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# Raman processes in free-free transitions in the presence of intense laser beam

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**Abstract.** The differential cross section for the electron scattering accompanied by simultaneous absorption or emisson of one Raman photon and the inverse bremsstrahlung process in an intense laser beam is calculated.

**Keywords.** Electron scattering; Raman photon; bremsstrahlung process in intense laser beam; transition probability.

## 1. Introduction

With the development of quantum optical devices, it has now become possible to study a number of new phenomena, *e.g.*, generation of harmonics of optical radiation, self-focussing of wave beams in a medium, inverse bremsstrahlung process (Man Mohan, 1974) multiphoton absorption, etc., due to the interaction of sufficiently intense electromagnetic radiation with matter (Man Mohan and Thareja 1972, 1973; Thareja and Haque, 1974). The study of these processes are very important as these are related to laser fusion reactions (Seely 1973, 1974) which have now become one of the most important field of investigations. In the present paper we have studied the stimulated bremsstrahlung process along with the emission or absorption of one-Raman photon of frequency $\omega'$ in the presence of an intense electromagnetic field of frequency $\omega$. When $\omega' < \omega$ then this is called Raman Stoke's process and when $\omega' > \omega$ then it is regarded as Raman anti-Stokes process. We use a semiclassical approach in which the electron motion is described quantum mechanically and the electromagnetic field is considered classically to calculate the cross section for the process.

## 2. Theory

The hamiltonian of a non-relativistic electron in the presence of an intense e.m. field of vector potential $A$ and of frequency $\omega$ and a weak field of vector potential $A'$ and frequency $\omega'$ is given by

$$H = \frac{1}{2m}\left[-i\hbar \vec{\nabla} - \frac{e}{c} A(t) - \frac{e}{c} A'(t)\right]^2 \tag{1}$$

where

$$A(t) = |A_0|\, \hat{\epsilon} \cos \omega t = \frac{-c\,|E_0|}{\omega}\, \hat{\epsilon} \cos \omega t$$

and

$$A'(t) = |A_0'|\, \hat{\epsilon}' \cos(\omega' t + \alpha) = \frac{-c\,|E_0'|}{\omega'}\, \hat{\epsilon}' \cos(\omega' t + \alpha)$$

where $\hat{\epsilon}$ and $\hat{\epsilon}'$ are the unit polarization vector for the intense field and weak field respectively and $\alpha$ is the phase between the two fields.

Corresponding the above hamiltonian, eq. (1), the exact wave function is

$$\psi_p = (2\pi\hbar)^{-3/2} \exp\left\{\frac{i}{\hbar}\left[p \cdot r - \int_0^t \left[p - \frac{e}{c}A(t) - \frac{e}{c}A'(t)\right]^2 \times (2m)^{-1}\, dt\right]\right\} \tag{2}$$

where $p$ is the electron-momentum.

Thus when the electron is scattered by a static potential, the probability amplitude for the transition of an electron situated in a state $\psi_p$ at $t = 0$ to $\psi_p$, using eq. (2) is given by the expression (Bunkin and Fedorov 1966) (Fedorov 1966, 1967)

$$\begin{aligned} T_{pp_0}(t) &= \left(\frac{-i}{\hbar}\right) \int_0^t \psi_p\, V(r)\, \psi_p\, dr\, dt \\ &= \left(\frac{-i}{\hbar}\right) \int_0^t dt' \exp\left\{\frac{i}{\hbar}\left[(p^2 - p_0^2)(2m)^{-1} t' \right.\right. \\ &\quad - \frac{e}{mc}(p - p_0)\frac{|A_0|}{\omega}\, \hat{\epsilon} \sin \omega t - \frac{e}{mc}(p - p_0)\frac{|A_0'|}{\omega'}\, \hat{\epsilon}' \\ &\quad \left.\left. \times \sin(\omega' t + \alpha) \int V(r)\right]\right\} \exp\left[\left(\frac{-i}{\hbar}\right)(p - p_0) \cdot r)\right] dr \end{aligned} \tag{3}$$

where $V(r) = -Ze^2/r$ is coulomb potential. As the initial free-electron with momemtum $p_0$ and energy $\epsilon_i = \frac{p_0^2}{2m}$ goes to final state with momentum $p$ and energy $\epsilon_f = p^2/2m$ after absorption of $N$ photons of intense field with absorption or emission of the Raman photon therefore by energy conservation we must have

$$\frac{p^2}{2m} = \frac{p_0^2}{2m} + N\hbar\omega \pm \hbar\omega'$$

which results in the change of the kinetic energy of the electron given by

$$\triangle\epsilon = \epsilon_f - \epsilon_i = \frac{p^2}{2m} - \frac{p_0^2}{2m} = N\hbar\omega \pm \hbar\omega'.$$

Now as the field $A'$ is assumed to be weak, we can expand

$$\exp\left\{-\frac{i}{\hbar}\frac{e}{mc}(p-p_0)\frac{|A_0'|}{\omega'}\hat{\epsilon}'\sin(\omega' t+\alpha)\right.$$

$$\simeq 1-\frac{i}{\hbar}\frac{e}{2mc}\frac{|A_0'|}{\omega'}\hat{\epsilon}'\left\{\frac{e^{i(\omega' t+\alpha)}-e^{-i(\omega' t+\alpha)}}{i}\right\}. \quad (4)$$

Substituting eq. (4) in eq. (3) and after integrating (Gradshteyn and Ryzhik 1965) over time we have

$$T^{\pm}_{pp_0}(t)=\sum_{N=-\infty}^{\infty}\frac{i}{\hbar}\frac{e}{2mc}\frac{A_0'}{\omega'}\hat{\epsilon}'(p_0-p)\,e^{i\alpha}$$

$$\times\left\{\exp\left[\left(\frac{i}{h}\right)(\triangle\epsilon+N\hbar\omega\pm\omega')\right]-1\right\}(\triangle\epsilon+N\hbar\omega\pm\omega')^{-1}$$

$$\times J_N\left\{\frac{eA_0\hat{\epsilon}}{mc\omega}(p_0-p)\right\}\int V(r)\exp\left[\left(\frac{-i}{\hbar}\right)(p-p_0)\cdot r\right]dr \quad (5)$$

where the positive sign refers to Raman photon absorption and the negative sign to the Raman photon emission process, $J_N$ is a Bessel function of order $N$.

The calculation of the transition probability per unit time, *i.e.*,

$$W_{pp0}=\mathrm{lt}_{t\to\infty}\left(|T_{pp0}(t)|^2/t\right) \quad (6)$$

leads to the differential cross-section for the simultaneous bremsstrahlung and the Raman processes following the scattering of an electron having velocity $v_0=p_0/m$ into a solid angle $d\Omega\,(=\sin\theta\,d\theta\,d\phi)$, we obtain as

$$\frac{d\sigma a,\ e^{(N\omega+\omega')}}{d\Omega}=\beta_{a,e}\frac{e^2c^2v_0^{\,2}}{\hbar^2 m\omega'^4}E_0'\{\hat{e}_0\cdot(\hat{n}_0-\beta_{a,e}\hat{n})\}^2$$

$$\times J_N^{\,2}[\gamma\hat{e}\cdot(\hat{n}_0-\beta_{a,e}\hat{n})]\frac{d\sigma_s}{d\Omega}(\hat{n}_0-\beta_{a,e}\hat{n})$$

where

$$\beta=[1+2(N\omega\pm\omega')\xi]^{1/2},\quad \xi=\hbar/mv_0^{\,2} \quad (7)$$

$$\gamma=\frac{ev_0E_0}{\hbar\omega^2},\qquad \hat{e}_0=\frac{E_0}{E},\qquad \hat{n}_0=\frac{\mathrm{v}_0}{\mathrm{v}_0}$$

with $\hat{n}$ taken as a unit vector in the direction of the scattered electron, and

$$\frac{d\sigma_s}{d\Omega}(\hat{n}_0-\beta\hat{n})=\left(\frac{m^2}{2\pi\hbar^2}\right)^2\left|\int V(r)\exp\left\{\frac{imv_0}{\hbar}(\hat{n}_0-\beta\hat{n})\cdot r\right\}dr\right|$$

$$=\left(\frac{m^2}{2\pi\hbar^2}\right)^2\left|\frac{2\pi\hbar^2Ze^2}{m^2v_0^{\,2}\beta\left|\cos\theta-\frac{1+\beta^2}{2\beta}\right|}\right| \quad (8)$$

is the differential cross-section for elastic scattering of the electron in the Born-approximation (Bunkin and Fedorov 1966, Fedorov 1969) and $V(r) = -Ze^2/r$ the Coulomb potential.

## Conclusion

The variation of the differential cross-section with $N$ the number of photons absorbed of intense laser field corresponding to Raman Stokes and antistokes component denoted by the frequency $\omega'$ and $\omega''$ is shown in the figure 1. The figure shows clearly that with increase of the number of photons absorbed the differential cross-section decreases which is the result expected physically also). For less number of photons the differential cross-section is quite large and is nearly equal to the elastic cross-section. This is because the initial and final energy of the electron is nearly equal for $N$ small and initial electron energy large.

Figure 2 shows that the differential cross-section increases with the increase of the intensity of the laser field which is also expected by expanding the Bessel function in eq. (7) in series as

$$J_n(Z) \simeq \frac{(\frac{1}{2} \cdot Z)^n}{\Gamma(n+1)} \left\{1 + 0(Z^2)\right\}; \; |Z| \ll 1 \tag{9}$$

which results

$$\frac{d\sigma_{e,e}^{(N\omega \pm \omega')}}{d\Omega} \simeq \gamma^{2N} \gamma'^{2} \tag{10}$$

$$\simeq \left(\frac{I}{I_0}\right)^N \left(\frac{I'}{I'_0}\right)$$

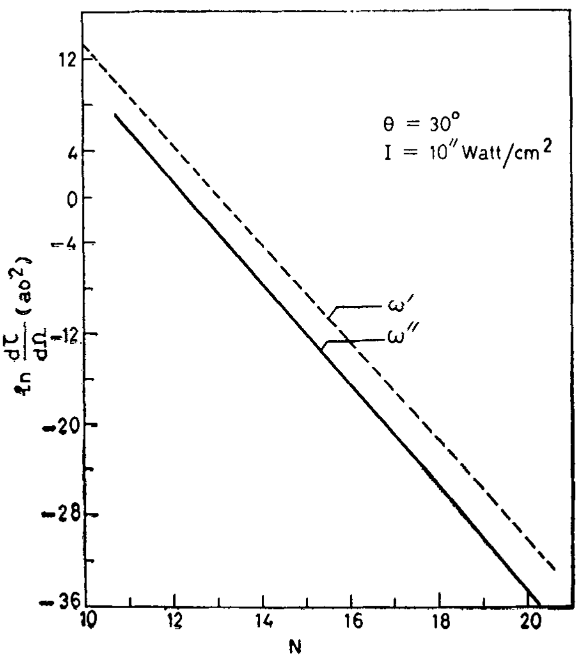


**Figure 1.** Variation of differential cross-section with $N$ for Raman stokes ($\omega'$) and Raman antistokes ($\omega''$) processes.

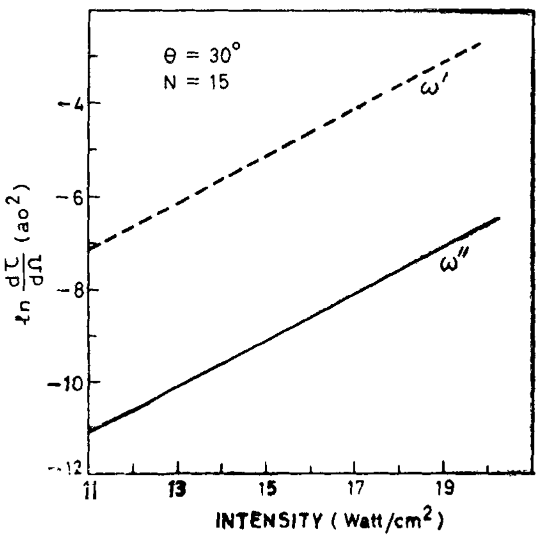


**Figure 2.** Variation of differential cross-section with intensity for Raman stokes ($\omega'$) and Raman antistokes ($\omega''$) processes.

where we have used

$$I = \frac{c\,|E|^2}{8\pi}, \qquad I' = \frac{c\,|E'|^2}{8\pi}$$

and

$$I_0 = \frac{c\hbar^2\,\omega^4}{8\pi e^2\,v_0^4}, \qquad I'_0 = \frac{c\hbar^2\,\omega'^4}{8\pi\,e^2\,v_0^4}$$

Further from eq. (10) we have

$$\frac{\partial \ln\,(\sigma_{a,e}^{(N\omega\pm\omega')})}{\partial \ln I} \simeq N$$

which is the usual perturbation result, that the powerless dependence of the cross section on intensity is proportional to the order of the process.

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## References

Bunkin F C and Fedorov M F 1965 *Z. Eksp. Teor. Fiz.* **49** 1215
Bunkin F C and Fedorov M F 1966 *Eng Trans. Sov. Phys. JETP* **22** 844
Fedorov M F 1966 *Z. Eksp. Teor. Fiz.* **51** 795
Fedorov M F 1967*a* *Z. Eksp. Teor. Fiz.* **52** 1434
Fedorov M F 1967*b* *Sov. Phys. JETP* **24** 529
Fedorov M F 1967*c* *Sov. Phys JETP* **25** 52
Fedorov M F 1967 *Sov. Phys. JETP* **29** 568

Gradshteyn I S and Ryzhik I M 1965 *Table of integrals, Series and Products* (New York and London: Academic Press)
Man Mohan and Thareja R K 1972 *J. Phys.* **B5** L134
Man Mohan and Thareja R K 1973 *a Phys. Rev.* **A7** 34
Man Mohan and Thareja R K 1973 *b J. Phys.* **B6** 809
Man Mohan and Thareja R K 1973 *c J. Phys.* **B6** 1218
Man Mohan 1974 *Phys. Lett* **A50** 283
Seely J F 1973 *Phys. Rev.* **A7** 1064
Seely J F 1974 *Am. J. Phys.* **42** 326
Thareja R K and Haque S N 1974 *Phys. Lett.* **48A** 231