Pramāṇa, Vol. 6, No 1, 1976, pp. 34–41. © Printed in India.

# Some neutral current effects in an SU(3) × U(1) gauge model

K MAHARANA and C V SASTRY
Department of Mathematics, Regional Engineering College, Rourkela 769008, Orissa

MS received 25 August 1975; in revised form 8 October 1975

**Abstract.** The anomalous magnetic moment of muon is calculated in an $SU(3) \times U(1)$ gauge model proposed by Gupta and Mani. We find the contribution due to the intermediate gauge bosons to be of the same order of magnitude as in Weinberg-Salam model. The deep-inelastic structure functions are also analysed in the same model and inequalities for the structure functions are obtained in the light-cone algebra approach.

**Keywords.** $SU(3)$; gauge model; muon magnetic moment; structure functions.

## 1. Introduction

Many models have been proposed for the unification of weak and electromagnetic interactions since the original suggestion of Weinberg (1967). Gupta and Mani (1974) have proposed an $SU(3) \times U(1)$ model in which the hadrons can be conveniently accommodated and the basic hadrons and leptons of the theory occur in a completely symmetrical fashion. The strangeness changing neutral currents are suppressed by the introduction of a new massive lepton.

In section 2 we review the relevant part of the model necessary for the calculation of muon magnetic moment and deep-inelastic structure functions. The muon magnetic moment is calculated in section 3 following the method of Jackiw and Weinberg (1972). Section 4 is devoted to the study of deep-inelastic lepton-hadron structure functions.

## 2. The model

For the leptons the triplet representation chosen is

$$\psi_R = \frac{1+\gamma_5}{2}\begin{pmatrix} e^+ \\ \bar{\nu}_e \\ \bar{\nu}'_e \end{pmatrix}, \; u = \frac{1}{3}$$

where $\bar{\nu}'_e$ is a new massive neutral lepton. The quantum number $u$ for $\psi_R$ comes from the $U(1)$ group and serves to give integral charges to the leptons. The electric charge

$$Q = T_3 + (Y/2) + u \tag{1}$$

where $T_3$ is the third component of isospin and $Y$ is the hyper charge. The left handed objects $\psi_L = \frac{1}{2}(1-\gamma_5)\, e^+$ and $\psi'_L = \frac{1}{2}(1-\gamma_5)\bar{\nu}'_e$ are $SU(3)$ singlets with $u = 1$ and 0 respectively. The assignment of $\mu^+, \bar{\nu}_\mu$ and $\bar{\nu}'_\mu$ is exactly the same. The requirement of Yang-Mills gauge invariance under the group $SU(3) \times U(1)$ will lead to the introduction of nine vector bosons $W^i_\mu$ which for $i = 1, \ldots, 8$ form a octet while $i = 0$ gives the singlet. The interaction of leptons with the nine gauge particles will contain two unknown coupling constants $g$ and $g'$ corresponding to $SU(3)$ and $U(1)$ groups. The electromagnetic field $A_\mu$ has to be a combination of the neutral vector bosons $W_\mu{}^3$, $W_\mu{}^8$ and $W_\mu{}^0$.

$$A_\mu = \frac{\sqrt{3}}{2}\left(W_\mu{}^3 + \frac{1}{\sqrt{3}}\, W_\mu{}^8\right) \sin\theta + W_\mu{}^0 \cos\theta. \tag{2}$$

The other physical neutral gauge bosons are taken as

$$B_\mu = \tfrac{1}{2}\,(W_\mu{}^3 - \sqrt{3}\; W_\mu{}^8) \tag{3}$$

$$C_\mu = \frac{\sqrt{3}}{2}\left(W_\mu{}^3 + \frac{1}{\sqrt{3}}\, W_\mu{}^8\right) \cos\theta - W_\mu{}^0 \sin\theta \tag{4}$$

The requirement that $A_\mu$ couples to $e^+$ alone with strength equal to its charge $e$ gives the relation

$$e = (\tfrac{3}{4})^{\frac{1}{2}}\, g \sin\theta = g' \cos\theta. \tag{5}$$

The part of the Lagrangian that gives the interaction of the leptons with the neutral gauge bosons is

$$\begin{aligned}\mathcal{L}_0 = {} & e\,(\bar{e}^+ \gamma_\mu\, e^+)\, A_\mu \\ & + \frac{e}{\sqrt{3}\sin\theta}\,(\bar{\bar{\nu}}'_e\, O_\mu{}^+\, \bar{\nu}'_e - \bar{\bar{\nu}}_e\, O_\mu{}^+\, \bar{\nu}_e)\, B_\mu \\ & - \frac{2e}{3\sin 2\theta}\,(\bar{\bar{\nu}}'_e\, O_\mu{}^+\, \bar{\nu}'_e + \bar{\bar{\nu}}_e\, O_\mu{}^+\, \bar{\nu}_e)\, C_\mu \\ & + \frac{e}{3}\left[(\cot\theta - 2\tan\theta)\,\bar{e}^+ \gamma_\mu\, e^+ + \frac{2}{\sin 2\theta}\,\bar{e}^+\,\gamma_\mu\,\gamma_5\, e^+\right] C_\mu\end{aligned} \tag{6}$$

where

$$O_\mu{}^+ = \frac{\gamma_\mu}{2}\,(1 + \gamma_1).$$

For the hadrons the triplet representation of $SU(3)$ chosen is

$$H_L = \frac{1}{2}\,(1 - \gamma_5)\begin{pmatrix}\mathscr{P}\\ n\\ \lambda\end{pmatrix}, \quad u = \frac{1}{3}$$

where $\mathscr{P}$, $n$, and $\lambda$ are the integrally charged quarks.

We define the hadronic currents generated by $\vec{\lambda}$ and $Q$ as

$$\vec{J}_\mu{}^a = i\bar{q}\,\frac{\lambda^a}{2}\,\gamma_\mu\;(1 - \gamma_5)\, q \tag{7}$$

$$J^{em}_\mu = i\bar{q}\left(\frac{1}{3} + \frac{\lambda_3}{2} + \frac{1}{\sqrt{3}}\,\frac{\lambda_8}{2}\right)\gamma_\mu q \tag{8}$$

where
$$q = \begin{pmatrix} \mathcal{P} \\ n \\ \lambda \end{pmatrix}$$

Now we can write the hadronic Lagrangian as

$$\mathcal{L}_h = \frac{1}{\sqrt{2}} g\; W_a^{\mu}\, J_{\mu}^{a} - \frac{1}{2} g'\; W_0^{\mu} \left( J_{\mu}^{3} + \frac{1}{\sqrt{3}} J_{\mu}^{8} - 2\, J_{\mu}^{em} \right) \tag{9}$$

In terms of the physical gauge bosons we can write

$$\mathcal{L}_h = \frac{2e}{\sqrt{3} \sin \theta} \left( W_{\mu}^{\pi} J_{\pi}^{\mu} + W_{\mu}^{K} J_{K}^{\mu} + \text{H. C} \right) + \frac{e}{2\sqrt{3} \sin \theta} B_{\mu} J_{B}^{\mu} + \frac{e}{\sin 2\theta} C_{\mu} J_{C}^{\mu} \tag{10}$$

where

$$J_B^{\mu} = J_3^{\mu} - \sqrt{3}\, J_8^{\mu} \tag{11}$$

and

$$J_C^{\mu} = J_3^{\mu} + \frac{1}{\sqrt{3}} J_8^{\mu} - 2 \sin^2 \theta\, J^{em\,\mu}. \tag{12}$$

### 3. The muon magnetic moment

Jackiw and Weinberg (1972), and Bars and Yoshimura (1972) have calculated the contribution to the muon magnetic moment from the intermediate boson Feynman graphs in the $U$-gauge for the $SU(2) \times U(1)$ model of Weinberg. Fujikawa, Lee and Sanda (1972) have also calculated the same in the models of Weinberg, Georgi-Glashow and Lee-Prentki-Zumino in the U, R and $t'$-Hooft-Feynman gauges. They find the contributions to one or two orders of magnitude too small to be measured. We get a result similar to theirs.

We follow the method of calculation of Jackiw and Weinberg (1972). The correction to the muon magnetic moment from figure (1 $a$) has already been calculated (Barnett *et al* 1967, Brodsky *et al* 1967) and is given by (in our notation)

$$(\triangle g_{\mu})_W = \frac{G m_{\mu}^2}{3\pi^2 \sqrt{2}} \left[ \frac{5}{3} + 3\,(g_W - 2) \ln \left( \frac{\Lambda}{M_W} \right) \right] \tag{13}$$

where $g_W$ is the gyromagnetic ratio of the W-meson and $\Lambda$ is an ultraviolet cut-off. In the proposed theory the $W_{\pi}$-lepton interactions are the same as in the conventional theories but with a $W$-gyromagnetic ratio $g_W = 2$. Hence this diagram is automatically finite here. In the absence of any information on $m_{\bar{\nu}'}$ and $m_{\phi}$, we assume the contribution from the diagrams 1 ($b$) and 1 ($d$) conspire to cancel each other (Primack and Quinn 1972).

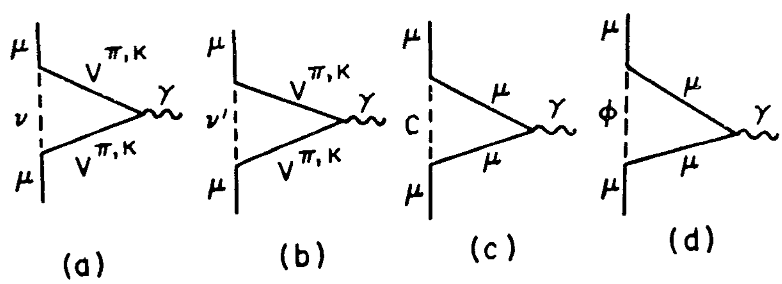


**Figure 1.** Feynman diagrams responsible for weak correction to the muon magnetic moment. *See* Gupta *et al.* (1974) for notations.

The contribution from figure 1 (*c*) in which the photon couples to a muon in the presence of a virtual *C* is calculated following Jackiw and Weinberg (1972). The magnetic form factor is finite:

$$[F(k^2)]_C = \frac{m_\mu^2 e^2}{18\pi^2 \sin^2 2\theta} \int_0^1 dx \int_0^x dy \, [m_\mu^2 x^2 + k^2 y (x - y) + m_C^2 (1 - x)]^{-1}$$
$$\times \left\{ (\cos^2 \theta - 2 \sin^2 \theta)^2 x (1-x) - (4-x)(1-x) - \frac{2m_\mu^2 x^2}{m_C^2} \right\} \tag{14}$$

The first and the second terms in the curly brackets arises from the $g_{\mu\nu}$ part of the photon propagator, the first being the contribution of the vector coupling and the second of the pseudovector coupling. The third term comes from the part of the *C*-meson propagator proportional to $q_\mu q_\nu$. Here, only the pseudovector coupling contributes. In evaluating the intergal we make the assumption that the *C*-meson line does not carry any external momenta. The contribution from the *C*-meson to muon gyromagnetic ratio comes out to be

$$(\triangle g_\mu)_C = 4\,[F(0)]_C \equiv \frac{2e^2 m_\mu^2}{9\pi^2 m_C^2 \sin^2 2\theta} \left[\frac{1}{3}(2 - 3\cos^2\theta)^2 - \frac{5}{3}\right] \tag{15}$$

Recently Gupta and Mani (Mani 1974) have given a bound for $m_C$

$$m_C^2 = \frac{4\sqrt{2}\, g^2}{3G \cos^2 \theta}\, x,\ 2 > x > 1 \tag{16}$$

With this we can write $(\triangle g_\mu)_C$ as

$$(\triangle g_\mu)_C = \frac{m_\mu^2 G}{16\pi^2 \sqrt{2}x} \left[\frac{1}{3}(2 - 3\cos^2\theta)^2 - \frac{5}{3}\right] \tag{17}$$

which may be compared with the result of Jackiw and Weinberg (1972),

$$(\triangle g_\mu)_Z = 4\,[F(0)]_Z = \frac{G m_\mu^2}{8\sqrt{2}\pi^2} \left[\frac{1}{3}(3 - 4\cos^2\theta_W)^2 - \frac{5}{3}\right] \tag{18}$$

Hence we obtain the bounds for $(\triangle g_\mu)_{W+C}$ as

$$3 \cdot 15 \times 10^{-9} \leq (\triangle g)_{\mu W+C} \leq 3 \cdot 42 \times 10^{-9}$$

for all $\theta$. However this number is too small to be detected at the present stage of experimental accuracy.

## 4. Deep-inelastic structure functions

The structure functions of deep-inelastic lepton hadron scattering have been analysed in the Salam-Weinberg model based on different quartet schemes by Fayyazuddin and Riazuddin (1972) and Budny and Scharbach (1972). We give a parallel analysis for structure functions with Gupta-Mani model (1974).

The reactions where the structure functions are measured are

$$e + N \to e + H \qquad (a)$$
$$\nu + N \to \mu + H \qquad (b)$$
$$\nu + N \to \nu + H \qquad (c)$$

where *N* is a nucleon and the final hadron states *H* and the nucleon spins are summed. The deep-inelastic structure functions and kinematic definitions used are those of Gross and Llewellyn Smith (1969) with an obvious extension to the neutral structure functions $F_j^C(x)$ and $F_j^B(x)$ where $x$ is the Bjorken variable $-q^2/2P.q$. The structure functions obey the Callan—Gross (1969) relation $F_2 = 2x F_1$.

We use the usual bilocal $SU(3)$ light cone algebra generated by generalizing the nonet of physical vector and axial vector currents to the bilocal operators $V^a(x, y)$ and $A^a(x, y)$ and assuming that these obey the commutators at light like separations found by substituting the free quark field expression.

$$V_\mu^a(x, y) \sim \tfrac{1}{2}\, i\, \bar{q}(x)\, \lambda^a \gamma_\mu\, q(y)$$
$$A^a(x, y) \sim \tfrac{1}{2}\, i\, \bar{q}(x)\, \lambda^a \gamma_\mu \gamma_5\, q(y) \tag{19}$$

where

$$\bar{q} = (\overline{\mathscr{P} n \lambda}).$$

Following Fritzseh and Gell-Mann (1971) we define the matrix element $\tilde{A}^c(p.z)$ and $\tilde{S}^c(p.z)$:

$$\langle p \mid V_\mu^c(x, y) + V_\mu^c(x, y) \mid p\rangle = 2\, \tilde{S}^c(p.z)\, p_\mu + \text{trace terms},$$
$$\langle p \mid V_\mu^c(y, x) - V_\mu^c(x, y) \mid p\rangle = 2\, \tilde{A}^c(p.z)\, p_\mu + \text{trace terms}. \tag{20}$$

where $z^2 = (x - y)^2 = 0$. The deep-inelastic structure functions are linear combinations of the functions $S^c(x)$ and $A^c(x)$, where

$$\tilde{S}^c(p.z) = \int_{-1}^{1} dx\, e^{-ix(p.z)}\, S^c(x),$$

$$\tilde{A}^c(p.z) = \int_{-1}^{1} dx\, e^{-ix(p.z)}\, A^c(x),$$

$$F_2^{ab}(x) = \frac{-1}{2}\, x\, (if^{abc} S^c - d^{abc} A^c),$$

$$F_3^{ab}(x) = \tfrac{1}{2}\, (if^{abc} A^c - d^{abc} S^c). \tag{21}$$

The weak current can now be written as

$$J_\mu^+ = i\, [\mathscr{P} \gamma_\mu (1 - \gamma_5)\, n \cos\theta_c + \bar{\lambda}\gamma_\mu (1 - \gamma_5)\, n \sin\theta_c]$$
$$= (V - A)^{4+i2} \cos\theta_c + (V - A)^{5+i5} \sin\theta_c \tag{22}$$

where $\theta_c$ is the Cabibbo angle.

The electromagnetic current is given by

$$J_\mu^{em} = \frac{2}{\sqrt{6}}\, V_\mu^0 + V_\mu^3 + \frac{1}{\sqrt{3}}\, V_\mu^8 \tag{23}$$

The neutral current $J_\mu^B$ and $J_\mu^C$ are

$$J_\mu^B = (V - A)_\mu^3 - \sqrt{3}\,(V - A)_\mu^8 \tag{24}$$

$$J_\mu^C = (V - A)_\mu^3 + \frac{1}{\sqrt{3}}\,(V - A)_\mu^8 - \frac{1}{2}\, \Omega\, J_\mu^{em} \tag{25}$$

where $\Omega = 4 \sin^2\theta$.

The deep-inelastic structure functions associated with each current (22), (23) (24) and (25) are given by commuting the current with its adjoint and calculating $F_2$ in (21) from the $VV$ and $AA$ commutators and $F_3$ from the $VA$ and $AV$ commutators (Fritzsch *et al* 1971). Six of the coefficients from the nucelon matrix elements are nonzero. They are $A^c$ and $S^c$ for $c = 0, 3$ and 8. The structure functions are given below:

$$\frac{F_2^e}{x} = \left(\frac{2}{3}\right)^{1/2} A^0 + A^3 + \frac{A^8}{\sqrt{3}}$$

$$\frac{F_2^B}{x} = 4\left(\frac{2}{3}\right)^{1/2} A^0 - 2\left(A^3 + \frac{1}{\sqrt{3}} A^8\right)$$

$$\frac{F_2^C}{x} = \frac{4}{3}\left(\frac{2}{3}\right)^{1/2} A^0 + \frac{2}{3}\left(A^3 + \frac{A^8}{\sqrt{3}}\right) - \left(\frac{2}{3}\Omega - \frac{\Omega^2}{4}\right)$$

$$\left[\left(\frac{2}{3}\right)^{1/2} A^0 + A^3 + \frac{A^8}{\sqrt{3}}\right]$$

$$F_3^B = 4\left(\frac{2}{3}\right)^{1/2} S_0 - 2\left(S^3 + \frac{1}{\sqrt{3}} S^8\right)$$

$$F_3^C = \frac{4}{3}\left(\frac{2}{3}\right)^{1/2} S_0 + \frac{2}{3}\left(S^3 + \frac{S^8}{\sqrt{3}}\right) - \frac{2\Omega}{3}\left[\left(\frac{2}{3}\right)^{1/2} S^0 + S^3 + \frac{S^8}{\sqrt{3}}\right]$$

$$\frac{F_2^\nu}{x} = \frac{2\sqrt{2}}{\sqrt{3}} A^0 + A^3 \sin^2\theta_c + \frac{A^8}{\sqrt{3}}(2 - 3\sin^2\theta_c) + 2A^6 \sin\theta_c \cos\theta_c$$

$$- [S^3 (2 - \sin^2\theta_c) + \sqrt{3}\, S^8 \sin^2\theta_c - 2S^6 \sin\theta_c \cos\theta_c]$$

$$\frac{F_2}{x} = \frac{2\sqrt{2}\, A^0}{\sqrt{3}} + A^3 \sin^2\theta_c + \frac{A^8}{\sqrt{3}}(2 - 3\sin^2\theta_c) + 2A^6 \sin\theta_c \cos\theta_c$$

$$+ [S^3 (2 - \sin^2\theta_c) + \sqrt{3}\, S^8 \sin^2\theta_c - 2S^6 \sin\theta_c \cos\theta_c]$$

$$F_3^\nu = \frac{2\sqrt{2}}{\sqrt{3}} S^0 + S^3 \sin^2\theta_c + \frac{S^8}{\sqrt{3}}(2 - 3\sin^2\theta_c) + 2S^6 \sin\theta_c \cos\theta_c$$

$$+ A^3 (2 - \sin^2\theta_c) + \sqrt{3}\, A^8 \sin^2\theta_c - 2A^6 \sin\theta_c \cos\theta_c$$

$$F_3^{\bar\nu} = \frac{2\sqrt{2}}{\sqrt{3}} S^0 + S^3 \sin^2\theta_c + \frac{S^8}{\sqrt{3}}(2 - 3\sin^2\theta_c) + 2S^6 \sin\theta_c \cos\theta_c$$

$$- [A^3 (2 - \sin^2\theta_c) + \sqrt{3}\, A^8 \sin^2\theta_c - 2A^6 \sin\theta_c \cos\theta_c]$$

Some of the linear relations that follow are

$$F_2^{(\nu+\bar\nu)p-(\nu+\bar\nu)n} = 2\sin^2\theta_c\, F_2^{ep-en}$$

$$xF_3^{\nu p-\bar\nu p-\nu n+\bar\nu n} = (4 - 2\sin^2\theta_c)\, F_2^{ep-en}$$

$$F_2^{Cp-Cn} = \left(\frac{2}{3} - \frac{2}{3}\Omega + \frac{\Omega^2}{4}\right) F_2^{ep-en}$$

$$xF_3^{Cp-Cn} = \left(\frac{2}{3} - \frac{2}{3}\Omega\right) F_2^{ep-en}$$

$$F_2^{Bp-Bn} = -2\, F_2^{ep-en}$$

$$xF_3^{Bp-Bn} = -2F_2^{ep-en}$$

where $F^{ab \pm cd} = F^{ab} \pm F^{cd}$ and $F^a$ denotes $F^{ap}$ or $F^{en}$. It may be noted that $A^3$, $A^8$, etc. would depend on the quantum numbers of the hadron, *e.g.*, $A^3$ would have opposite signs for the proton and neutron states.
If we expand $S^c(x)$ in Taylor's series of bilocal operators.

$$S^c(x) = S_1^c\, \delta(x) - \frac{1}{2!} S_3^c\, \delta''(x) + \ldots \tag{26}$$

the moments of the structure functions are given by the expansion coefficient of (26). From the definition of $S_1^c$ where $c = 0, 3$ and $8$, we have

$$S_1^c \langle p \mid p\rangle = 2 \langle p \mid \int d^3x\, V_0^c(x, x) \mid p\rangle.$$

The baryon current in this model can be written as

$$B_\mu = \sqrt{6}\, V_\mu^0 .$$

Hence, *e.g.*,

$$S_1^0 \langle p \mid p\rangle = 2 \langle p \mid \int d^3x\, V^0 \mid p\rangle = \frac{\sqrt{2}}{\sqrt{3}} B \langle p \mid p\rangle$$

where $B$ is the baryon number of the target. Then, since

$$F_3^{\nu+\bar{\nu}} = 4\left(\frac{2}{3}\right)^{1/2} S^0 + 2S^3 \sin^2\theta_c + \frac{2}{\sqrt{3}} S^8 (2 - 3\sin^2\theta_c)$$

one has the sum rule

$$\int_0^1 dx\, F_3^{\nu+\nu} = \frac{8B}{3} + 4T_3 \sin^2\theta + \frac{4}{3} Y\left(1 - \frac{3}{2}\sin^2\theta_c\right)$$

$$\int_0^1 \frac{dx}{x} F_2^{\bar{\nu}-\nu} = 2\left[2T_3(2 - \sin^2\theta_c) + Y\sin^2\theta_c\right]$$

$$\int_0^1 dx\, F_3^C = \frac{8}{9} B + \frac{2}{3}\left(2T_3 + \frac{Y}{3}\right) - \frac{2\Omega}{3}\left(2B + 2T_3 + \frac{Y}{3}\right).$$

Other conclusive tests results from the Nachtman's $SU(2)$ positivity conditions. They imply

$$\left(\frac{2}{3}\right)^{1/2} A^0 + \left(\frac{1}{3}\right)^{1/2} A^8 \pm A^3 = u_\pm \geq 0$$

$$\left(\frac{2}{3}\right)^{1/2} A^0 - 2\left(\frac{1}{3}\right)^{1/2} A^8 = v \geq 0 \tag{27}$$

Hence

$$F_2^{ep} = u_+$$

$$F_2^{en} = u_-$$

$$F_2^{Cp} = \left(\frac{8}{9}\right) u_+ + \left(\frac{2}{9}\right) u_- + \left(\frac{2}{9}\right) v - \left[\left(\frac{2}{3}\right)\Omega - \left(\frac{\Omega^2}{4}\right)\right] u_+$$

$$F_2^{Bp} = 2(u_- + v) \tag{28}$$

Therefore we obtain

$$0 \leq \frac{F_2^{en}(x)}{F_2^{ep}(x)} \leq \infty.$$

This is the same bound obtained by Budny and Scharbach (1972) for integrally charged quartet quark scheme in $SU(2) \times U(1)$ model.

The structure function $F_2^{\nu\nu p}$ is associated with the current $xJ_\mu^B + J_\mu^C$ where $x = (m_C^2/m_B^2)\cos^2\theta$ and we obtain

$$F_2^{\nu\nu p} = \left(\frac{8}{9} - \frac{2}{3}\Omega + \frac{\Omega^2}{4}\right)u_+ + \frac{2}{9}(u_- + v) + \frac{1}{3}x\left(4 - \frac{\Omega}{2}\right)(u_- - v) + 2x^2(u_- + v).$$

As $\frac{2}{9}(u_- + v) + \frac{x}{3}\left(4 - \frac{\Omega}{2}\right)(u_- - v) + 2x^2(u_- + v) \geq 0$ for all values of $\Omega$

$$F_2^{\nu\nu p} \geq \left[\left(\frac{2}{3} - \frac{\Omega}{2}\right)^2 + \left(\frac{4}{9}\right)\right]u_+ \geq \frac{4}{9}u_+,$$

which implies

$$\frac{F_2^{\nu\nu p}}{F_2^{ep}} \geq \frac{4}{9}.$$

This may be compared with the result of Fayyazuddin and Riazuddin (1972), and Budny (1972) in the $SU(2) \times U(1)$ model with a fractionally charged quartet quark scheme where they get

$$\frac{F_2^{\nu\nu p}}{F_2^{ep}} \geq \frac{9}{16}.$$

The inequality we obtain is an interesting prediction in Gupta-Mani's model in the light-cone algrbra approach with $SU(2)$ positivity conditions. It may be feasible to test it in future.

## Acknowledgement

We express our sense of deep gratitude to V Gupta for his valuable correspondence which helped us in improving the manuscript.

## References

Barnett L and Levine M J 1967 *Phys. Lett.* **24B** 467
Bars I and Yoshimura M 1972 *Phys. Rev.* **D6** 374
Brodsky S and Sullivan J 1967 *Phys. Rev.* **156** 1644
Budny R and Scharbach P N 1972 *Phys. Rev.* **D6** 3651
Callan C G and Gross D J 1969 *Phys. Rev. Lett.* **25** 156
Fayyazuddin and Riazuddin 1972 *Phys. Rev.* **D6** 2032
Fritzsch H and Gell-Mann M In Proceedings of the International Conference on Duality and Symmetry in Hadron Physics, edited by E Gotsman (Weizmann Science Press, Jerusalem, 1971)
Fujikawa K, Lee B W and Sanda A I 1972 *Phys. Rev.* **D6** 2923
Gross D J and Llewellyn Smith C H 1969 *Nucl Phys.* **B14** 337
Gupta V and Mani H S 1974 *Phys. Rev.* **D10** 1310
Jackiw R and Weinberg S 1972 *Phys. Rev.* **D5** 2396
Mani H S Reported in the Second All India High Energy Physics Conference at Visva Bharati 1974
Primack J R and Quinn H R 1972 *Phys. Rev.* **D6** 3171
Weinberg S 1967 *Phys. Rev. Lett.* **19** 1264