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Dependence of pion-nucleus total cross-section on the nuclear density and pion-nucleon off-shell amplitude

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Abstract. Total cross-section for π^+, using the high energy approximation, have been calculated for ^{12}C and ^{32}S at various energies around the (3, 3) resonance in the free pion-nucleon system. It is observed that the shift in energy of the position of the maximum in the pion-nucleus cross-section with respect to that in the free pion-nucleon system depends sensitively on the shape of the nuclear density. Using the realistic density it is further found that the shift due to the off-shell behaviour of the pion-nucleon scattering amplitude is very small. This observation is different from that reported in the literature by other authors.

Keywords. Pion; total cross-section; (3, 3) resonance; off-shell; nuclear-density.

The measured total cross-section for positive pions on nuclei show a broad peak around 155 MeV pion lab. energy. This occurs due to the (3, 3) resonance in the pion-nucleon system. The position of the peak, however, is shifted down with respect to that of the (3, 3) resonance (190 MeV). This shift is attributed to the effect of the nuclear medium (Silbar and Sternheim 1974). One of these nuclear effects is the occurrence of pion-nucleon off-shell scattering amplitude in describing the pion nucleus scattering. In fact, various authors (Ericson and Hufner 1970) have concluded that the main contribution to the shift arises due to the off-shell nature of the scattering amplitude. This conclusion, however, is based on using some simple form, like sharp cut-off, for the nuclear density. In the present note we show that the shift in the maximum of pion-nucleus cross-section with respect to that of the (3, 3) resonance depends sensitively on the form of the nuclear density. For realistic nuclear density, in fact, the shift due to the off-shell behaviour of the pion-nucleon amplitude is very small, most of the observed energy shift is produced due to the nuclear density itself.

According to optical theorem, the total cross-section is given by

$$\sigma_{\text{tot}} = (4\pi/k)\ \text{Im}\ f(\boldsymbol{k}, \boldsymbol{k}) \tag{1}$$

where $f(\boldsymbol{k}, \boldsymbol{k})$ is the scattering amplitude in the forward direction and $\boldsymbol{k}$ is the incident wave vector. In the high energy approximation (Glauber 1959) the scattering amplitude can be written as

$$f(\boldsymbol{k}', \boldsymbol{k}) = -ik/(4\pi) \int_0^\infty b\,db \int_0^{2\pi} d\phi\, e^{i\boldsymbol{q}\cdot\boldsymbol{b}} \{e^{i\chi(b)} - 1\} \tag{2}$$

where b is the impact parameter and q is the momentum transfer. If we associate a refractive index $n(b, z)$ with the nucleus, $\chi(b)$ is given by

$$\chi(b) = \frac{ik}{2} \int_{-\infty}^{+\infty} \{n^2(b, z) - 1\}\, dz. \tag{3}$$

Substituting eq. (2) in eq. (1)

$$\sigma_{\text{tot}} = 2 \text{ Re} \int_0^{\infty} bdb \int_0^{2\pi} d\phi \{1 - e^{i\chi(b)}\}. \tag{4}$$

Assuming that the nuclear medium is homogeneous and the main contribution to the pion nucleus scattering comes from the forward pion-nucleon scattering, the refractive index can be given by the dispersion relation

$$K^2(r, E_\pi) = k^2 + 4\pi\rho(r) f_{\pi N}(K, k)$$

or

$$n^2(r, E_\pi) \equiv K^2(r, E_\pi)/k^2 = 1 + 4\pi\rho(r) f_{\pi N}(K, k)/k^2 \tag{5}$$

where $r^2 = b^2 + z^2$, $K(r, E_\pi)$ is the local wave number of pion in the nucleus, $\rho(r)$ the nuclear density and $f_{\pi N}(K, k)$ the off-shell pion-nucleon scattering amplitude in the pion-nucleus c.m. system. If in eq. (5) we approximate $f_{\pi N}$ by its on-shell value at k, expression for n^2 becomes

$$[n^2 - 1)]_{\text{on shell}} = 4\pi\rho(r) f_{\pi N}(k, k)/k^2. \tag{6}$$

To investigate the effect of the off-shell behaviour of the pion-nucleon scattering amplitude we use a simple form (earlier used by Ericson and Hufner 1970)

$$f_{\pi N}(K, k) = f_{\pi N}(k, k)\, K^2/k^2. \tag{7}$$

With this prescription for the off-shell behaviour

$$[n^2 - 1]_{\text{off shell}} = 4\pi\rho(r) f_{\pi N}(k, k)/[k^2 - 4\pi\rho(r) f_{\pi N}(k, k)]. \tag{8}$$

To describe the energy dependence of the on-shell forward scattering amplitude around the (3, 3) resonance first we transform it to the c.m. system of pion-nucleon system and then use the Breit-Wigner form for the latter. That is, if $f_{\pi N}(k')$ is the scattering amplitude in the pion-nucleon c.m. system,

$$f_{\pi N}(k') = Ck'^2/(E' - E_R + i\Gamma(E')/2)$$

where E_R (= 1236 MeV).

The constant C is fixed through the optical theorem at resonance energy

$$\sigma_{\text{tot}}^{\pi N}(E_R) = 4\pi \text{Im} f_{\pi N}(E_R)/k_R$$

with

$$\sigma_{\text{tot}}^{\pi N} = (Z + N/3)\, \sigma_{\text{tot}}^{\pi + p}$$

where Z, N and A are the proton, neutron and mass number of the nucleus. The energy dependence of Γ is taken such that it reproduces the shape of the pion-nucleon (3, 3) resonance.

We have calculated the total cross-section for ^{12}C and ^{32}S using expressions (6) and (8) for the refractive index. For density of these nuclei we have used the two parameter Fermi shape,

$$\rho_{I}(r) = \rho_0^{I}/\{1 + \exp(r - c)/t\}$$

and the one parameter sharp cut—off form,

$$\rho_{II}(r) = \begin{cases} \rho_0^{II}, & \text{for } r \leqslant R_0 \\ 0, & \text{for } r > R_0. \end{cases}$$

Parameters c, t and R_0 of these density form are taken from the electron scattering analysis (Hofstader and Collard 1967). Results are plotted in figures 1 and 2,

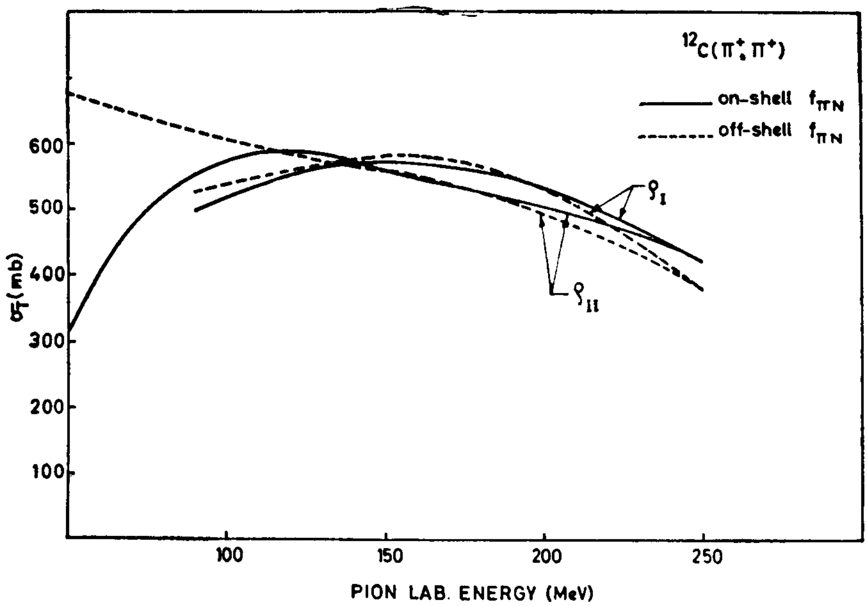

Figure 1. Pion-nucleus total cross-section for ^{12}C around the (3, 3) resonance.

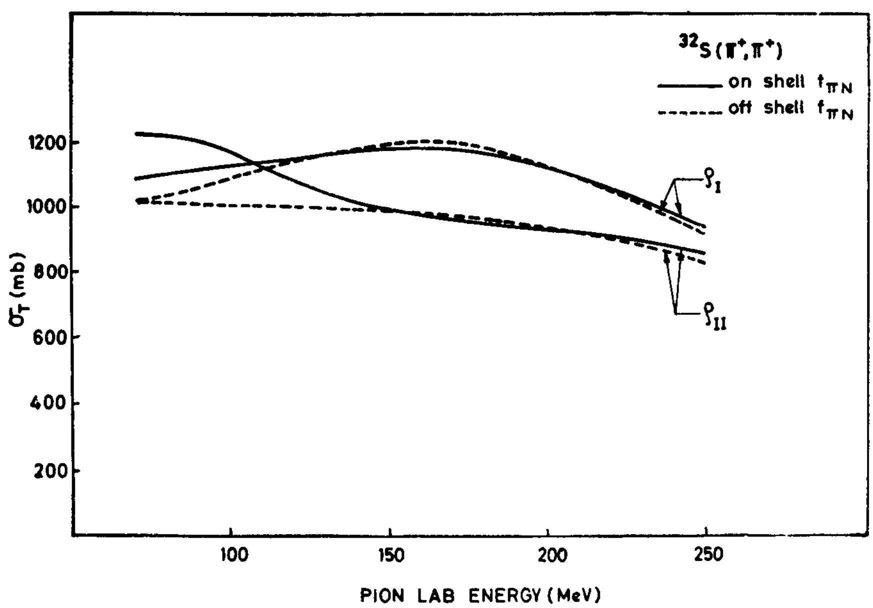

Figure 2. Pion-nucleus total cross-section for ^{32}S around the (3, 3) resonance.

wherein it may be observed: (i) the energy shift in the maximum of the cross-section due to the off-shell behaviour of $f_{\pi N}$ depends sensitively on the form of the nuclear density; (ii) for the two parameter density form (which is the realistic description) the shift due to off-shell behaviour of $f_{\pi N}$ is negligible. The magnitude of the cross-section also does not change much. This suggests that, contrary to earlier conclusion (Ericson and Hufner 1970, Silbar and Sternheim 1972) based on using density with a sharp cut-off, the off-shell nature of $f_{\pi N}$ does not play a significant role in accounting for the pion-nucleus total cross-section data; (iii) the position of the maxima in the calculated cross-sections with the Fermi shape density form occurs around 155 MeV. Experimentally also the peak occurs around the same energy.

In conclusion it may be stated that, in determining the energy position of the maximum in the total cross-section the shape of the nuclear density is more important than the off-shell behaviour of the pion-nucleon scattering amplitude.

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