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Sum rules for the baryon $\underline{20}$, baryon $\underline{20}'$ and meson 16-plet coupling constants in broken SU (4) and SU (3) symmetry

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Abstract. Sum rules for the coupling constants for D $(\underline{20})$, B $(\underline{20}')$ and P $(\underline{16})$ are given taking into account first order breaking of SU (4) and SU (3) symmetries. The D $(\underline{20})$ and B $(\underline{20}')$ contain the usual $3/2^+$ baryon decuplet and $1/2^+$ baryon octet of SU (3), while the P $(\underline{16})$ contains the usual pseudoscalar octet of pions, etc. These sum rules generalize the decuplet → octet + octet sum rules of broken SU (3 to a broken SU (4) symmetry scheme, in particular the charm SU (4) for hadrons. It is pointed out that, of the many sum rules, it may be possible to check some of them experimentally (see Section 5) and thus provide a test for an underlying SU (4 symmetry for strong interactions.

Keywords. Broken SU (4), charmed hadrons, coupling constants, sum rules.

1. Introduction

The recent discovery of the new vector resonances at 3·1 GeV (Aubert *et al* 1974; Augustin *et al* 1974; Bucci *et al* 1974) and 3·7 GeV (Abrams *et al* 1974) has been a dramatic development in particle physics. The basic reason for this is their extremely narrow width for decay into hadrons. This narrow width is at present understood on the basis a new quantum number which implies the existence of a larger symmetry group, than SU(3), underlying the strong interactions of hadrons. In this note we consider some consequences of the larger symmetry group being SU (4) (Amati *et al* 1964; Bjorken and Glashow 1964; Hara 1964; De Rujula and Glashow 1975; Das *et al* 1975). The sum rules for the strong coupling constants which we present here will be valid for most SU (4) generalizations, however we will use the language of the more popular SU (4) model in which the additional quantum number is called ' charm ' for definiteness. The weak decays of the charmed hadrons (yet to be found) have been discussed by many authors (Altarelli *et al* 1975, Gaillard *et al* 1975). These results provide tests of the SU (4) symmetry, however they are based in addition on a particular choice of the weak interactions (Glashow *et al* 1970). A direct test, independent of the choice of the weak interactions, of the underlying SU (4) symmetry, apart from mass relations, would be relations between strong interaction coupling constants. In considering such relations it is necessary to deal with broken SU (4) as well as broken SU (3) symmetry. Our experience with SU (3) shows that first order SU (3) breaking works well. Though one expects SU (4) to be broken more badly compared to SU (3), as a first step we consider a simple first order breaking of SU (4) as well as SU (3).

In this note we consider sum rules for the coupling constants of the $J^P = 3/2^+$ baryons D to the $1/2^+$ baryons B and the pseudoscalar mesons P. The usual SU (3) octet of the $1/2^+$ baryons (N, Λ, Σ, Ξ) is accommodated in the twenty dimensional SU (4) representation (1, 1, 0) and will be denoted by **B** ($\underline{20}$). The usual SU (3) decuplet of the $3/2^+$ baryons (N^* (1232), Σ^* (1385), Ξ^* (1530), Ω^- (1670)) is accommodated in the completely symmetric twenty dimensional representation (3, 0, 0) of SU (4) and will be denoted by D (20). The mesons are accommodated in the reducible SU (4) representation $\underline{15} + \underline{1}$. The mixing of the SU (4) singlet with the $\underline{15}$ is required to avoid unacceptably small masses for charmed pseudoscalar and vector mesons. In case of the pseudoscalar (P) mesons the mixing pattern is not clear and the mixed states are not unambiguously identified so far, though for the vector (V) mesons (ρ, ω, ϕ, etc.) with the discovery of the 3·1 GeV resonance, ψ, one knows the mass scale and knows that the mixing is 'ideal' though all the states of the 16-plet have not been found by experiment so far. The baryon-baryon-meson couplings of interest are (DBP) and (BBP). From the point of view experimental checking of the coupling constant sum rules the DBP couplings are of greater interest as the D ($\underline{20}$) will contain B and P resonances (*e.g.* N^* (1232)) and in general the baryons in D ($\underline{20}$) will have higher mass than those in B ($\underline{20}'$). Consequently one may expect to have relations between decays $D \to B + P$ which can be checked directly by experiment, as turned out to be the case in broken SU (3) for the N^*, Σ^* and Ξ^* decays (Gupta and Singh 1964; Becchi *et al* 1964). Further in first order broken SU (4) and SU (3) symmetry the (BBP) couplings are given in terms of a total of 20 parameters while the (DBP) couplings are given in terms of only 9 parameters, thus implying sum rules between fewer coupling constants for the (DBP) couplings and hence more easily confronted with experiment. For this reason we only consider sum rules for (DBP)-couplings in this paper and indeed find some directly checkable relations among about 60 or so sum rules.

In section 2 we establish our notation and in section 3 we give the exact SU (4) results. In section 4 we give the sum rules with first order breaking of SU (4) and SU (3). In the last section we give a brief discussion of the sum rules and point out the sum rules one may hope to check experimentally, in the near future, thus providing tests for an underlying SU (4) symmetry.

2. States and notation

To construct the tensors representing the D, B, and P in SU (4) it is convenient, like SU (3) to construct them out of the fundamental SU (4) representation $\underline{4}$ (1, 0, 0) consisting of the four quarks q_i ($q_1 = p$, $q_2 = n$, $q_3 = \lambda$ and $q_4 = c$) with baryon number 1/3. The first three quarks p, n and λ form a SU (3) triplet and carry the usual fractional charge Q and hypercharge Y (Gell-Mann 1964) and charm $C_h = 0$. The charmed quark 'c' is a SU (3) singlet and has $Q = 2/3$, $Y = 1/3$ and charm $C_h = 1$. The p, n have isospin $I = 1/2$ while $I = 0$ for λ and

c. Strangeness $S \equiv$ Hypercharge — Baryon Number is zero for p, n and c, while $S = -1$ for λ. Clearly baryons will be made out of qqq and mesons out of $q\bar{q}$. Since $\underline{4} \times \underline{4} \times \underline{4} = \underline{4}^* + \underline{20}' + \underline{20}' + \underline{20}$, the D $(\underline{20})$ and B $(\underline{20}')$ are represented by the tensors D_{ijk} and B_{ijk} where $i, j, k = 1, 2, 3, 4$. The tensor D_{ijk} is completely symmetric in i, j and k, while B_{ijk} has the following symmetry properties (Okubo 1975)

$$B_{ijk} = B_{jik}$$

$$B_{ijk} + B_{jki} + B_{kij} = 0. \tag{1}$$

The indices indicate the quark content of the state and thus the quantum numbers of the state, *e.g.*, the $N^{*-} = D_{222}$ will have $I_3 = -3/2$, etc. We will use the usual symbols for the known particles, *e.g.*, N^*, etc., which have $C_h = 0$. For the yet to be discovered charmed baryons we will use the notation $D^Q (N_3, I)$ and $B^Q (N_3, I)$ where N_3 denotes the dimensionality of its SU (3) representation and I its isospin while its charge Q will indicate its I_3 value. Further, $D (N_3, I)$ and $B (N_3, I)$ will denote the isomultiplet in a given SU (3) multiplet denoted by $D (N_3)$ and $B (N_3)$ respectively. This should not cause confusion with the SU (4) assignment as here we are concerned only with the 20 and 20′ baryon representations. In this notation, for example, the proton will be $B_{112} = B^+ (8, \frac{1}{2})$ the nucleon $B (8, \frac{1}{2})$ and the baryon octet simply B (8). The SU (3) content of D $(\underline{20})$ is D (*10*) D (6), D (3) and D (1) with $C_h = 0, 1, 2$ and 3 respectively while B $(\overline{20'})$ contains B (8), B (6), B $(\bar{3})$ and B (3) with $C_h = 0, 1, 1$ and 2 respectively.

The 16-plet of the pseudoscalar mesons is given by

$$P_j^{\,i} = \begin{pmatrix} P_1^{\,1} & -\pi^+ & K^+ & \bar{D}^0 \\ \pi^- & P_2^{\,2} & K^0 & D^- \\ -K^- & \bar{K}_0 & P_3^{\,3} & F^- \\ -D^0 & D^+ & F^+ & P_4^{\,4} \end{pmatrix} \tag{2}$$

where the upper index numbers the columns and the lower index numbers the rows, for example, $P_1^{\,2} = -\pi^+$, $P_1^{\,3} = K^+$, etc. The diagonal elements are given by

$$P_1^{\,1} = \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}P_8 + \frac{1}{\sqrt{12}}P_{15} + \frac{1}{2}P_0, \tag{3 a}$$

$$P_2^{\,2} = -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}P_8 + \frac{1}{\sqrt{12}}P_{15} + \frac{1}{2}P_0, \tag{3 b}$$

$$P_3^{\,3} = -\frac{2}{\sqrt{6}}P_8 + \frac{1}{\sqrt{12}}P_{15} + \frac{1}{2}P_0, \tag{3 c}$$

$$P_4^{\,4} = -\frac{3}{\sqrt{12}}P_{15} + \frac{1}{2}P_0 \tag{3 d}$$

where P_8 is the eighth component of the SU (3) octet in the SU (4) $\underline{15}$, P_{15} is the fifteenth component of the *15* and P_0 is a SU (4) singlet. The $C_h = -1$ mesons $(\bar{D}^0, D^-, F^-)$ transform as *3* under SU (3) while their antiparticles $(D^+, -D^0, F^+)$, with $C_h = +1$ transform as $\underline{3}^*$ under SU (3). Clearly if $P_0 = 0$, P_j^i will be traceless giving us a pure $\underline{15}$-plet. Denote by P_1 the SU (3) singlet which mixes with P_8 and the mixing angle by θ. Let X denote the SU (4) mixing angle between P_0 and P_{15}, which are both SU (3) singlets. Then the physical states are defined by (the mixing angles θ and X are in the first quadrant)

$$\eta \equiv \sin\theta P_1 - \cos\theta P_8, \tag{4 a}$$

$$\eta_1 \equiv \cos\theta P_1 + \sin\theta P_8, \tag{4 b}$$

$$\eta_2 \equiv \sin X P_0 - \cos X P_{15}, \tag{4 c}$$

where $P_1 \equiv \cos X P_0 + \sin X P_{15}$. Experimentally η is known, for η_1 both the η' (958) and E (1420) are possible candidates and η_2 is not yet firmly established. The vector meson matrix V_j^i can be obtained by the replacements $\pi \to \rho$, $K \to K^*$, $D \to D^*$, $F \to F^*$, $\eta \to \phi$, $\eta_1 \to \omega$ and $\eta_2 \to \psi$. For ideal mixing, *i.e.*, $\sin\theta = 1/\sqrt{3}$ and $\sin X = \frac{1}{2}$ for the vector mesons, one has $V_1^1 = 1/\sqrt{2}\,(\rho^0 + \omega)$, $V_2^2 = 1/\sqrt{2}\,(-\rho^0 + \omega)$, $V_3^3 = \phi$ and $V_4^4 = \psi$.

In the DBP-couplings we will use the notation established above for the mesons and further we give them in terms of P_0, P_8 and P_{15} which can be translated in terms of the physical states using equations (3) and (4). The isoscalar coupling constants will be denoted by G (D (N_3, I) B (N_3', I') K), etc., with the isospin Clebsch-Gordon coefficients absorbed as was done in the SU (3) case (Gupta and Singh 1964).

3. Exact SU (4) relations

In SU (4), since $\underline{20}' \times \underline{15} = \underline{4}^* + \underline{20} + \underline{20}' + \underline{20}' + \underline{36}^* + \underline{60}^* + \underline{140}''$, there is only one coupling constant G_0 characterising the (DBP)-coupling constants. [We use the notation of Amati *et al* (1964) to distinguish different irreducible SU (4) representations with the same dimensionality]. The exact SU (4) coupling is given by

$$\tfrac{1}{3}\, G_0 D_{ijk}\,(B_{ijm} P_k^m + B_{ikm} P_j^m + B_{jkm} P_i^m), \tag{5}$$

where all repeated indices are summed over and take the values 1, 2, 3 and 4. In exact SU (4) there are twelve ' SU (3)-scalar ' couplings of the form G (D (N_3), B (N_3') (P (N_3'')) where N_3, N_3' and N_3'' denote the dimensionality of the SU (3) representation to which the baryon or meson belongs. Instead of the isoscalar coupling constants (*e.g.* G (N^*, $N\Pi$)) it is more convenient to deal with the quantities denoted by X which are defined from them by absorbing the Clebsch-Gordan factors, so that in the exact SU (4) limit all the X's are equal to G_0. The X's so defined are listed in table 1 for each of the 62 isoscalar coupling and have been classified under sub-heads for each of the twelve SU (3) couplings possible in exact SU (4). In addition to these there are 13 more isoscalar couplings which vanish in exact SU (4) but become non-zero for either first-order breaking of SU (4) alone or of SU (3) alone or both. These are defined and listed in table 2. Note that all DBP_0 couplings are zero in the exact SU (4) limit since $Tr\,(DB) = 0$.

There are 74 sum rules in exact SU (4) since 75 isoscalar couplings are given in terms of the single parameter G_0. Of these 61 sum rules are just that all the X's

Table 1. The 62 isoscalar couplings for $D \to B + P$ which exist in the exact SU (4) limit. The X's defined in terms of them are used to give the various sum rules. Note that the numbering of the X's is not entirely sequential.

1. $G(D(10), B(8)\,P(8))$

$X_1 = -G(N^*, N\pi)$
$X_2 = G(N^*, \Sigma K)$
$X_3 = \sqrt{3}\,G(\Sigma^*, N\bar{K})$
$X_4 = -\sqrt{3}\,G(\Sigma^*, \Xi K)$
$X_5 = -\sqrt{2}G(\Sigma^*, \Sigma\pi)$
$X_6 = \sqrt{2}G(\Sigma^*, \Lambda\pi)$
$X_7 = -\sqrt{2}G(\Sigma^*, \Sigma P_8)$
$X_8 = \sqrt{2}G(\Xi^*, \Xi\pi)$
$X_9 = \sqrt{2}\,G(\Xi^*, \Xi P_8)$
$X_{10} = \sqrt{2}G(\Xi^*, \Sigma\bar{K})$
$X_{11} = -\sqrt{2}G(\Xi^*, \Lambda\bar{K})$
$X_{12} = -\sqrt{\frac{1}{2}}G(\Omega, \Xi\bar{K})$

2. $G(D(10), B(6)\,P(3))$

$X_{13} = G(N^*, B(6, 1)\,\bar{D})$
$X_{14} = \sqrt{3}\,G(\Sigma^*, B(6, 1)\,\bar{F})$
$X_{15} = \sqrt{\frac{3}{2}}\,G(\Sigma^*, B(6, \frac{1}{2})\,\bar{D})$
$X_{16} = \sqrt{3}\,G(\Xi, B(6, 0)\,\bar{D})$
$X_{17} = \sqrt{\frac{3}{2}}\,G(\Xi, B(6, \frac{1}{2})\,\bar{F})$
$X_{18} = G(\Omega, B(6, 0)\,\bar{F})$

3. $G(D(6), B(8)\,P(3^*))$

$X_{23} = \sqrt{3}\,G(D(6, 1), ND)$
$X_{24} = \sqrt{3}\,G(D(6, 1), \Sigma F)$
$X_{25} = 2G(D(6, \frac{1}{2}), \Sigma D)$
$X_{26} = -2G(D(6, \frac{1}{2}), \Lambda D)$
$X_{27} = -\sqrt{6}G(D(6, \frac{1}{2}), \Xi F)$
$X_{28} = -\sqrt{\frac{3}{2}}\,G(D(6, 0), \Xi D)$

4. $G(D(6), B(6)\,P(8))$

$X_{29} = -\sqrt{3}G(D(6, 1), B(6, 1)\,\pi)$
$X_{30} = -3\sqrt{2}G(D(6, 1), B(6, 1)\,P_8)$
$X_{31} = -\sqrt{6}G(D(6, 1), B(6, \frac{1}{2}, K)$
$X_{32} = 2G(D(6, \frac{1}{2}), B(6, 1)\,\bar{K})$
$X_{33} = -\sqrt{8}\,G(D(6, \frac{1}{2}), B(6, \frac{1}{2})\,\pi)$
$X_{34} = 6\sqrt{2}\,G(D(6, \frac{1}{2}), B(6, \frac{1}{2})\,P_8)$
$X_{35} = -\sqrt{6}G(D(6, \frac{1}{2}), B(6, 0)\,K)$
$X_{36} = \sqrt{3}\,G(D(6, 0), B(6, \frac{1}{2})\,\bar{K})$
$X_{37} = \sqrt{\frac{3}{2}}\,G(D(6, 0), B(6, 0)\,P_8)$

5. $G(D(6), B(6)\,P(1))$

$X_{38} = -3/2G(D(6, 1), B(6, 1)\,P_{15})$
$X_{39} = -3/2G(D(6, \frac{1}{2}), B(6, \frac{1}{2})\,P_{15})$
$X_{40} = -3/2\,G(D(6, 0), B(6, 0)\,P_{15})$

6. $G(D(6), B(3^*)\,P(8))$

$X_{41} = \sqrt{2}\,G(D(6, 1), B(3^*, 0)\,\pi)$
$X_{42} = \sqrt{2}G(D(6, 1), B(3^*, \frac{1}{2})\,K)$
$X_{43} = -\sqrt{\frac{8}{3}}\,G(D(6, \frac{1}{2}), B(3^*, \frac{1}{2})\,\pi)$
$X_{44} = -\sqrt{\frac{8}{3}}\,G(D(6, \frac{1}{2}), B(3^*, \frac{1}{2})\,P_8)$
$X_{45} = -2\,G(D(6, \frac{1}{2}), B(3^*, 0)\,\bar{K})$
$X_{46} = G(D(6, 0), B(3^*, \frac{1}{2})\,\bar{K})$

7. $G(D(6), B(3)\,P(3))$

$X_{47} = -\sqrt{3}G(D(6, 1), B(3, \frac{1}{2})\,\bar{D})$
$X_{48} = -\sqrt{6}G(D(6, \frac{1}{2}), B(3, \frac{1}{2})\,\bar{F})$
$X_{49} = -\sqrt{6}G(D(6, \frac{1}{2}), B(3, 0)\,\bar{D})$
$X_{50} = -\sqrt{3}G(D(6, 0), B(3, 0)\,\bar{F})$

8. $G(D(3), B(6)\,P(3^*))$

$X_{52} = \sqrt{2}\,G(D(3, \frac{1}{2}), B(6, 1)\,D)$
$X_{53} = -\sqrt{6}\,G(D(3, \frac{1}{2}), B(6, \frac{1}{2})\,F)$
$X_{54} = \sqrt{3}\,G(D(3, 0), B(6, \frac{1}{2})\,D)$
$X_{55} = -\sqrt{3}G(D(3, 0), B(6, 0)\,F)$

9. $G(D(3), B(3^*)\,P(3^*))$

$X_{56} = -\sqrt{2}G(D(3, \frac{1}{2}), B(3^*, 0)\,D)$
$X_{57} = \sqrt{2}\,G(D(3, \frac{1}{2}), B(3^*, \frac{1}{2})\,F)$
$X_{58} = G(D(3, 0)\,B(3^*, \frac{1}{2})\,D)$

10. $G(D(3), B(3)\,P(8))$

$X_{59} = \sqrt{2}\,G(D(3, \frac{1}{2}), B(3, \frac{1}{2})\,\pi)$
$X_{60} = 3\sqrt{2}\,G(D(3, \frac{1}{2}), B(3, \frac{1}{2})\,P_8)$
$X_{61} = \sqrt{3}\,G(D(3, \frac{1}{2}), B(3, 0)\,K)$
$X_{62} = -\sqrt{\frac{3}{2}}G(D(3, 0), B(3, 0)\,P_8)$
$X_{63} = -\sqrt{\frac{3}{2}}\,G(D(3, 0), B(3, \frac{1}{2})\,\bar{K})$

11. $G(D(3), B(3)\,P(1))$

$X_{64} = 3/2\,G(D(3, \frac{1}{2}), B(3, \frac{1}{2})\,P_{15})$
$X_{65} = 3/2\,G(D(3, 0), B(3, 0)\,P_{15})$

Table 2. The 13 isoscalar couplings for $D \to B + P$ which are non-vanishing for first order breaking of either SU (4) or SU (3) or both.

12. $G(D(1), B(3)\,P(3^*))$

$X_{66} = -\sqrt{\frac{1}{2}}\,G(D(1,0), B(3,\tfrac{1}{2})\,D)$

$X_{67} = G\text{-}D(1,0), B(3,0)\,F)$

2A. $G(D(10), B(8)\,P(1))$

$X_{19} = 6G(\Sigma^*, \Sigma P_{15})$
$X_{20} = 6G(\Xi^*, \Xi P_{15})$

2 B. $G(D(10), B(3^*, P(3))$

$X_{21} = \sqrt{8}\,G(\Xi^*, B(3^*, \tfrac{1}{2})\,\bar{D})$

$X_{22} = \sqrt{8}G(\Xi^*, B(3^*, \tfrac{1}{2})\,\bar{F})$

7 A. $G(D(6), B(3^*)\,P(1))$
$X_{51} = 4\sqrt{3}G(D(6,\tfrac{1}{2}), B(3^*, \tfrac{1}{2}), P_{15})$

13 A. $G(D(10), B(8)\,P_0)$

$X'_{19} = \sqrt{12}\,G(\Sigma^*, \Sigma P_0)$
$X'_{20} = \sqrt{12}\,G(\Xi^*, \Xi P_0)$

13 B. $G(D(6), B(6)\,P_0)$

$X'_{38} = \sqrt{12}\,G(D(6,1), B(6,1)\,P_0)$
$X'_{39} = \sqrt{12}\,G(D(6,\tfrac{1}{2}), B(6,\tfrac{1}{2})\,P_0)$
$X'_{40} = \sqrt{12}\,G(D(6,0), B(6,0)\,P_0)$

13 C. $G(D(6), B(3^*)\,P_0)$

$X'_{51} = 4G(D(6,\tfrac{1}{2}), B(3^*, \tfrac{1}{2})\,P_0)$

13 D. $G(D(3), B(3)\,P_0)$

$X'_{64} = \sqrt{12}\,G(D(3,\tfrac{1}{2}), B(3,\tfrac{1}{2})\,P_0)$
$X'_{65} = \sqrt{12}\,G(D(3,0), B(3,0)\,P_0)$

in table 1 are equal to G_0 while the other 13 sum rules are the vanishing of the X's in table 2. These latter sum rules can be converted in terms of the physical states using equations (4). For example, $X_{19} = X_{19}' = 0$ and $X_7 = 0$ gives $\sec\theta G(\Sigma^*, \Sigma\eta) = -\operatorname{cosec}\theta G(\Sigma^*, \Sigma\eta_1) = -(2)^{-1/2}\,G_0$ and $G(\Sigma^*, \Sigma\eta_2) = 0$. The exact SU (4) relations are expected to be poor as even the exact SU (3) relations for the experimentally known couplings, X_1, X_5, X_6 and X_8 do not work too well. We go on to consider broken symmetry effects in the next section.

4. Broken symmetry

For SU (4) breaking we assume that the symmetry breaking interaction transforms like the fifteenth component of a 15-plet that is like S_4^4 in tensor notation. Such a term will break SU (4) while preserving SU (3) symmetry intact. It is convenient to think of the symmetry-breaking interaction as a spurion $S\,(\underline{15})$ and consider $S + D \to B + P$. Comparing $\underline{20} \times \underline{15} = \underline{20} + \underline{20}' + \underline{120}^* + \underline{140}''$ with the reduction of $\underline{20}' \times \underline{15}$ given earlier one finds there are only four SU (4) breaking terms to first order. The SU (3) breaking interaction will be assumed to transform as the $I = 0$, $Y = 0$ member of a SU (3) octet which is contained in a SU (4) $\underline{15}$-plet, that is it will transform as S_3^3. As for SU (4), we will have four SU (3) breaking parameters to first order. The breaking parameters are defined by

$$\sum_{s=3}^{4} [a_s D_{ijs} B_{ijk} P_s^{\ k} + \tfrac{1}{3}\, b_s D_{ijk}\,(B_{ijs} P_k^{\ s} + B_{iks} P_j^{\ s} + B_{jks} P_i^{\ s})$$
$$+ \tfrac{1}{2}\, c_s D_{ijs}\,(B_{ksi} P_j^{\ k} + B_{ksj} P_i^{\ k}) + \tfrac{1}{2}\, d_s D_{ijs}\,(B_{kis} P_j^{\ k} + B_{kjs} P_i^{\ k})] \qquad (6)$$

where a_4, b_4, etc., and a_3, b_3, etc., are the four SU (4) and SU (3) breaking parameters. This breaking reduces the symmetry group from SU (4) to SU (2) $\times$ U (1) $\times$ U (1) where the SU (2) refers to isospin and two U (1)'s to the conservation of hypercharge and charm. As a consequence of charm conservation the following couplings: (D (3), B (8) P (3)), (D (1), B (8) P (8)), (D (1), B (3*) P (3)), (D (1). B (6) P (3)), (D (6), B (8) P (3)) and (D (6), B (3) P (8)) are forbidden even though allowed by SU (3) or broken SU (3). At this point we summarise how the various couplings are affected by first order breaking of SU (4) and SU (3):

(i) All the couplings (*i.e.* X's) in table 1 are affected and would receive contributions from the 8 breaking parameters.

(ii) The X's under the sub-heads 2 A, 2 B, 7 A, 13 A and 13 C, in table 2, remain zero despite SU (4) breaking as they are forbidden by exact SU (3), while the couplings under sub-heads 13 B and 13 D became non-zero due to SU (4) breaking alone.

We first give the sum rules for SU(4) breaking alone, *i.e.*, $a_3 = b_3 = c_3 = d_3 = 0$ and then for combined SU (4) and SU (3) breaking for the 75 couplings listed in tables 1 and 2. For SU (4) breaking but exact SU(3) one expects 70 sum rules while for both SU (4) and SU (3) breaking one would get 66 sum rules in all.

4.1 *Broken SU* (4)

It is clear that for SU (4) breaking alone all the couplings will be given in terms of five parameters G_0, a_4, b_4, c_4 and d_4 and the symmetry is reduced to SU (3) $\times$ U (1). Since SU (3) remains exact all the isoscalar couplings (or X's) under each SU (3)-scalar coupling sub-heads, *e.g.*, G (D (10), B (8) P (8)) in the two tables will receive a fixed contribution from the SU (4) breaking parameters so that all the X's under each subhead will remain equal. For example all the 9 X's under sub-head 4 are equal to $(G_0 + \frac{1}{2} c_4 - d_4)$, and yield eight exact SU (3) sum rules. Table 1 yields thus 50 exact SU (3) sum rules leaving only one independent isoscalar coupling (or X) from each SU (3) sub-set. We choose these twelve to be $\mathbf{X}_1$, X_{13}, X_{23}, X_{29}, X_{38}, X_{41}, X_{47}, X_{52}, X_{56}, X_{59}, X_{61}, and X_{66}. In terms of these table 1 yields the following 7 broken SU (4) sum rules since these 12 X's are given in terms of five parameters. The sum rules are

$$\tfrac{1}{2} (X_{13} + X_{52}) = \tfrac{1}{3} (2X_{38} + X_{29}), \tag{7.1}$$

$$\tfrac{1}{2} (X_{47} + X_{66}) = \tfrac{1}{3} (2X_{64} + X_{59}), \tag{7.2}$$

$$2 (X_1 + X_{59}) = 3X_{41} + X_{29}, \tag{7.3}$$

$$2 (X_{23} + X_{66}) = 3X_{56} + X_{52}, \tag{7.4}$$

$$X_{29} + X_{64} = X_{38} + X_{59}, \tag{7.5}$$

$$X_{29} + X_{56} = X_{41} + X_{52}, \tag{7.6}$$

$$X_{47} + X_{52} = X_{13} + X_{66}. \tag{7.7}$$

The way the symmetry breaking parameters are defined by eq. (6), these twelve X's are such that they do not receive any contribution from the SU (3) breaking parameters. This means the above seven sum rules will remain valid in the presence of the SU (3) breaking considered here. Furthermore, G_0, a_4, etc. can be solved in terms of five of them, giving

$$G_0 = X_1,\ a_4 = X_{23} - X_1,\ b_4 = X_{13} - X_1,$$
$$c_4 = 2 (X_1 - X_{41}),\ d_4 = 2 (X_{41} - X_{59}). \tag{8}$$

Notice that X_1 is already known experimentally and the couplings X_{29}, X_{41} and X_{59} would hopefully turn out to be decays since the meson involved is a pion and thus determinable. In that case it would be possible to confront the sum rule (7.3) with experiment.

In addition to the above sum rules, table 2 yields 13 sum rules, namely,

$$X_{19} = X_{20} = X_{21} = X_{22} = X_{51} = X'_{19} = X'_{20} = X'_{51} = 0; \tag{9.1}$$

$$X'_{38} = X'_{39} = X'_{40};\ X'_{64} = X'_{65} \tag{9.2}$$

$$X'_{38} = -X'_{64} = 4/3\,(X_{38} - X_{29}) \tag{9.3}$$

Of these, the sum rules in (9.1) and (9.2) are exact SU (3) sum rules. The two sum rules in (9.3) are the broken SU (4) sum rules which remain valid for broken SU (3) also.

4.2. *Broken SU (3) and SU (4) symmetry*

As pointed above, when both SU (3) and SU (4) symmetries are broken according to equation (6) then the 75 isoscalar couplings are given in terms of 9 parameters and one would expect 66 sum rules in all of which 53 are among the couplings in table 1 and the rest between those in table 2. We first consider the sum rules arising from table 1 which are of three kinds:

(i) *Broken SU (3) sum rules.*—These are 24 in number and involve relations between various isoscalar couplings for a given type of SU (3) coupling, *e.g.*, G (D (10), B (8) P (8)) and which are generally independent of the nature of the SU (4). The sum rules so obtained are a consequence of a first order SU (3) breaking alone, except for the two cases G (D (6), B (6) P (8)) and G (D (3), B (3) P (8)). We point out the reason for the difference below.

(ii) *Broken SU (3) and SU (4) sum rules.*—These are sum rules connecting isoscalar couplings under two different SU (3) coupling sub-heads. These are 22 in number. We exhibit them below in such a manner that they are independent of the SU (4) breaking parameteres and further the unknown couplings are given in terms of the five isoscalar couplings X_1, X_3, X_4, X_6 and X_8 of the SU (3) coupling G (D (10), B (8) P (8)). Our choice of these five is the same as that of Gupta and Singh (1964). Note that X_1, X_6 and X_8 are known experimentally from the corresponding decays $N^* \to N\pi$, etc. Further it may be useful to know the SU (3) breaking parameters in terms of these, namely

$$a_3 = X_3 - X_1,\ b_3 = X_1 + X_4 - 3X_6 + X_8 \tag{10}$$

$$c_3 = 2\,(X_1 - X_6),\ d_3 = 2\,(X_6 - X_8).$$

(iii) *Broken SU (4) sum rules.*—These are seven in number and as pointed out earlier are just given by the equations (7.1) through (7.7).

We now proceed to give the broken SU (3) some rules and the broken SU (3) and SU (4) sum rules, arising from table 1, in sections 4.3 and 4.4 below. In section 4.5 we give the 13 sum rules arising from table 2.

4.3. *Broken SU (3) sum rules*

We list these for each of the twelve SU (3) couplings separately below.

(i) $D\,(10) \to B\,(8) + P\,(8)$.—The seven sum rules in this case are not new (Becchi *et al* 1964, Gupta and Singh 1964) but we list them in our notation for the sake of completeness.

$$2(X_1 + X_8) = 3X_6 + X_5 \tag{11.1}$$

$$2(X_2 + X_{10}) = 3X_7 + X_5 \tag{11.2}$$

$$2(X_3 + X_{12}) = 3X_{11} + X_{10} \tag{11.3}$$

$$2(X_4 + X_{12}) = 3X_9 + X_8 \tag{11.4}$$

$$X_1 + X_{10} = X_3 + X_5 \tag{11.5}$$

$$X_2 + X_8 = X_4 + X_5 \tag{11.6}$$

$$X_8 + X_{10} = X_5 + X_{12}. \tag{11.7}$$

The sum rule in (11.1) involves observed decays of the baryon resonances in the decuplet and thus all the coupling constants can be determined experimentally. In fact, this sum rule is extremely well satisfied (Particle Data Group, 1974).

(ii) $D(10) \rightarrow B(6) + P(3)$.—The three independent sum rules are

$$X_{13} + X_{16} = 2X_{15} \tag{12.1}$$

$$X_{13} + X_{17} = X_{14} + X_{15} \tag{12.2}$$

$$2X_{13} + X_{18} = X_{14} + 2X_{15}. \tag{12.3}$$

These sum rules are independent of SU (4) symmetry and would result at the level of SU (3) symmetry, from first order breaking which transforms as the eighth component of an octet.

(iii) $D(6) \rightarrow B(8) + P(3^*)$.—The two broken SU (3) sum rules independent of SU (4) are

$$X_{24} + X_{28} = X_{25} + X_{27} \tag{13.1}$$

$$2(X_{23} + X_{28}) = 3X_{26} + X_{25}. \tag{13.2}$$

Note that sum rule 13.2 has the structure of the Gell-Mann-Okubo mass formula for the baryon octet.

(iv) $D(6) \rightarrow B(6) + P(8)$.—First order SU (3) breaking gives the 9 couplings in terms of 5 parameters and one expects 4 sum rules. However, one of the SU (3) breaking parameters vanishes since the SU (3) breaking interaction is in a SU (4) $\underline{15}$-plet. Consequently we obtain five sum rules.

$$X_{29} = X_{30} \tag{14.1}$$

$$X_{31} + X_{36} = X_{29} + X_{37} \tag{14.2}$$

$$3X_{33} + X_{34} = 2(X_{29} + X_{37}) \tag{14.3}$$

$$X_{29} + X_{35} = X_{31} + X_{33} \tag{14.4}$$

$$X_{29} + X_{36} = X_{32} + X_{35}. \tag{14.5}$$

The last three sum rules are independent of SU (4). For first order SU (3) breaking, without SU (4), the two sum rules in (14.1) and (14.2) reduce to the single sum rule

$$X_{31} + X_{36} = X_{37} + \tfrac{1}{2}(X_{29} + X_{30}) \tag{14.6}$$

Thus the sum rule (14.1) could provide a test of SU (4).

(v) The SU (3) couplings numbered 5, 6, 7 and 8 in table 1 provide five broken SU (3) sum rules independent of SU (4), namely

$$(\mathrm{D}\,(6), \mathrm{B}\,(6)\,\mathrm{P}\,(1)): \qquad 2X_{39} = X_{38} + X_{40} \tag{15}$$

$$(\mathrm{D}\,(6), \mathrm{B}\,(3^*)\,\mathrm{P}\,(8)): \qquad 2\,(X_{42} + X_{46}) = X_{43} + 3X_{44} \tag{16.1}$$

$$X_{41} + X_{46} = X_{43} + X_{45} \tag{16.2}$$

$$(\mathrm{D}\,(6), \mathrm{B}\,(3)\,\mathrm{P}\,(3)): \qquad X_{47} + X_{50} = X_{48} + X_{49} \tag{17}$$

$$(\mathrm{D}\,(6), \mathrm{B}\,(6)\,\mathrm{P}\,(3^*)): \qquad X_{52} + X_{55} = X_{53} + X_{54} \tag{18}$$

(vi) $D\,(3) \rightarrow B\,(3) + P\,(8)$.—First order SU (3) breaking gives the five couplings in terms of 4 parameters. However, one of these vanishes due to SU (4), giving us two sum rules:

$$X_{59} = X_{60} \tag{19.1}$$

$$X_{59} + X_{62} = X_{61} + X_{63}. \tag{19.2}$$

Without SU (4) the first order SU (3) breaking sum rule is

$$\tfrac{1}{2}\,(X_{59} + X_{60}) + X_{62} = X_{61} + X_{63}. \tag{19.3}$$

Thus the simple equality in (19.1) may provide a test of SU (4).

(vii) For the couplings (D (3), B (3*) P (3*)), (D (3), B (3) P (1)) and (D (1), B (3) P (3*)) there are no broken SU (3) sum rules.

4.4. *Broken SU (3) and SU (4) sum rules*

The twenty two sum rules connecting different SU (3) couplings in terms of (D (8), B (8) P (8)) couplings are

$$X_3 - X_1 = X_{14} - X_{13} = X_{32} - X_{29} = X_{45} - X_{41} = X_{48} - X_{47} = X_{63} - X_{59}\,; \tag{20.1}$$

$$X_1 + X_4 - 3X_6 + X_8 = X_{24} - X_{23} = X_{31} - X_{29} = X_{42} - X_{41} = X_{53} - X_{52} = X_{57} - X_{56} = X_{61} - X_{59} = X_{67} - X_{66}\,; \tag{20.2}$$

$$X_1 - X_6 = X_{23} - X_{26},\; X_1 - X_8 = 2\,(X_{13} - X_{15}); \tag{20.3}$$

$$X_6 - X_8 = X_{26} - X_{23} = X_{41} - X_{43} = X_{56} - X_{58}; \tag{20.4}$$

$$3X_6 - 2X_1 - X_8 = X_{33} - X_{29} = X_{54} - X_{52}\,; \tag{20.5}$$

$$X_1 + 2X_8 - 3X_6 = X_{49} - X_{47}\,; \tag{20.6}$$

$$X_3 + X_4 - 5X_1 + 3X_8 = 8\,(X_{39} - X_{38}); \tag{20.7}$$

$$3X_1 + X_3 + X_4 - 12X_6 + 7X_8 = 4\,(X_{65} - X_{64}). \tag{20.8}$$

Note that from the decuplet decays one knows the left hand sides in (20.3) through (20.6) from experiment. So that one has to measure two more unknowns to be able to check these sum rules.

4.5. *Sum rules arising from table 2*

The thirteen isoscalar couplings in table 2 are given in terms of the eight symmetry breaking parameters which are known in terms of the other X's (see (8) and (10)) and thus will give rise to 13 sum rules. We obtain

$$X_3 + X_4 - X_1 - X_8 = X_{19} = -X_{20} = X'_{19} = -X'_{20} = X'_{51} \tag{21.1}$$

$$4X_6 - 2X_1 - 2X_8 = X_{21} = X_{22}; \quad X_{51} = X_{19} - 2X_{21} \tag{21.2}$$

$$2X'_{39} = X'_{38} + X'_{40} \tag{22.3}$$

$$X'_{38} = -X'_{64} = 4/3\,(X_{38} - X_{29}) \tag{22.4}$$

$$X'_{40} = -X'_{65} = X'_{38} - X_{19}. \tag{22.5}$$

Apart from the two relations to the (D (10), B (8) P (8)) couplings, (21.1), (21.2) and (22.3) give 7 broken SU (3) relations, while the other 4 are broken SU (3) and SU (4) sum rules. Note that (22.4) is identical to the broken SU (4) relation (9.2).

4.6. *A more restrictive symmetry breaking interaction*

So far we have considered SU (4) breaking followed by SU (3) breaking. This means that the SU (4) and SU (3) breaking interactions transforming like S_4^4 and S_3^3 are taken to be independent. However, one could implement SU (4) and SU (3) breaking at one stroke by taking the symmetry breaking interaction to be $(S_4^4 + yS_3^3)$, where y is an unknown parameter. Such a possibility has been considered in deriving mass relations (Okubo 1975; Gaillard *et al* 1975; Borchardt *et al* 1975). Group theoretically it means that S_4^4 and S_3^3 are assumed to be in the same SU (4) 15-plet. For such a symmetry breaking interaction it is clear that the SU (4) and SU (3) symmetry breaking parameters in equation (6) will be proportional, that is $a_3 = ya_4$, $b_3 = yb_4$, $c_3 = yc_4$ and $d_3 = yd_4$. Eliminating $y = a_3/a_4$, etc., from these, gives us three relations, which can be obtained directly from eqs (8) and (10), namely

$$y = \frac{X_3 - X_1}{X_{23} - X_1} = \frac{X_1 + X_4 - 3X_6 + X_8}{X_{13} - X_1} = \frac{X_1 - X_6}{X_1 - X_{41}} = \frac{X_6 - X_8}{X_{41} - X_{59}}. \tag{23}$$

It is clear that these three sum rules are in addition to the sum rules given earlier and that the latter are unaffected by the more restrictive form of the symmetry breaking interaction which leads to (23). In case, X_{41} and X_{59} are determinable by experiment (see section 5) and since X_1, X_6 and X_8 are known experimentally it will be possible to check the last relation in (23) thus providng a test of the restrictive breaking hypothesis considered here as well as an estimate of y which represents the ratio of SU (3) to SU (4) breaking in the couplings.

5. Discussion of the sum rules

At present, without the discovery of charmed baryons or mesons and thus not knowing their masses it is a little hard to discuss which of the sum rules may be amenable to experimental check. However, one can make some educated guesses about which of the DBP-couplings may turn out to be decays $D \to B + P$ thus enabling the corresponding coupling constant (or X) to be eventually determined from the experimental width. Let us denote by D_c and B_c any of the charmed baryons in the D (20) and B (20′) respectively. One may guess that the DBP-couplings in the following cases may turn out to be decays:

(i) The coupling $(D_c B_c P)$ where the meson is the pion offer the best chance o being decays, just as in the case for SU (3) decuplet decays (see sum rule 11.1).

There are five such couplings, namely, X_{29}, X_{33}, X_{41}, X_{43} and X_{59}. This means one should be able to test the SU (3) and SU (4) breaking sum rule (7.3) and the sum rules $X_6 - X_8 = X_{41} - X_{43}$ and $3X_6 - 2X_1 - X_8 = X_{33} - X_{29}$ from (20.4) and (20.5) respectively. Recall X_1, X_6 and X_8 are already known from experiment. Further, note that using (8), one would also be able to determine c_4 and d_4.

(ii) The other couplings $D_e B_e P(8)$ offer a chance of being decays, more so for K and $\bar{K}$ (mass $\sim$ 500 MeV) members of P (8) and possibly for η which has a mass of 550 MeV. However, the latter, though mainly P_8, is a mixture of P_0, P_{15} and P_8 and may not give a clean test. Assuming the K and $\bar{K}$ couplings are decays then X_{31}, X_{35}, X_{42} and X_{61} as well as X_{32}, X_{36}, X_{45}, X_{46} and X_{63} can be determined experimentally. One can then test the broken SU (3) relations (14.4), (14.5) and (16.2).

(iii) From mass relations, to lowest order in SU (4) breaking, Gaillard *et al* (1975) have suggested the possibility of strong decays of the form $B_c \to B(8) + P_c$, where P_c refers to P (3) or P (3*). Since one would expect that, in general, the mass of D_c would be greater than that of the corresponding B_c one has the possibility that all (D_c, B (8) P_c) couplings would also be strong decays. In addition couplings like (N^*, B (6, 1) $\bar{D}$) may be decays (*i.e.*, $B(6, 1) \to N^* + \bar{D}$) since mass of N^* is only 1232 MeV. If these speculative conjectures about the masses are correct then one should be able to determine experimentally X_{13} (possibly X_{14} and X_{15}) and X_{23} through X_{28}. In that case all the five parameters (see (i) above) in equation (8) will be determined thus enabling us to determine X_{38}, X_{47}, X_{52}, X_{56} and X_{64} using the SU (3) and SU (4) breaking sum rules in (7.1) through (7.7) as well as X'_{38} and X'_{64} from (9.3). It is clear such a possibility would provide a rich testing ground for a broken SU (4) symmetry.

It is clear from the above discussion that at least a few of the many sum rules for (DBP)-couplings will provide a test of a SU (4) symmetry for hadrons, in the future. Further such a test would depend directly on the underlying SU (4) symmetry for strong interactions and not depend on the choice of the weak interactions for the new hadrons. Finally the sum rules given here are clearly valid for (DBM)-couplings where M may be a 16-plet of vector or tensor mesons with the translation $P \to M$.

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