu (x) \equiv S(\theta)\, U_\mu\, S^{-1} (\theta) + \frac{2}{if} S(\theta) (\partial_\mu S^{-1} (\theta),) \tag{3.19}$$

$$\text{Column}\, (\nu_e, e_L, E_L) \equiv \mathscr{L}_1 (x) \equiv S(\theta)\, \psi_L (x). \tag{3.20}$$

We further define

$$G_{\mu\nu a} = \partial_\mu W_{\nu a} - \partial_\nu W_{\mu a} - _{er} f f_{abc} W_{\mu b} W_{\nu c}. \tag{3.21}$$

Then we have

$$L_1 = -\tfrac{1}{4} G_{\mu\nu a} G_{\mu\nu a} - \tfrac{1}{4} F_{\mu\nu} F_{\mu\nu} - \bar{\mathscr{L}}_1 \gamma_\mu (D_\mu \mathscr{L}_1) - \mathrm{Tr}\, \{(D_\mu \Psi)^\dagger (D_\mu \Psi)\} - V(\psi), \tag{3.22}$$

where

$$(D_\mu \bar{\mathscr{L}}_1) \equiv \left\{ \partial_\mu + \frac{i}{2} f\, W_\mu + if \left(-\frac{2}{3}\right) B_\mu \right\} \bar{\mathscr{L}}_1, \tag{3.23}$$

$$(D_\mu \Psi) \equiv \partial_\mu \Psi + \frac{if}{2} (W_\mu \Psi + B_\mu\, \Psi\, t_0), \tag{3.24}$$

and $V(\psi)$ stands for the potential function in terms of the triplets $\psi^{(i)}$.

Now from among the infinitely degenerate vacua we choose the physical world vacuum to be such that

$$\langle \Psi \rangle_0 = \eta \mathbf{1}, \quad \eta \text{ real}. \tag{3.25}$$

The theory may be recast in terms of displaced fields, *i.e.*, in terms of Ψ' where, $\Psi \equiv \Psi' + \eta \mathbf{1}$ and $\langle \Psi' \rangle_0 = 0$. To keep the notation simple we drop the prime, *i.e.*, we make the displacement

$$\Psi \rightarrow \Psi + \eta \mathbf{1}. \tag{3.26}$$

The resulting potential function displays masses generated for the ten real fields ρ_a, ρ_0 and σ_0, the 8 massless fields θ_a having disappeared (see appendix A). Also as a result

$$\mathrm{Tr}\, \{(D_\mu \Psi)^\dagger (D_\mu \Psi)\} \rightarrow \mathrm{Tr}\, \{(D_\mu \Psi)^\dagger (D_\mu \Psi)\} + + \frac{f^2 \eta}{4} \mathrm{Tr}\, \{W'^2_\mu (\rho + \lambda_0 \rho_0)\} + \frac{f^2 \eta^2}{4} \mathrm{Tr} (_\mu W'^2_\mu), \tag{3.27}$$

where

$$W'_\mu \equiv W_\mu + B_\mu t_0. \tag{3.28}$$

We thus see the complete diappearance of the 8 real (Goldstone) fields θ_a from the theory while the vector gauge-bosons attain through the last term of eq. (3.27) a mechanical mass term:

$$L_{\text{mass}}(W) = -\frac{f^2\eta}{4}\,\text{Tr}\,(W'^2_\mu)$$
$$= -f^2\eta^2\left[W_\mu^+W_\mu^- + R_\mu^+R_\mu^- + H_\mu H^*_\mu + \tfrac{1}{2}(1+\sigma^2)\,Z^2_{1\mu} + \tfrac{1}{2}\,Z^2_{2\mu}\right], \tag{3.29}$$

where we have defined (the orthonormal mass diagonalizing combinations):

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}(W_{\mu 1} \mp iW_{\mu 2}),$$
$$R_\mu^\pm \equiv \frac{1}{\sqrt{2}}(W_{\mu 4} \mp iW_{\mu 5}),$$
$$H_\mu \equiv \frac{1}{\sqrt{2}}(W_{\mu 6} - iW_{\mu 7}),\; H_\mu^* \equiv \frac{1}{\sqrt{2}}(W_{\mu 6} + iW_{\mu 7}),$$
$$Z_{1\mu} = \frac{\sqrt{3}}{2\sqrt{1+\sigma^2}}\left[W_{\mu 3} + \frac{1}{\sqrt{3}}\,W_{\mu 8} - \frac{2\sigma}{\sqrt{3}}B_\mu\right],$$
$$Z_{2\mu} \equiv -\tfrac{1}{2}\,W_{\mu 3} + \frac{\sqrt{3}}{2}\,W_{\mu 8},$$
$$A_\mu \equiv \frac{\sqrt{3}\sigma}{2\sqrt{1+\sigma^2}}\left[W_{\mu 3} + \frac{1}{\sqrt{3}}\,W_{\mu 8} + \frac{2}{\sqrt{3}\sigma}\,B_\mu\right]. \tag{3.30}$$

The vector field A_μ remains massless, whereas the other eight vector fields attain non-zero masses. We shall see in section 5 that A_μ couples to the minimal electric-charge current and is to be identified with the photon field. We thus have finally for the masses of the vector gauge bosons in terms of η, f, and $\sigma \equiv (2f'/\sqrt{3}f)$:

$$m^2(W^\pm) = m^2(R^\pm) = m^2(H, \bar{H}) = m^2(Z_2) = f^2\eta^2,$$
$$m^2(Z_1) = f^2\eta^2(1+\sigma^2). \tag{3.31}$$

The weak and electromagnetic interactions of all the leptons and the quarks in terms of these intermediate boson fields will be written in section 5.

4. Fermion masses, Cabibbo-angle

To generate mechanical masses for the fermions we now introduce $U_3(W)$ invariant non-derivative Yukawa (Y) coupling of the fermions with the scalar triplets $\phi^{(i)}$ into the Lagrangian density. On implementing the spontaneous symmetry breaking described in section 3; *i.e.*, by making the substitutions Ψ for Φ, etc., and making the displacement $\psi_j^{(i)} \to \psi_j^{(i)} + \eta\delta_{ij}$, we will generate mass-like terms for the fermions. With the imposition of suitable relations among the various coupling constants we shall then obtain the desired mass terms.

Consider first the quark masses. Here the coupling constants are chosen to ensure first that the L- and R- components combine correctly in the resulting mass terms. As a requirement we shall also impose the further condition that in the resulting mass-matrix the " light " quarks do not mix with the " heavy " quarks. Thus the two negatively charged quarks d and s may mix among themselves on the one hand and the positively charged heavy quarks c, t and g among themselves on the other. Let us simply write down the resulting general coupling term $L_Y(q)$:

$$\begin{aligned}
-L_Y(q) = {} & a_1 \bar{u}_R (q_1 \cdot \psi^{(2)}) + a_2 \bar{d}'_R (q_1 \cdot \psi^{(1)}) + a_3 \bar{s}'_R (q_2 \cdot \psi^{(1)}) \\
& + A_1 \bar{g}'_R \; (q_2 \cdot \psi^{(3)}) + A_2 \bar{t}'_R (q_1 \cdot \psi^{(3)}) + A_3 \bar{c}'_R (q_2 \cdot \psi^{(2)}) \\
& + b\, [\bar{s}'_R (q_1 \cdot \psi^{(1)}) + \bar{d}'_R (q_2 \cdot \psi^{(1)})] \\
& + B_1 [\bar{t}'_R (q_2 \cdot \psi^{(3)}) + \bar{g}'_R (q_1 \cdot \psi^{(3)})] \\
& + B_2 [\bar{g}'_R (q_2 \cdot \psi^{(2)}) + \bar{c}'_R (q_2 \cdot \psi^{(3)})] \\
& + B_3 [\bar{t}'_R (q_2 \cdot \psi^{(2)}) + \bar{c}'_R (q_1 \cdot \psi^{(3)})] + \text{h.c.},
\end{aligned} \tag{4.1}$$

where a_1, a_2, a_3, A_1, A_2, A_3, b, B_1, B_2 and B_3 are all real coupling constants and where, *e.g.*, $(q_1 \cdot \psi^{(1)})$ stands for the $SU_3(W)$ invariant which in matrix notation has the form $(\tilde{q}_1 \psi^{(1)})$. Now on making the replacement $\psi_j^{(i)} \to \psi_j^{(i)} + \eta \delta_{ij}$, we obtain

$$-L_Y(q) \to -L_Y(q) - L_{\text{mass}}(q), \tag{4.2}$$

$$\begin{aligned}
-L_{\text{mass}}(q) = \eta \Bigg[& a_1 (\bar{u}u) + (\bar{d}', \bar{s}') \mathcal{M}_2 \begin{pmatrix} d' \\ s' \end{pmatrix} + \\
& + (\bar{g}', \bar{t}', \bar{c}') \mathcal{M}_3 \begin{pmatrix} g' \\ t' \\ c' \end{pmatrix} \Bigg]
\end{aligned} \tag{4.3}$$

$$\mathcal{M}_2 \equiv \begin{pmatrix} a_2 & b \\ b & a_3 \end{pmatrix}, \qquad \mathcal{M}_3 \equiv \begin{pmatrix} A_1 & B_1 & B_2 \\ B_1 & A_2 & B_3 \\ B_2 & B_3 & A_3 \end{pmatrix} . \tag{4.4}$$

We now diagonalize the real symmetric mass-matrices $\eta\mathcal{M}_2$ and $\eta\mathcal{M}_3$. To diagonalize $\mathcal{M}_2$ we use a 2×2 real orthogonal matrix S_2 and identify the eigenstates of definite mass with the d and the s quarks. The general 2×2 real orthogonal matrix is specified in terms of one real angle, which we shall identify with the *Cabibbo-angle* θ. We can thus take

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = S_2 \begin{pmatrix} d \\ s \end{pmatrix}, \qquad S_2 \equiv \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} . \tag{4.5}$$

Writing for the diagonal mass-matrix:

$$\eta S_2^{-1} \mathcal{M}_2 S_2 = \text{diag}\,(m_d, m_s), \tag{4.6}$$

and denoting the mechanical masses of the quarks u, d, s by m_u, m_d, m_s, we have the result:

$$\begin{aligned}
\eta a_1 = m_u, \quad \eta a_2 &= m_d \cos^2\theta + m_s \sin^2\theta, \\
\eta a_3 &= m_d \sin^2\theta + m_s \cos^2\theta, \\
\eta b &= (m_s - m_d) \cos\theta \sin\theta.
\end{aligned} \tag{4.7}$$

Note that $\bar{d}' d' + \bar{s}' s' = \bar{d}d + \bar{s}s$, a combination that will appear in the neutral currents, avoiding any strangeness-changing terms in them (Glashow *et al* 1970).

To diagonalize $\mathcal{M}_3$ we use a 3×3 real orthogonal matrix S_3 and identify the eigenstates by the g, t, c quarks. Now S_3 requires three angles for its general specification. In order to avoid the introduction of too many parameters into the theory at this stage, we shall make here almost the simplest phenomenological *approximation* for illustration (the more general case may be considered in future if need be):

$$\begin{pmatrix} g' \\ t' \\ c' \end{pmatrix} = S_3 \begin{pmatrix} g \\ t \\ c \end{pmatrix}, \quad S_3 \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{pmatrix}, \tag{4.8}$$

so that

$$\eta S_3^{-1} \mathcal{M}_3 S_3 \simeq \operatorname{diag}(m_g, m_t, m_c), \tag{4.9}$$

and

$$\begin{aligned} &\eta A_1 \simeq m_g; \qquad B_1 \simeq B_2 \simeq 0; \\ &\eta A_2 \simeq m_t \cos^2\phi + m_c \sin^2\phi, \quad \eta A_3 \simeq m_t \sin^2\phi + m_c \cos^2\phi; \\ &\eta B_3 \simeq (m_t - m_c) \cos\phi \sin\phi, \end{aligned} \tag{4.10}$$

and

$$g' \simeq g, t' \simeq t\cos\phi - c\sin\phi, \quad c' \simeq (c\cos\phi + t\sin\phi). \tag{4.11}$$

Note that $\bar{g}'g' + \bar{t}'t' + \bar{c}'c' = \bar{g}g + \bar{t}t + \bar{c}c$ quite exactly. Thus we have identified the primed quarks of section 2 appearing in the weak interaction theory.

Coming now to the lepton masses, we proceed in the same manner as above except that now we impose the condition that there be *no* mixing of the leptons in the mass term at all, *i.e.*, the leptons as introduced are already the mass-eigenstates. If we assume that ν_e, ν_μ are the only purely left-handed leptons (which if need be, can be relaxed) we obtain mass terms for each of the remaining leptons, independent of one another. To simplify writing we illustrate this just for one type of leptons: n_1, l_1, L_1. We can write the $U_3(W)$ invariant Yukawa term in the Lagrangian density $L_Y(\mathcal{L}_3)$:

$$\begin{aligned} -L_Y(\mathcal{L}_3) = k_3 \bar{n}_{1R} (\mathcal{L}_3 \cdot \psi^{(1)\dagger}) + k'_3 \bar{l}_{1R} (\mathcal{L}_3 \cdot \psi^{(2)\dagger}) + \\ + k''_3 \bar{L}_{1R} (\mathcal{L}_3 \cdot \psi^{(3)\dagger}) + \text{h.c.}, \end{aligned} \tag{4.12}$$

where k_3, k_3', k_3'' are all real coupling constants. Making the replacement $\psi_j^{(i)} \to \psi_j^{(i)} + \eta\delta_{ij}$; we get

$$-L_Y(\mathcal{L}_3) \to -L_Y(\mathcal{L}_3) - L_{\text{mass}}(\mathcal{L}_3), \tag{4.13}$$

$$-L_{\text{mass}}(\mathcal{L}_3) \equiv m(n_1)(\bar{n}_1 n_1) + m(l_1)(\bar{l}_1 l_1) + m(L_1)(\bar{L}_1 L_1), \tag{4.14}$$

where the masses are thus identified as

$$m(n_1) = k_3\eta, \; m(l_1) = k'_3\eta, \; m(L_1) = k''_3\eta. \tag{4.15}$$

Thus all the fermions can be given mechanical mass terms as a result of the spontaneous symmetry breaking of the $U_3(W)$ gauge symmetry.

5. Interaction of the vector-bosons with the quarks and the leptons

Having indicated the essential technical steps in the previous sections we can now write down the weak and electromagnetic interactions including all the fermions. It is to be remembered that each quark term really stands for a sum over the three colours.

We may write the interactions of the fields $W_{\mu a}$ and B_{μ} introduced in section 3 with the quarks and the leptons as $L_{\text{int}}(q, \mathcal{L})$:

$$-L_{\text{int}}(q, \mathcal{L}) = f\, W_{\mu a}\, J_{\mu a} + f'\, B_{\mu} j_{\mu}, \tag{5.1}$$

where $J_{\mu a}$ are the G_a-currents of the fermions and j_{μ} the G_0-current. Thus

$$-iJ_{\mu a} \equiv \sum_{i=1}^{6} \bar{\mathcal{L}}_i \gamma_{\mu} \left(\frac{\lambda_a}{2}\right) \mathcal{L}_i + \sum_{i=1}^{2} \bar{q}_i \gamma_{\mu} \left(-\frac{\tilde{\lambda}_a}{2}\right) q_i, \tag{5.2}$$

$$\begin{aligned}-ij_{\mu} \equiv\; & \sum_{i=1}^{4} \bar{\mathcal{L}}_i \gamma_{\mu} (-\tfrac{2}{3}) \mathcal{L}_i + \sum_{i=5}^{6} \bar{\mathcal{L}}_i \gamma_{\mu} (\tfrac{1}{3}) \mathcal{L}_i + \sum_{i=1}^{2} \bar{q}_i \gamma_{\mu} (\tfrac{1}{3}) q_i \\ & + [\bar{p}_{1R} \gamma_{\mu}\, p_{1R} + (1 \to 2)] - [\bar{e}_R \gamma_{\mu} e_R + (e \to \mu, E, M, l_1, l_2, L_1, L_2)] \\ & + \tfrac{2}{3}\, [\bar{u}_R \gamma_{\mu} u_R + (u \to c, t, g)] - \tfrac{1}{3}\, [\bar{d}_R \gamma_{\mu} d_R + (d \to s)].\end{aligned} \tag{5.3}$$

In connection with the neutral currents it is to be remembered (section 4) that $\bar{d}'d' + \bar{s}'s' = \bar{d}d + \bar{s}s$ and $\bar{g}'g' + \bar{t}'t' + \bar{c}'c' = \bar{g}g + \bar{t}t + \bar{c}c$.

Now for discussions of the phenomenological implications of the theory it is important to remember that the neutral bosons which are the eigenstates of the mechanical mass-matrix generated by spontaneous gauge symmetry breaking are the A_{μ} (photon), $Z_{1\mu}$ and $Z_{2\mu}$ introduced in eq. (3.30). We should thus write the interactions in terms of these fields as well as the fields $W_{\mu}^{\pm}$, $R_{\mu}^{\pm}$, H_{μ}, H_{μ}^{*} introduced in section 3. Their masses are given in eq. (3.31). The interaction of eq. (5.1) may thus be recast into the form:

$$\begin{aligned}-L_{\text{int}}(q, \mathcal{L}) = & \frac{f}{\sqrt{2}} \{W_{\mu}^{+} J_{\mu}(W^{+}) + R_{\mu}^{+} J_{\mu}(R^{+}) + H_{\mu} J_{\mu}(H) + \text{h.c.}\} + \\ & + eA_{\mu} J_{\mu}(em) + f_1\, Z_{1\mu} J_{\mu}(Z_1) + f_2\, Z_{2\mu} J_{\mu}(Z_2),\end{aligned} \tag{5.4}$$

where, defining $\tan\chi \equiv \sigma \equiv (2f'/\sqrt{3}f)$, we have

$$e = \frac{\sqrt{3}\sigma f}{2\sqrt{1+\sigma^2}} = \frac{\sqrt{3}}{2} f \sin\chi;\quad f_1 = \frac{\sqrt{3}}{2} f \cos\chi;\; f_2 = \frac{1}{2} f. \tag{5.5}$$

Here $J_{\mu}(em)$ stands for the electromagnetic current of $Q = \left(G_3 + \frac{1}{\sqrt{3}} G_8\right) + G_0$, and $J_{\mu}(Z_1)$ ahd $J_{\mu}(Z_2)$ stand for the hermitian neutral currents of $\left[\left(G_3 + \frac{1}{\sqrt{3}} G_8\right) - \sigma^2 G_0\right]$ and of $[-G_3 + \sqrt{3}\, G_8]$, respectively. These neutral currents do *not* change strangeness.[||] Defining

$$\Gamma_{\lambda} \equiv \tfrac{1}{2}\, \gamma_{\lambda} (1 + \gamma_5), \tag{5.6}$$

[||]In achieving this we have essentially used the idea first put forward by Glashow, Illiopoulos and Maiani (1970).

and remembering eqs (4.5) and (4.11), we write below the expressions for the currents appearing in eq. (5.4):

$$-iJ_\lambda(W^+) = \bar{\nu}_e \Gamma_\lambda e + \bar{\nu}_\mu \Gamma_\lambda \mu + \bar{n}_1 \Gamma_\lambda l_1 + \bar{n}_2 \Gamma_\lambda l_2 + \bar{p}_1 \Gamma_\lambda v_1 + \bar{p}_2 \Gamma_\lambda v_2$$
$$-\bar{u}\,\Gamma_\lambda (d\cos\theta + s\sin\theta) - (\bar{c}\cos\phi + \bar{t}\sin\phi)\,\Gamma_\lambda(-d\sin\theta + s\cos\theta), \qquad (5.7\,a)$$

$$-iJ_\lambda(R^+) = \bar{\nu}_e \Gamma_\lambda E + \bar{\nu}_\mu \Gamma_\lambda M + \bar{n}_1 \Gamma_\lambda L_1 + \bar{n}_2 \Gamma_\lambda L_2 + \bar{p}_1 \Gamma_\lambda V_1 + \bar{p}_2 \Gamma_\lambda V_2$$
$$-\bar{t}'\,\Gamma_\lambda d' - \bar{g}'\,\Gamma_\lambda s', \qquad (5.7\,b)$$

$$-iJ_\lambda(H) = \bar{e}\Gamma_\lambda E + \bar{\mu}\,\Gamma_\lambda M + \bar{l}_1 \Gamma_\lambda L_1 + \bar{l}_2 \Gamma_\lambda L_2$$
$$+ \bar{v}_1 \Gamma_\lambda V_1 + \bar{v}_2 \Gamma_\lambda V_2 - \bar{t}'\,\Gamma_\lambda u - \bar{g}'\,\Gamma_\lambda c', \qquad (5.7\,c)$$

$$-iJ_\lambda(em) = [\bar{p}_1\gamma_\lambda p_1 + (1\to 2)] - [\bar{e}\gamma_\lambda e + (e\to \mu, E, M, l_1, l_2, L_1, L_2)]$$
$$+ \tfrac{2}{3}[\bar{u}\gamma_\lambda u + (u\to c, t, g)] - \tfrac{1}{3}[\bar{d}\gamma_\lambda d + (d\to s)], \qquad (5.7\,d)$$

$$-iJ_\lambda(Z_1) = \tfrac{2}{3}[1+\sigma^2][\bar{\nu}_e\Gamma_\lambda \nu_e + (\nu_e \to \nu_\mu, n_1, n_2)]$$
$$-\tfrac{1}{6}[\bar{e}\{(1-5\sigma^2)\gamma_\lambda + (1+\sigma^2)\gamma_\lambda\gamma_5\} e + (e\to \mu, E, M, l_1, l_2, L_1, L_2)]$$
$$+\tfrac{1}{3}[\bar{p}_1\{(1-2\sigma^2)\gamma_\lambda + (1+\sigma^2)\gamma_\lambda\gamma_5\} p_1 + (p_1\to p_2)]$$
$$-\tfrac{1}{3}[\bar{v}_1\Gamma_\lambda v_1 + (v_1\to v_2, V_1, V_2)](1+\sigma^2)$$
$$-\tfrac{1}{3}[\bar{d}\{\gamma_\lambda + (1+\sigma^2)\gamma_\lambda\gamma_5\} d + (d\to s)]$$
$$+\tfrac{1}{6}[\bar{u}\{1-3\sigma^2)\gamma_\lambda + (1+\sigma^2)\gamma_\lambda\gamma_5\} u + (u\to c, t, g)], \qquad (5.7\,e)$$

$$-iJ_\lambda(Z_2) = [\bar{e}\Gamma_\lambda e + (e\to \mu, l_1, l_2, v_1, v_2)]$$
$$-[\bar{E}\Gamma_\lambda E + (E\to M, L_1, L_2, V_1, V_2)]$$
$$+(-\bar{u}\Gamma_\lambda u + \bar{t}'\,\Gamma_\lambda t' - \bar{c}'\,\Gamma_\lambda c' + \bar{g}'\,\Gamma_\lambda g'). \qquad (5.7\,f)$$

Note that $J_\lambda(H)$ is also an electrically neutral but non-hermitian current.

In terms of the vector boson masses of eq. (3.31), we may now identify the standard four-fermion Fermi coupling constant G as

$$G = \frac{f^2}{4\sqrt{2}\,m^2(W)}, \qquad G = 1.026\times 10^{-5} m_p^{-2}, \qquad (5.8)$$

so that

$$\frac{1}{\eta^2} = 4\sqrt{2}\,G. \qquad (5.9)$$

We further find using eqs (5.5);

$$m^2(W) = m^2(R) = m^2(H) = m^2(Z_2) \gtrsim (43\text{ GeV})^2;$$
$$m^2(Z_1) \gtrsim (86\text{ GeV})^2. \qquad (5.10)$$

6. General phenomenological discussion

In this section we make some general phenomenological remarks on the implications of the theory proposed in the present work.

(1) It is clear that the $W^{\pm}$ -mediated weak interaction accounts for the standard low energy weak interaction phenomena according to the conventional V–A theory with CVC and Cabibbo-suppression. It is interesting to note the rather natural manner in which the Cabibbo-angle θ (and other possible mixing angles) get incorporated in the theory through the diagonalization of the quark mechanical mass matrix generated by the spontaneous symmetry breakdown mechanism.

(2) The Z_1-mediated weak interaction should describe (*e.g.*,) the neutral current weak processes of the neutrinos. The relative amounts of V and A structures will depend on the value of the parameter σ (or $\tan\chi$) to be determined from experiments. The isospin behaviour will also be determined by this parameter. There are, of course, *no* strangeness-changing terms in the neutral current.

(3) The Z_2-mediated neutral current weak interaction will be clearly far more subtle for easy disentanglement in present experiments. Note that the current $J_\lambda(Z_2)$ does not involve the neutrinos.

(4) Assuming that the leptons occurring in the triplets $\mathcal{L}_4$, $\mathcal{L}_5$, $\mathcal{L}_6$ are all too heavy for showing up at the present energies, we have only a theoretical reason for their introduction so far. However, at least some of the other new leptons might be playing important experimental roles already as indicated below.

(5) The charged leptons $l_1^{\pm}$ may very well be the leptons recently suggested in experiments on e^+e^- annihilation (Perl *et al* 1975). Thus we can have their production by $e^+e^- \to$ 'γ' $\to l_1^+ l_1^-$, followed by subsequent decays through the $W^{\pm}$-mediated weak interaction: $l_1^+ \to \mu^+ + \nu_\mu + \bar{n}_1$, $l_1^+ \to e^+ + \nu_e + \bar{n}_1$, etc., assuming that the neutral counter-part of l_1, the n_1, is of a suitably lower mass.

(6) The recently observed "*dilepton events*" in neutrino interactions in high energy machine experiments (Benvenuti *et al* 1975 *bc*; Barish *et al* 1975; Deden *et al* 1975; Blietschau *et al* 1976; von Krogh *et al* 1976) are to be accounted for by the production ' of the c and the t quarks along with a μ^- in ν_μ absorption and the subsequent decays of these into $\mu^+ + \nu_\mu$ or $e^+ + \nu_e$ plus hadrons, both steps via the $W^{\pm}$-mediated weak interactions. The processes envisaged are rather similar to those [see *e.g.*, Sehgal and Zerwas (1976), Lee (1976)] in the well known U_2 gauge theory with four quarks [Glashow *et al* (1970)].

(7) A particularly interesting feature of the theory is the possible explanation it offers for the recently observed *Kolar events* [Krishnaswamy *et al* (1975)] in the underground cosmic ray experiments.[||] We assume that the Kolar particle is *charged* and is to be identified in our theory with either the lepton E or the lepton M. This, in fact, is the reason for putting the lepton triplets in the 3-representation and not in the $\underline{3}^*$-representation of $SU_3(W)$. If the Kolar particle is our lepton $E(\bar{E})$, then it can only be produced by the electron-neutrinos ν_e ($\bar{\nu}_e$) and not by the muon-neutrinos ν_μ ($\bar{\nu}_\mu$). Its production in machine experiments will thus still be difficult. However, in cosmic rays a sizable fraction of the neutrinos are of the electron-type, so that there is no difficulty there for its production. The production of the Kolar particle will be due, in our theory, to the new weak interaction mediated by the $R^{\pm}$ vector bosons [eq. (5.7 *b*)] along with hadrons carrying the heavy quarks t, g and c. There will thus be a correspondingly high threshold. The most spectacular decay of the E, with three final charged particle

[||]In this connection see also the interesting discussion given by Sarma and Wolfenstein (1976) (and references made therein).

tracks, will be through the $H(\bar{H})$ vector-boson mediated new weak interaction [eq. (5.7 c)]: $E^- \to M^- + \mu^+ + e^-$, $E^+ \to M^+ + \mu^- + e^+$, assuming that $m(E) > m(M)$. The M would then subsequently decay into a μ (or ν_μ) along with new hadrons. It is also possible, of course, that the Kolar particle is the $M(\bar{M})$, that $m(M) > m(E)$, and that it is produced by $\nu_\mu(\bar{\nu}_\mu)$ absorption and decays spectacularly as $M^+ \to E^+ + \mu^+ + e^-$, $M^- \to E^- + \mu^- + e^+$ (among other less spectacular decays), followed by the decays of the E into e (or ν_e) and new hadrons. It is important to *note* in this connection that, since according to eq. (3.31) the masses $m(H)$ and $m(R)$ are equal to $m(W)$, the effective weak coupling constants relevant here have the same value as the Fermi coupling constant. This should account for the large number of Kolar events in relation to the older conventional events initiated by neutrinos in cosmic rays.

It is thus clear that, if the theory proposed here turns out to be relevant to physics, it will imply the existence of a whole realm rich in new weak interaction phenomena, some of which are perhaps already showing up. It is also clear that a proper quantitative evaluation of the implications of the theory can only be made when a great deal of experimental work in the future (hopefully) establishes the new quark-flavours and lepton-types, and further determines the detailed characteristics of the new phenomena with reasonable finality. For the moment we shall have to be content with the foregoing rather general qualitative remarks.

Acknowledgements

The author has greatly profited from numerous conversations with his colleagues. He records his thanks particularly to V Singh, K V L Sarma, G Rajasekaran, T Das, P P Divakaran, Probir Roy, V S Narasimham and M G K Menon.

Appendix A

We give here for illustration a particularly simple example of the potential function $V(\psi)$:

$$V(\psi) = -m_0^2 \operatorname{Tr}(\Psi^\dagger \Psi) + h_1 [\operatorname{Tr}(\Psi^\dagger \Psi)]^2 - h_2 (\epsilon_{ijk} \psi_i^{(1)} \psi_j^{(2)} \psi_k^{(3)} + \text{h.c.}). \tag{A.1}$$

We make the displacement (assuming η to be a real positive number):

$$\Psi \to \Psi + \eta \mathbf{1} : \quad \psi_i^{(j)} \to \psi_i^{(j)} + \eta \delta_{ij}, \tag{A.2}$$

and demand for consistency (condition for the needed minimum) that no terms linear in the fields appear, so that we must put

$$m_0^2 = 6\eta^2 h_1. \tag{A.3}$$

Remembering the representation

$$\Psi \equiv \tfrac{1}{2} \lambda_a \rho_a + \tfrac{1}{2} \lambda_0 (\rho_0 + i\sigma_0), \tag{A.4}$$

we then obtain (dropping the constant terms)

$$V(\psi) \rightarrow h_1 [\mathrm{Tr}(\Psi^\dagger \Psi)]^2 + 2h_1\eta [\mathrm{Tr}(\Psi^\dagger \Psi)] [\mathrm{Tr}(\Psi + \Psi^\dagger)] - h_2 [\epsilon_{ijk}\psi_i^{(1)} \psi_j^{(2)} \psi_k^{(3)} + \mathrm{h.c.}] + \tfrac{1}{2} h_2\eta\rho_a{}^2 + 2h_2\eta\sigma_0{}^2 + 2(3h_1\eta - h_2)\,\eta\rho_0{}^2. \qquad (A.5)$$

By eq. (A.3) h_1 is positive, and if we take h_2 also a positive number such that $h_2 < 3h_1\eta$, we obtain masses for all the ten real scalar fields ρ_a, ρ_0, and σ_0, while the Goldstone fields θ_a do not make an appearance.

Needless to say, we can take $V(\psi)$ to be far more general than the simple expression of eq. (A.1) given here to illustrate the various relevant points made in section 3.

Note added in proof—Two earlier attempts at constructing $SU_3 \otimes U_1$ gauge theories have come to our notice (Schechter and Ueda 1973; Gupta and Mani 1974). Our model differs from these in motivation, implementation and results.

References

Bardakci K and Halpern M 1972 *Phys. Rev.* **D6** 696
Barish B C *et al* 1975 Talk given by B C Barish at the Lepton-Hadron Symposium, Stanford (August 1975).
Bég M A B and Sirlin A 1974 *Ann. Rev. Nucl. Sci.* **24** 379
Benvenuti A *et al* 1975 *a* *Phys. Rev. Lett.* **35** 1486
Benvenuti A *et al* 1975 *b* *Phys. Rev. Lett.* **34** 419, 597
Benvenuti A *et al* 1975 *c* **35** 1199, 1203, 1249
Blietschau J *et al* 1976 *Phys. Lett.* **60B** 207
Dass Tulsi 1973 *Advances in High Energy Physics* eds S M Roy and V Singh TIFR Vol. 1 p. 48
Deden H *et al* 1975 *Phys. Lett.* **58B** 361
Faissner H *et al* 1976 *Phys. Lett.* **60B** 401
Gell-Mann M 1962 *Phys. Rev.* **125** 1067
Georgi H and Glashow S 1972 *Phys. Rev.* **D6** 429
Glashow S, Illiopoulos J and Maiani L 1970 *Phys. Rev.* **D2** 1285
Gupta V and Mani H S 1974 *Phys. Rev.* **D10** 1310
Krishnaswamy M R *et al* 1975 *Phys. Lett.* **57B** 105
Krishnaswamy M R *et al* 1975 *Pramāṇa* **5** 59
Lee B W 1972 *Proc. XVI Int. Conf. High Energy Physics* Vol. 4 p. 249
Lee B W 1976 Fermilab-Conf. 75/78-THY
Perl M L *et al* 1975 *Phys. Rev. Lett.* **35** 1489
Salam A 1968 *Proc. Eighth Nobel Symp.* (New York: John Wiley and Sons)
Sarma K V L and Wolfenstein L 1976 *Phys. Lett.* **61B** 77
Schechter J and Ueda Y 1973 *Phys. Rev.* **D8** 484.
Sehgal L M and Zerwas P M 1976 *Phys. Rev. Lett.* **36** 399
von Krogh J *et al* 1976 preprint
Weinberg S 1967 *Phys. Rev. Lett.* **19** 1264
Zumino B 1972 *Proc. XVI Int. Conf. High Energy Physics* Vol 2 p. 285