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Intensity ratios of mesonic x-ray lines K_{α_1}, K_{α_2} and L_{α_1}, L_{β_1}

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Abstract. Intensity ratios of μ-mesonic x-rays in some heavy elements are discussed. Intensity ratios of the $K_{\alpha_1}/K_{\alpha_2}$ and L_{α_1}/L_{β_1} lines have been calculated for ^{206}Pb, ^{209}Bi and ^{238}U with relativistic wavefunctions and retardation effect. Though for a refined calculation, it is necessary to take into consideration several features that are peculiar to mesonic atoms, the present calculations have shown that the observed intensity ratios of mesonic x-rays are not anomalous.

Keywords. Mesonic x-rays ; K and L lines ; anomalous intensity ratios.

1. Introduction

The study of mesonic x-rays, together with scattering experiments, has made significant contribution to nuclear structure studies (Electromagnetic Conf., Ottawa, 1967, Kim 1972). The intensity ratios of the mesonic x-ray $K_{\alpha_{1,2}}$ doublets of heavy elements are considered to be anomalous because they are far below the expected theoretical value of 1·92 as suggested by considerations of the statistical weight alone (Anderson and Telegdi 1967, Coté *et al* 1967, Wu 1967, 1969). Especially, intensity ratio of bismuth is of considerable significance.

An explanation has been put forward by Hüfner (1967) that this anomaly might have its origin in nuclear excitations. According to Hüfner, a part of the anomaly is due to some " atomic effect " which is independent of the property of the host nucleus. The intensity ratios of Bi lines and of all the other heavy elements show the anomaly, and to account for them effects other than nuclear excitation cannot be ruled out.

Coté *et al* (1969) have conducted exhaustive studies on the energies of the prominent K, L, M and N lines of ^{238}U and ^{232}Th. Their intensities have been calculated with relativistic Dirac wavefunctions. Other corrections, such as skin thickness, vacuum polarization effects, etc., have been studied. Hyperfine structure of the K, L, M lines of ^{238}U and ^{232}Th has been studied by McKee (1969) to determine nuclear parameters such as skin thickness and nuclear quadrupole parameters. Chen (1970) finds that nuclear polarization makes a contribution of the order of 10% in the intensities of both K and L x-rays. Hitlin *et al* (1970) in their studies of dynamic hyperfine structure and isotope effects find that the results are sensitive to isomer shift, Lamb shift and nuclear polarization factor. According to the

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present authors, all the above effects are not enough to give a reasonable interpretation of the relative intensities of the K and L lines.

An anomaly in the intensity ratios of K and L lines has been known to be present in the case of electronic spectra of heavy elements also, in the sense that these ratios are quite different from those given by statistical weight considerations alone. For uranium, for instance, the intensity ratio of K_{α_1} to K_{α_2} is about 1·67 instead of 2. The intensities of electronic x-rays have been calculated by various authors to different degrees of sophistication and agreements with experimental values have been obtained (Babushkin 1964, 1965 *a, b*, 1967, 1969, Scofield 1969, 1974 *a, b*, Rosner and Bhalla 1970, Krishnan 1971, Krishnan and Nigam 1973).

2. Calculations

In the present work, we have calculated the relative intensities of the K_{α_1} and K_{α_2} lines, *i.e.*, $2p_{3/2} \to 1s_{1/2}$ and $2p_{1/2} \to 1s_{1/2}$ transitions and L_{α_1} and L_{β_1} lines, *i.e.*, $3d_{5/2} \to 2p_{3/2}$ and $3d_{3/2} \to 2p_{1/2}$ transitions for lead, bismuth and uranium. In our earlier studies on electronic x-ray spectra, we found that the application of field theoretical corrections to energy levels, together with a relativistic determination of intensities with retardation effect (Krishnan 1971, Krishnan and Nigam 1973) give good agreement with the experimental results. In our present studies, we have made use of retardation effect with relativistic calculation of intensities. However, instead of using the calculated energy values, the experimentally observed energies which are readily available from literature are made use of. In his study on the effects of nuclear motion on the energy eigenvalues of muonic atoms, among other interactions, Fricke (1973) has studied the effect of retardation. However, we are studying the effect of retardation on the intensities of lines. Screening by outer electrons is expected to be small for the inner shells of the mesonic atoms and hence is neglected.

The expressions used in the calculation of the intensities of the K and L lines are given below. Babushkin (1965 *b*) has given the oscillator strength of the K lines as follows:

$$f_{1s\to np_{1/2}} = \frac{1}{2}\frac{mc^2}{\hbar\omega} \mid R_1 + R_2 - R_3 - 3R_4 \mid^2$$

$$f_{1s\to np_{3/2}} = \frac{mc^2}{\hbar\omega} \mid R_1 + R_2 + 2R_3 \mid^2$$

where

R_1, R_2, R_3, R_4 are given by

$$R_1 = \int_0^\infty g_2 \, J_L(\omega r) \, g_1 \, r^2 \, dr$$

$$R_2 = \int_0^\infty f_2 \, J_L(\omega r) \, f_1 \, r^2 \, dr$$

$$R_3 = \int_0^\infty g_2 \, j_{L-1}(\omega r) f_1 \, r^2 \, dr$$

$$R_4 = \int_0^\infty f_2 \, j_{L-1}(\omega r) \, g_1 \, r^2 \, dr$$

where f_1, f_2, g_1, g_2 are the corresponding small and large components of the Dirac wavefunctions.

These wavefunctions are given by Mizushima (1970) as follows. When the nuclear charge is Ze, α should be replaced by $Z\alpha$ and γ by $Z\gamma$.

$$g = \left[\frac{\Gamma(2\lambda + n' + 1)}{\{\Gamma(2\lambda + 1)\}^2 \, \Gamma(n' + 1)} \, \frac{1 + (\epsilon nj / \mu c^2)}{4N(N - k)} \left(\frac{2\gamma}{N}\right)^3\right]^{1/2}$$

$$\times \left\{-n' F\left(-n' + 1, \, 2\lambda + 1; \, \frac{2r\gamma}{N}\right) + (N - k)\right.$$

$$\left. \times F\left(-n', \, 2\lambda + 1; \, \frac{2r\gamma}{N}\right)\right\} \left(\frac{2r\gamma}{N}\right)^{\lambda - 1}$$

$$\times \exp\left(-\frac{r\gamma}{N}\right)$$

$$f = -\left[\frac{\Gamma(2\lambda + n' + 1)}{\{\Gamma(2\lambda + 1)\}^2 \, \Gamma(n' + 1)} \times \frac{1 - (E_{nj}/\mu c^2)}{4N(N - k)} \left(\frac{2r\gamma}{N}\right)^3\right]^{1/2}$$

$$\times \left\{n' F\left(-n' + 1, \, 2\lambda + 1; \, \frac{2r\gamma}{N}\right) + (N - k)\right.$$

$$\left. \times F\left(-n', \, 2\lambda + 1; \frac{2r\gamma}{N}\right)\right\} \times \left(\frac{2r\gamma}{N}\right)^{\lambda - 1}$$

$$\times \exp\left(-\frac{r\gamma}{N}\right)$$

where

$$k = \begin{cases} -(j + \frac{1}{2}) & \text{when } j = l + \frac{1}{2} \\ +(j + \frac{1}{2}) & \text{when } j = l - \frac{1}{2} \end{cases}$$

$$\lambda = [(i + \tfrac{1}{2})^2 - \alpha^2)^{1/2}$$

$$n = n' + (j + \tfrac{1}{2})$$

$$N = [n^2 - 2n'(j + \tfrac{1}{2} - \lambda)]^{1/2}$$

$$E_{nj} = \frac{\mu c^2}{[1 + \alpha^2 / (\lambda + n')^2]^{1/2}}$$

and $\gamma = 1/r_B$ where r_B is the first Bohr radius.

$F_{nj}(r)$ and $G_{nj}(r)$ of Mizushima have been replaced by g and f respectively according to the notation given by Bethe and Salpeter (1957).

The explicit expressions for the oscillator strengths of the L lines are given by Babushkin (1969) as follows:

$$f_{2p_{3/2}-nd_{5/2}} = \frac{2}{5}\,\frac{mc^2}{\hbar\omega}\left|2R_3' + \frac{1}{2}R_5' - \frac{3}{2}R_6'\right|^2$$

$$f_{2p_{1/2}-nd_{5/2}} = \frac{4}{9}\,\frac{mc^2}{\hbar\omega}\left|2R_4' - \frac{3}{2}R_5' + \frac{1}{2}R_6'\right|^2$$

$$f_{2p_{3/2}-nd_{3/2}} = \frac{2}{45}\,\frac{mc^2}{\hbar\omega}\left|2R_3' + 5R'_4 - 3R'_5 - R'_6\right|^5$$

where

$$R'_3 = \int_0^\infty g_2\, j_{L-1}^{(\omega r)}\, f_1\, r^2\, dr$$

$$R_4' = \int_0^\infty f_2\, j_{L-1}^{(\omega r)}\, g_1\, r^2\, dr$$

$$R'_5 = \int_0^\infty g_2\, j_{L+1}^{(\omega r)} f_1\, r^2\, dr$$

$$R'_6 = \int_0^\infty f_2\, j_{L+1}^{(\omega r)}\, g_1\, r^2\, dr$$

(We are using the symbols R'_3, R'_4, etc. to distinguish them from R_3, R_4, etc., issued for the K lines).

While comparing transitions from the levels of common j values, such as K_{α_1} and K_{α_2} or L_{α_1}, and L_{α_2} the statistical weight is obtained as usual by $(2j+1)$. But when comparing transitions such as L_{α_1} and L_{β_1}, where both the initial and final levels are different, the statistical weights for both the levels are used. This also brings the constants in conformity with those obtained by Compton and Allison (1960) by the application of the Burger-Dorgelo rules.

The frequency factor used in our calculations are according to Bethe and Salpeter (1957). The frequency of a transition plays a very important part in its intensity as the intensity is proportional to the fourth power of the frequency. For example, when comparing the intensities of K_{α_1} and K_{α_2} lines, the frequency of K_{α_1} is greater than that of K_{α_2} line and the ratio of the fourth power of frequency of K_{α_1} line to that of K_{α_2} varies from 1·00 to 1·16 for elements of atomic number 20 to 92, in the case of the electronic x-ray spectra. The same ratio for L_{α_1} to L_{β_1} varies from 0·96 to 0·41 for the same range of elements because the frequency of L_{α_1} is less than that of L_{β_1}. This means that the contribution of the ν^4 factor makes the L_{β_1} line much more intense in the case of the heavy elements. One should not be surprised to find an unusually low value for L_{α_1}/L_{β_1} intensity ratios for the electronic x-ray spectra of the very heavy elements. For μ-mesonic x-rays, however, the variation of the ratio for the same L lines is not so large and even for ^{238}U, the ratio is only 0·81 for L_{α_1}, L_{β_1} lines. This is almost twice that of the electronic spectra. Hence the intensity ratio of L_{α_1}/L_{β_1} will not be affected by the frequency factor to the same degree in the case of the μ-mesonic x-rays.

3. Results

The results are given in table 1.

Table 1. Intensity ratios of μ-mesonic x-rays.

Element	Intensity ratios of K_{α_1} to K_{α_2}		Intensity ratios of L_{α_1} to L_{β_1}	
	Calculated	Observed	Calculated	Observed
^{206}Pb	1·69	1·65 ± 0·20^b	1·23	1·57 ± 0·15^a
^{238}Bi	1·69	1·39 ± 0·10^a	1·23	1·51 ± 0·20^c
^{209}U	1·53	..	1·30	..

The μ-atomic hyperfine structure of K, L, M lines of ^{238}U and ^{232}Th has been studied by McKee (1969) to determine nuclear parameters such as skin-thickness and nuclear quadrupole parameters. We are not comparing our results with those of McKee as the intensities of K_{α_1}, K_{α_2}, L_{α_1}, L_{β_1} lines are not given by him explicity and the purpose of his study is hyperfine structure and nuclear parameters whereas we are concerned with the relative intensities of the fine structure components.

We have also not compared our results with those of Coté *et al* (1969) as they have not given assignments to the prominent lines observed, in an explicit manner. Either extrapolation from the unperturbed energies for the different levels or comparison of the intensities of lines corresponding to the energies given by different authors will not be proper. The energy values given by the different authors vary and thus there is the possibility of the wrong lines being taken up for comparison.

The observed value of the intensity ratio, 1·65, for the K_{α_1} to K_{α_2} quoted for ^{206}Pb is the mean of three values 1·45, 1·72 and 1·79 given by Anderson (1967). Anderson's earlier value of 2 has not been taken but only his more accurate later value because the error in the earlier experiment was 0·3 whereas it was only 0·04 in his later experiment. The values quoted by Kim (1971) are lower than those of Anderson. The value for the intensity ratio of L_{α_1} to L_{β_1} of ^{209}Bi is also the mean of 1·52 and 1·54 given by Wu (1971) and 1·56 given by Hüfner (1967). The results show that the difference between the calculated and observed values are in very good agreement with those given by Anderson (1967). The CERN value of 1·45, (quated in the same paper) appears to be rather low in comparison. Even for the L lines, though the calculated values are less than the observed values, the calculations are not very much off the mark. However, in order to obtain a true picture of the intensity ratios, not only more experimental data with greater degree of accuracy than is available at present, are necessary, but also calculations based on more realistic models with further refinements due to mixing of states, nuclear polarization and field theoretical corrections to energies calculated *ab initio* taking the various parameters concerning the host nucleus will be necessary. As in the case of the electronic x-rays, a relativistic calculation taking the retardation effect into account is sufficient to show that the intensity ratios of the mesonic x-rays are not as anomalous as they appear to be at first sight.

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^{a} Kim (1971), ^{b} Wu (1971), ^{c} Hüfner (1967), Cotè *et al* (1969).

References

Anderson H L 1967 *Proc. Int. Conf. on Electromagnetic Sizes of Nuclei* p 53–71 Ottawa

Babushkin F A 1964 *Acta Phys. Polon* **25** 749

Babushkin F A 1965 *a Opt. Spek.* **19** 3

Babushkin F A 1965 *b Soviet Physics JETP* **21** 372

Babushkin F A 1967 *Acta Phys. Polon* **31** 459

Babushkin F A 1969 *Acta Phys. Polon* **35** 883

Bethe H A and Salpeter E E 1957 *Handbuch der Physik* **35** Springer-Verlag Berlin

Chen M Y 1970 *Phys. Rev.* **C1** 1167.

Compton A H and Allison S K 1960 *X-rays in Theory and Experiment* p 650 D Van Nostrand Co Inc., Princeton, New Jersey (reprinted)

Coté R E *et al* 1967 *Proc. Int. Conf. on Electromagnetic Sizes of Nuclei* p 41 Ottawa

Coté R E, Prestwich W V, Gaigalas A K, Raboy S, Trail C C, Carrigan Jr. R A, Gupta P D› Sutton R B, Suzuki M N and Thompson A C 1969 *Phys. Rev.* **179** 1134

Fricke B 1973 *Phys. Rev. Lett.* **30** 119

Hitlin D, Bernow S, Devons S, Duerdoth I, Kast J W, Macagno E R, Rainwater J, Wu C S and Barret R C 1970 *Phys. Rev.* **C1** 1184

Hüfner J 1967 *Phys. Lett.* **25B** 189

Kim Y N 1971 *Mesic Atoms and Nuclear Structure* North Holland Pub. Co., Amsterdam London, p. 69.

Krishnan T V 1971 Effect of field theoretical corrections on x-ray spectra and some new experi. mental observations Ph.D Thesis, University of Bombay

Krishnan T V and Nigam A N 1973 *Proc. Indian Acad. Sci.* **77** A 26

McKee R J 1969 *Phys. Rev.* **180** 1139

Mizushima M 1970 *Quantum Mechanics of Atomic Spectra and Atomic Structure* p 201 W A Ben• jamin Inc. New York

Proc. Int. Conf. Electromagnetic Sizes of Nuclei Ottawa 1967 eds. D J Brown, M K Sundaresan and R D Barton (Carleton University, Department of Physics, Ottawa).

Rosner H R and Bhalla C P 1970 *Z. Physika* **231** 347

Scofield J H 1974 *a Phys. Rev.* **179** 9

Scofield J H 1974 *b Phys. Rev.* **A10** 1507

Wu C S 1971 *IV Int. Conf. on High Energy Physics and Nuclear Sructure* p 370 Dubna

Wu C S 1969 *Physics of the One and Two-Electron Atoms* eds. F Bopp and H Kleinpoppen p 474 North Holland