Pramāṇa, Vol. 5, No. 5, 1975, pp 249–259. © Printed in India.

Charge transfer in *pp* collisions according to a mixed two component model

V GUPTA, P K MALHOTRA and D S NARAYAN
Tata Institute of Fundamental Research, Bombay 400005

MS received 19 March 1975; in revised form 15 September 1975

Abstract. The charge transfer from one hemisphere to the other, observed in *pp*-collisions is explained on the basis of a 'mixed two component' model, which has been proposed previously to account for the multiplicity distribution of charged particles. Results of the calculations, based on the model, for various measurable quantities relating to charge transfer are compared with the available experimental data.

Keywords. *pp* collisions; charge transfer; leading and non-leading particles; poisson distribution and geometric distribution.

1. Introduction

The new measurements [Bromberg *et al* (1973) and (1975); Cohen 1973; Derrick 1974] on charge transfer, in *pp* collisions at 102 and 205 GeV/*c*, form a significant addition to our knowledge of the detailed features of multiparticle production. The charge transfer u, an integer, is defined by $u = \frac{1}{2}(Q_F - Q_B)_{\text{final}} - \frac{1}{2}(Q_F - Q_B)_{\text{initial}}$, where Q_F and Q_B are the algebraic sum of charges moving in the forward and backward hemispheres respectively. Attempts have been made to understand charge transfer by Chow and Yang (1973) on the basis of a diffractive model and by Quigg and Thomas (1973) on a multiperipheral model.

The aim of this paper is to show that the model, proposed previously by Chaudhary, Gupta and Narayan (1973) to explain the charged particle multiplicity distribution and referred to hereafter as ' mixed two component model ', can be elaborated to explain quantitatively the charge transfer. We present here the results of the calculations, based on such a model, for various measurable quantities relating to the charge transfer.

2. A mixed two component model with independently produced clusters

By calling our model a mixed two component model, we want to stress the distinction between our version and the usual version of a two component model (Wilson 1970, Quigg and Jackson 1972; Lach and Malamud 1973; Fialkowski 1972; Van Hove 1973; Harari and Rabinovici 1973). In the mixed two component model, both the pionization and fragmentation components are present in each event in contrast to the other models in which the two components correspond to two different types of events. These components originate in the decay of independently produced clusters, composed of a certain number of central clusters of low mass and neutral charge and two leading clusters, which carry the quantum numbers of the incident nucleons. The central clusters, which are slow in the CM system, give rise to the pionization component and the leading clusters give rise to the fragmentation component. This picture of multiparticle production in hadron collisions was proposed by one of the authors (Narayan 1971). In this model, the interaction between the colliding hadrons gets localized to small cells or point like interactions and different parts get shattered independently into fragments or clusters with the proviso, that there is conservation of energy, momentum and other quantum numbers at each local centre. The distribution of mass and velocities amongst the clusters would depend crucially on the spacial distribution of mass (equivalently spacial distribution of momentum fractions) inside a hadron. A nucleon is pictured as having a massive constituent or a core and a meson cloud of light constituents. In a peripheral collision of two nucleons, the core of one nucleon collides with a light constituent in the pion cloud of the other nucleon and gives rise to a leading cluster, while the collisions between the light constituents in the clouds give rise to the central clusters. In a further elaboration of the model, Chavda and Narayan (1974) have explained the production of particles with large transverse momenta and the copious production of antiprotons at ISR energies, as being due to central collisions involving core-core interactions.

Recently Pokorsky and Van Hove (1974) have proposed an independent cluster production model, which has a content similar to that in the model described above for the peripheral collisions. It may also be possible to realize the production of independent clusters of the above description, in a multiperipheral model with Pomeran exchanges, under special assumptions. In this connection, the so called polyperipheral picture (Bishari *et al* 1972) in which there are a series of double Pomeran exchanges as in figure 1 may be worth studying. The details of the model may be so adjusted that each double Pomeran exchange leads to the emission of a central cluster, while the nucleons emerge out as the leading clusters.

In the mixed two component model, we consider only a two body decay of a central cluster into a pair of charged or neutral particles. The resulting particles which are slow in the CM system, are referred to hereafter as non-leading particles. Their number distribution can be expected to be a Poisson in the number of pairs. One has, however, no such *a priori* feeling for the number distribution in the fragmentation component, which accounts for the leading particles. It

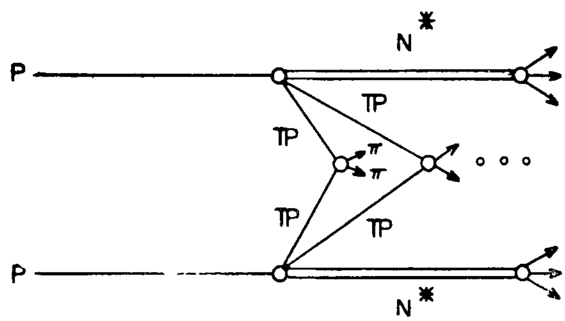

Figure 1. Poly-peripheral mechanism for the simultaneous production of fragmentation and pionization components.

is postulated that the number distribution of the leading particles is given by a finite geometric distribution with R terms, where R is a function of s. It has been shown that the production of particles according to a simple cascade process (Fujiwara and Kitazoe 1970) leads to a geometric distribution. It is therefore reasonable to assume a geometric distribution for the leading particles resulting from a cascade decay of the leading excited baryons.

As the emission of leading and non-leading particles is assumed to be unrelated, the probability P_{2m} of having $2m$ charged particles in an event can be written as

$$P_{2m} = \sum_{r=1}^{m} a_r \, \bar{P}_{m-r}, \qquad \text{if } m \leq R$$

$$= \sum_{r=1}^{R} a_r \, \bar{P}_{m-r}, \qquad \text{if } m > R \tag{1}$$

$\bar{P}_m$ is the probability of having $2m$ charged non-leading particles and a_r is the probability of having $2r$ charged leading particles in an event. From the assumptions regarding the number distributions of leading and non-leading particles, one can write

$$\bar{P}_m = \exp\,[-\langle n_{NL} \rangle/2]\, \frac{[\langle n_{NL} \rangle/2]^m}{m\,!} \tag{2}$$

$$a_r = \frac{1-q}{1-q^R}\, q^{r-1} \tag{3}$$

and

$$\langle n_L \rangle = \frac{2}{1-q} - \frac{2Rq^R}{1-q^R} \tag{4}$$

where $\langle n_L \rangle$ and $\langle n_{NL} \rangle$ are the average numbers of leading and non-leading charged particles. Further, the average number of charged particles $\langle n \rangle = \langle n_L \rangle + \langle n_{NL} \rangle$. If we regard $\langle n_{NL} \rangle$ and R as free parameters and the value of $\langle n \rangle$ is taken from experiment, then equations (1) to (4) completely determine the charged particle distribution P_n. The parameters $\langle n_{NL} \rangle$ and R are fixed at each energy by carrying out a χ^2 fit to the experimental data on P_n. Alternately, one may parametrize $\langle n_L \rangle$, $\langle n_{NL} \rangle$ and R as functions energy and calculate the multiplicity distributions.

3. Charge transfer

A realistic treatment of charge transfer in pp-collisions would require detailed information on the velocity distribution of the clusters as well as the momentum distribution of the particles emitted from a cluster in the latter's rest frame. We first consider however a simplified approach, which does not require the introduction of new parameters in the mixed two component model, and yet brings out the essential features of charge transfer. A more realistic treatment is presented in section 4.

3 *a*. *Simplified treatment of charge transfer*

The two simplifying assumptions we make are

(A) Charge transfer arises only from the non-leading particles. This assumption requires that there is no spill-over of particles from the leading clusters. This is a reasonable assumption at high energies but at lower energies it is expected to give an underestimate of the charge transfer.

(B) As the central clusters are slow in the CM system, we ignore their velocity distribution and assume, as a first approximation, that all the central clusters are produced at rest. It may be noted that this assumption could lead to an overestimate of the charge transfer.

On the basis of these assumptions, we calculate the charge transfer in the mixed two component model. Since each central cluster emits only a pair of charged particles, the two particles would be emitted in opposite hemispheres in the CM system. This implies that one pair of charged particles will give a charge transfer $u = \pm 1$, two pairs will contribute to $u = \pm 2$, 0 and so on. The probability, that m pairs of non-leading particles (from m central clusters) would contribute to $u = m$ is simply equal to the probability that all the m positively charged particles are emitted in the forward hemisphere, which also implies that the negatively charged particles are emitted in the backward hemisphere. Similarly the probability that $u = m - 2$ would be equal to the probability that $(m - 1)$ positively and one negatively charged particle are emitted in the forward hemisphere and so on. Due to the hypothesis of random emission of the particles, the above probabilities are simply related to the probabilities of having a given number of heads and tails in the toss of m coins. As is well known, these probabilities are given by the binomial coefficients.

One can then show that the cross-section $6^{(u)}$ for charge transfer $u \geqslant 0$ is given by

$$6^{(u)}(s) = 6_{\text{in}}(s) \sum_{m=u} \frac{1}{2^m} \, {}^{m}C_{\frac{m-u}{2}} \, \bar{P}_m \tag{5}$$

where ${}^nCr$ are the binomial coefficients and $6_{\text{in}}(s)$ is the total in elastic cross-section at CM energy $\sqrt{s}$. Using the expression for $\bar{P}_m$ given by (2), one finds the probability for a charge transfer u is

$$P^{(u)} = \frac{6^{(u)}}{6_{\text{in}}} = e^{-\triangle} I_u(\triangle), \tag{6}$$

where

$$\triangle = \tfrac{1}{2} \langle n_{NL} \rangle, \tag{7}$$

and

$$I_u(x) = \sum_{l=0}^{\infty} \frac{(x/2)^{n+2l}}{l!\, n+l!} \tag{8}$$

is the modified Bessel function of the first kind. Using $\sum_{n=0}^{\infty} n^2 I_n(x) = \frac{1}{2} x e^{x}$ and since $P^{(u)} = P^{(-u)}$ one finds that the mean square charge transfer

$$\langle u^2 \rangle = \sum_{-\infty}^{+\infty} u^2 P^{(u)} = \triangle. \tag{9}$$

It is also interesting to consider the cross-sections $\sigma^{(u)}_{2n}$ for a charge transfer n when $2n$ charged particles (leading and non-leading) are produced. One finds

$$\sigma^{(2k)}_{2n} = \sigma_{\text{in}} \sum_{j=k}^{(n-1)/2} \frac{1}{2^{2j}}\, {}^{2j}C_{j-k}\, Q^{n}_{n-2j} \quad \text{if } n \text{ is odd,} \tag{10 a}$$

$$= \sigma_{\text{in}} \sum_{j=k}^{(n-2)/2} \frac{1}{2^{2j}}\, {}^{2j}C_{j-k}\, Q^{n}_{n-2j} \quad \text{if } n \text{ is even,} \tag{10 b}$$

$$\sigma^{(2k+1)}_{2n} = \sigma_{\text{in}} \sum_{j=k}^{(n-2)/2} \frac{1}{2^{2j+1}}\, {}^{2j+1}C_{j-k}\, Q^{n}_{n-2j-1} \quad \text{of } n \text{ is even,} \tag{10 c}$$

$$= \sigma_{\text{in}} \sum_{j=k}^{n-3/2} \frac{1}{2^{2j+1}}\, {}^{2j+1}C_{j-k}\, Q^{n}_{n-2j-1} \quad \text{if } n \text{ is odd} \tag{10 d}$$

where $Q_i^n = a_i \bar{P}_{n-i}$ for $i < R$ and $i < n$, otherwise it is zero. In equations (10), k takes integral values 0, 1, 2,... and for a given k, $n = k+1$, $k+2, \ldots$

The result of the simplified treatment for $\langle u^2 \rangle$ given in equation (9) is compared with the experimental values in table 1. The values of $\triangle$ at various energies are those computed by Chaudhary, Gupta and Narayan (1974). One finds that the simplified treatment underestimates the value of $\langle u^2 \rangle$ at energies below 20 GeV and overestimates it for energies above 100 GeV. This is not unexpected in light of the two assumptions (A) and (B) above.

4. A detailed treatment of charge transfer

In this section we present a more realistic treatment of charge transfer by incorporating the effect of a velocity distribution for the central clusters and by including the contribution of the leading clusters to the charge transfer.

4 *a*. Let v be the velocity of a cluster in the CM system and let $F(v)$ be the velocity distribution of the central clusters. Let $\boldsymbol{v}^*$ be the common velocity of the charged pair resulting from the decay of a cluster of mass m. In general there would also be a mass distribution $P(m)$ of the clusters, resulting in a distribution for the velocities $\boldsymbol{v}^*$. The two distributions are normalized so that

Table 1. Comparison of our mixed two component model with the experimental data on $\langle u^2 \rangle$.

P_{lab} (GeV/c)	$\langle n_L \rangle$	$\triangle$	$\langle u^2 \rangle_L$	$\langle u^2 \rangle_{NL}$	$\langle u^2 \rangle$	$\langle u^2 \rangle_{\text{EXPT.}}$
12·8	2·89	0·30	0·44	0·20	0·64	0·65±0·05
28	3·05	0·76	0·20	0·50	0·70	0·71±0·05
69	3·37	1·265	0·08	0·84	0·92	0·91±0·02
102	3·87	1·32	0·06	0·87	0·93	0·90±0·04
205	4·87	1·435	0·02	0.95	0·97	1·04±0·04
405	5·6	1·8	0·00	1·19	1·19	1·12±0·05

$$\int_0^c F(v)\,dv = 1 \text{ and } \int_{2m_\pi}^{\infty} P(m)\,dm = 1. \tag{11}$$

Now a cluster will not contribute to charge transfer if $v > v^*$ as both the charged particles from its decay will be emitted in the same hemisphere in the CM system. Further for $v < v^*$, a forward (backward) moving cluster will only contribute to the charge transfer if a charged particle is emitted in the angular interval $\cos^{-1}(-v/v^*) < \theta^* < \pi$ $(0 < \theta^* < \cos^{-1} v/v^*)$. Call Ω_c the solid angle corresponding to these angular intervals; then the probability that one of the particles in the pair is emitted into Ω_c is $2\Omega_c/_{4\pi} = (1 - v/v^*)$. So the fraction, f, of the clusters which will contribute to the charge transfer is given by

$$f = \int_{2m_\pi}^{\infty} P(m)\,dm \int_0^{v^*} (1 - v/v^*)\,F(v)\,dv \tag{12}$$

where $v^* = (1 - 4m_\pi^2/m^2)^{1/2}$. So the average number of clusters which will contribute to charge transfer would be $f\triangle$. If we assume that the distribution in the number of central clusters, which contribute to charge transfer, is also a Poisson distribution with an average equal to $f\triangle$, then the analysis of section 3 goes through for the central clusters with $\triangle$ replaced by $f\triangle$. Thus the probability $p_{NL}^{(u)}$ for a charge transfer u due to non-leading particles alone is given by

$$P_{NL}^{(u)} = e^{-\triangle'} I_u(\triangle'), \tag{13}$$

with

$$\triangle' = f\triangle. \tag{14}$$

Of course a knowledge of $P(m)$ and $F(v)$ would enable us to calculate f. However in the absence of this knowledge we treat the fraction f as a parameter.

4 *b*. We now consider the problem of charge transfer from the leading clusters. To keep the analysis simple, we assume that a leading cluster has an average mass $\bar{M}$ and an average Lorentz factor $\bar{\gamma}$ (velocity $\bar{v}$) such that

$$2\bar{M}\bar{\gamma} = \bar{E}s^{1/2}, \tag{15}$$

where $\bar{E}$ is the average fraction of the incident energy retained by the two leading clusters. In models of independent cluster production, $\bar{M}$ is supposed to have a value ~ 2 to 3 GeV. Also, since $\bar{\gamma} \to 1$ as $s \to 4M^2$, one expects that $\bar{M}/\bar{E}$ should exhibit some energy dependence, so we write

$$\bar{M}/\bar{E} = M + M_0(1 - s/4M^2) \tag{16}$$

where M_0 is a constant and M is the proton mass.

The contribution of the leading clusters is likely to be important only at lower energies. Consequently we consider only the case when each leading cluster emits either one or three charged particles besides some neutral particles. So the different decay channels of a leading cluster, with one or two neutrals pions, are (i) p, (ii) $n\pi^+$ (iii) $p\pi^+\pi^-$ and (iv) $n\pi^+\pi^-\pi^+$. Let P_a be the probability for the emission of a π^+ accompanied by a neutron and let p_b be the probability for the emission of a $\pi^+\pi^-$ pair, from the decay of a cluster. Each leading cluster decays on the average into $\frac{1}{2}\langle n_L\rangle$ charged particles and this should be equal to average number of charge particles from the decays (i) to (iv) which is $(1 - p_b)$, $+ 3p_b$. This gives

$$p_b = \tfrac{1}{2} (\tfrac{1}{2} \langle n_L \rangle - 1). \tag{17}$$

To fix p_a we note that experimentally there is considerable evidence [Antinucci *et al* (1973)] to show that the average number of protons emitted in a *pp*-collision is $\sim 1 \cdot 4$. This implies that a leading cluster emits on the average 0·7 protons and 0·3 neutrons, giving $p_a = 0 \cdot 3$.

The protons from the leading clusters are always fast in the CM and will not contribute to the charge transfer. The pions emitted from the leading clusters would give rise to charge transfer. To calculate this, we associate an average velocity $\bar{v}^*$ (Lorentz factor $\bar{r}^*$) to the pions in the rest system of a leading cluster. A pion from a forward (backward) leading cluster will contribute to the charge transfer if $\bar{v}/\bar{v}^* < 1$ and the pion is emitted into the angular interval $\cos^{-1}(-\bar{v}/\bar{v}^*) < \theta^* < \pi$ $(0 < \theta^* < \cos^{-1} \bar{v}/\bar{v}^*)$. The solid angle $\bar{\Omega}$ corresponding to this interval is $2\pi(1 - \bar{v}/\bar{v}^*)$. For isotropic and uncorrelated emission, the probability that the pion is emitted into this solid angle $\bar{\Omega}$ is $\delta = \frac{1}{2}(1 - \bar{v}/\bar{p}^2)$. Now $\bar{v}^*$ can be determined in terms of $\bar{p}_T$, the average transverse momentum of the pions. For isotropic emission, the mean square momentum $\bar{p}^2 = 3/2\, \bar{p}_T{}^2$ while $\bar{p}_T{}^2 = 3/2\, (\bar{p}_T)^2$ for a p_T distribution of the form $p_T\, e^{-ap_T}\, dp_T$. One has therefore $m_\pi \bar{r}^* = (9/4\, \bar{p}_T{}^2 + m_\pi{}^2)^{1/2}$. Since experimentally $\bar{p}_T$ is known this fixes $\bar{v}^*$. By equations (15) and (16) δ becomes a known function of s in terms of one parameter M_0.

The pattern of decay for a leading cluster considered above can lead to the emission of charged pions into $\bar{\Omega}$ with a total charge $Q = 2, 1, 0, -1$. The probabilities $D^{(Q)}$ for emission of Q units of charge into $\bar{\Omega}$ are given by

$$D^{(2)} = xy(1 - z) \tag{18 a}$$

$$D^{(1)} = xyz + x(1 - y)(1 - z) + y(1 - x)(1 - z) \tag{18 b}$$

$$D^{(0)} = yz(1 - x) + xz(1 - y) + (1 - x)(1 - y)(1 - z) \tag{18 c}$$

$$D^{(-1)} = z(1 - x)(1 - y), \tag{18 d}$$

where $x = \delta\, p_a$ is the probability of emission of π^+ (from proton $\rightarrow \pi^+$ neutron) into $\bar{\Omega}$ while y and z are the probabilities for the emission of the π^+ and the π^-, from the $\pi^+\pi^-$ pair, into $\bar{\Omega}$. Clearly $y = z = \delta p_b$. The different combinations of charges emitted simultaneously by the two leading clusters (one backward and the other forward) can give rise to charge transfers up to 3 units. Denoting by $P_L{}^{(u)}$ the probability for charge transfer u by the leading clusters, one has

$$P_L{}^{(0)} = (D^{(0)})^2 + (D^{(1)})^2 + (D^{(-1)})^2 + (D^{(2)})^2, \tag{19 a}$$

$$P_L{}^{(1)} = P_L{}^{(-1)} = D^{(0)}\, D^{(1)} + D^{(0)}\, D^{(-1)} + D^{(1)}\, D^{(2)} \tag{19 b}$$

$$P_L^{(2)} = P_L^{(-2)} = D^{(0)} D^{(2)} + D^{(1)} D^{(-1)} \tag{19 c}$$

$$P_L^{(3)} = P_L^{(-3)} = D^{(2)} D^{(-1)}. \tag{19 d}$$

Since the leading and non-leading clusters are emitted independently the probability $P^{(u)} = 6^{(u)}/6_{\text{in}}$ for a net charge transfer u, due to both the leading and non-leading clusters, is

$$P^{(u)} = P^{(-u)} = \sum_{k=-3}^{3} p_{NL}^{(\mu-k)} \, p_L^{(k)} \tag{20}$$

and

$$\langle u^2 \rangle = \sum_{-\infty}^{+\infty} u^2 P^{(u)} = \langle u^2 \rangle_{NL} + \langle u^2 \rangle_L \tag{21 a}$$

where from (13), (14) and (19),

$$\langle u^2 \rangle_{NL} = \triangle' \tag{21 b}$$

$$\langle u^2 \rangle_L = 2P_L^{(1)} + 8P_L^{(2)} + 18 \, P_L^{(3)}. \tag{21 c}$$

5. Results and comparison with data

The equations (21) for the mean square charge transfer depends on two parameters. One is the fraction f of the average number of central clusters which contribute to charge transfer and determines $\triangle'$ through (14). We regard f to be independent of energy. The second parameter is M_0 which fixes the value of $\bar{v} = (1 - 1/r^2)^{1/2}$ through eqs (15) and (16). The value of $\bar{v}^*$ is related to $\bar{p}_T$, the average transverse momentum of pions from the leading clusters. The values of $\langle n_L \rangle$ occurring in eq. (17) have already been given by Chaudhary, Gupta and Narayan (1974) up to 300 GeV/c and are reproduced in table 1. The values of $\langle n_L \rangle$ and $\langle m_{NL} \rangle$ at 400 GeV/c are calculated using the parametrization:

$$\langle n_L \rangle = -2 \cdot 58 + 1 \cdot 24 \log s$$

$$\langle n_{NL} \rangle = 0 \cdot 368 + 0 \cdot 598 \, s^{1/4} \tag{22}$$

for energies above 50 GeV/c, given by the above authors.

To get a fit to the experimental data, the values of f and M_0 are fixed to be $0 \cdot 66$ and $1 \cdot 68$ GeV. Further we have used the value $\langle p_T \rangle = 0 \cdot 45$ to calculate the value of $\bar{v}^*$. This larger value of $\langle p_T \rangle$ would be consistent with experiment (Capilupp *et al* 1974) if the pions from the leading clusters are identified with the fragmentation component. The values of $\langle u^2 \rangle$ calculated from (21), using the above parameters, are given in table 1. With the same parameters, we have also calculated the cross-sections $6^{(u)}$ for different values of u at 102 GeV/c and 205 GeV/c, according to equation (20). The results of the calculation are shown in figure 2. A detailed test of the model lies in comparing the predictions of the model with the experimental data on the cross-sections $6_{2n}^{(u)}$ for a fixed charge transfer u as a function of the charged particle multiplicity $2n$. For this purpose, we consider the experimental data at 205 GeV/c [Derrick (1974)]. One notices from table 1 that the contribution of the leading clusters to charge transfer has become

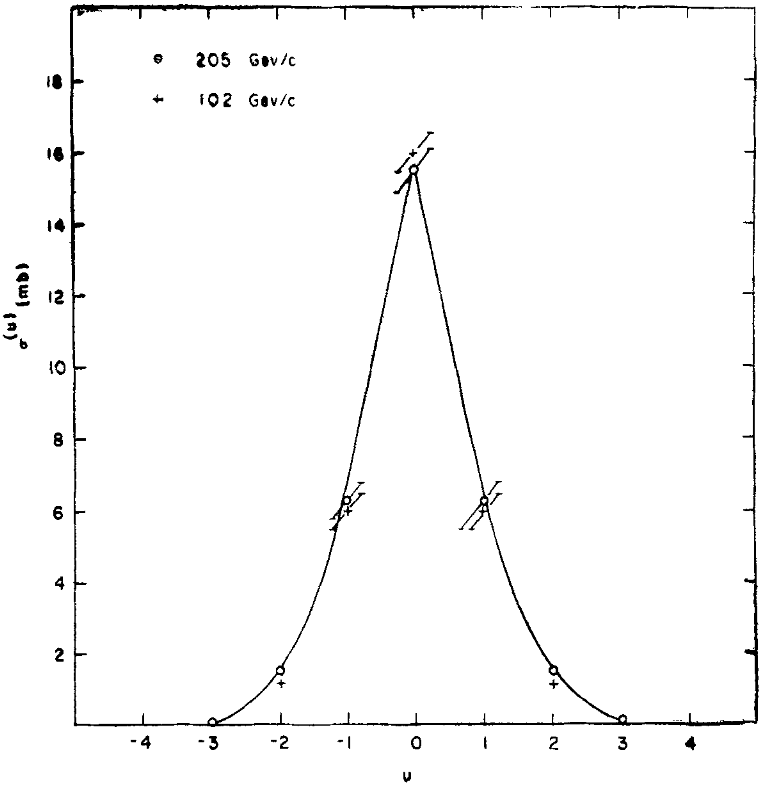

Figure 2. Charge transfer cross-sections $\sigma^{(u)}$. The solid curve is the predicted distribution at 205 GeV/c according to the model. The predicted distribution at 102 GeV/c almost coincides with that at 205 GeV/c and is not plotted for sake of clarity. The experimental points have been taken from Bromberg *et al.* (1973) and Cohen *et al* (1973).

negligible at 205 GeV/c and at higher energies. So it would be adequate to use expressions (10), wherein $\triangle$ is replaced by $\triangle'$, to calculate $\sigma_{2n}^{(u)}$ at 205 GeV/c. The results of these calculations are presented in figure 3 along with the experimental data.

For the particular case $n = u$, the expressions (10) would give the cross-sections $\sigma_{2u}^{(u)}$ to be identically zero. Experimentally the cross-sections $\sigma_{2u}^{(u)}$ are, in fact, quite small. One also notices a drop by a factor $\sim$20 in the experimental values of the cross-sections in going from $\sigma_{2u}^{(u-1)}$ to $\sigma_{2u}^{(u)}$. This drop is understood in our model as being due to the absence of non-leading particles, which are, mostly responsible for charge transfers at 205 GeV/c, in the configuration, charged multiplicity $= 2u$ and charge transfer $= u$. We make an estimate of the cross-sections $\sigma_{2u}^{(u)}$ by calculating the contribution of the leading clusters according to section (4) and equations (18). We have also given these estimated values for $\sigma_{2u}^{(u)}$ in figure 3.

As one can see, our results in table 1 and figures 2 and 3 are generally in good agreement with experiment.

6. Asymptotic behaviour of $\langle u^2 \rangle$ and concluding remarks

As the charge transfer due to leading clusters goes to zero at high energies, the

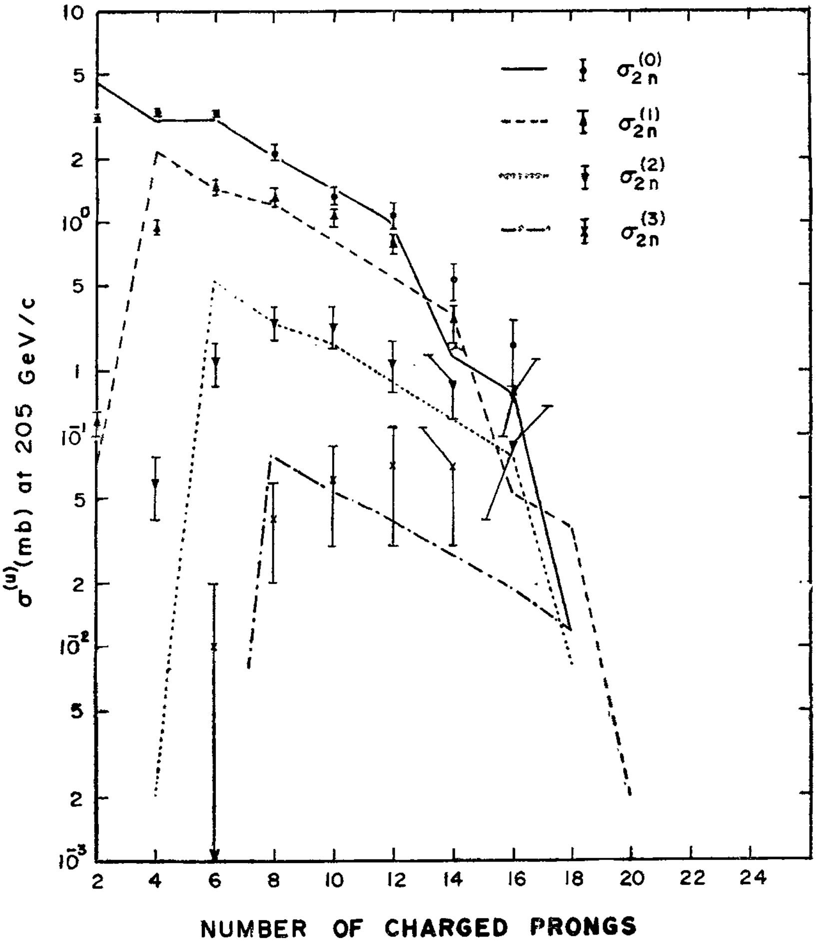

Figure 3. Predicted cross-sections for a given charge transfer as a function of the charged prong multiplicity at 205 GeV/c. Experimental points are from Derrick (1974).

asymptotic behaviour of $\langle u^2\rangle$ is the same as the asymptotic behaviour of $\langle u^2\rangle$ due to the central clusters. According to equations (21) and t he parametrization of $\triangle = \langle n_{NL}\rangle/2$ in equation (22), one finds that $\langle u^2\rangle \sim s^{1/4}$ for $s\to\infty$. An alternative parametrization of $\langle n_{NL}\rangle = a + b\log s$ $(a = -1{\cdot}87,\ b = 0{\cdot}863)$ was given by Chaudhary, Gupta and Narayan. In this case $\langle u^2\rangle \sim \log s$ for $s\to\infty$. In any case, one obtains an indefinite slow increase of $\langle u^2\rangle$ in the present model. This result may be compared with the multiperipheral picture (Quigg and Thomas 1973) in which $\langle u^2\rangle$ tends to a constant asymptotically.

The difference in the asymptotic behaviors of $\langle u^2\rangle$ in the two models is intimately related to the differences in the rapidity distribution of the clusters around $y = 0$. In the present model, the velocity distribution of the central clusters is independent of s but their average number increases as $\log s$ or $s^{1/4}$. This results as we have seen, in an increase of $\langle u^2\rangle$ with s in the same manner. In the multiperipheral model, the distribution of clusters develops at high energies a plateau of constant height at $y = 0$. One finds that $\langle u^2\rangle$ also tends asymptotically to a constant. It seems plausible that, in an independent cluster production model, the inclusive charged particle cross-section $d6/dy$ at $y = 0$ and $\langle u^2 \rangle$ would have the same asymptotic behaviour.

The present experimental evidence (Bromberg *et al* 1975) does not suggest an approach to a constant value for (u^2) at FNAL energies. In this connection, the recent measurements (The British-MIT-Scandinavian Collaboration (1975) at CERN on the pion inclusive cross-section are of great interest. It has been found that the inclusive cross-section ratios R, as a function of the CM energy, relative to the lowest ISR data, show an approximate linear rise of about $(40 \pm 5)/\%$ between the lowest and the highest values of $s^{1/2}$. It would be very interesting to see experimentally if $\langle u^2 \rangle$ also shows a similar increase.

Acknowledgement

We would like to thank M. Derrick for providing us with data on charge transfer in *pp* collisions at 205 GeV/c.

References

Antinucci M *et al.* 1973 *Nuovo Cimento Lett.* **6** 121
Bishari M, Horn D and Nussinov S 1972 *Nucl. Phys.* **B36** 109
British-MIT-Collaboration H Boggild *et al.* 1975 Paper submitted to the Palermo Conference
Bromberg C *et al.* 1973 Preprint UMBC 73-19, UR-460
Bromerg C *et al.* 1975 Preprint UMBC 75-1 Coo-3065-111
Capiluppi P *et al.* 1974 *Nucl. Phys.* **B70** 1
Chavda L K and Narayan D S 1974 *Phys. Lett.* **51B** 359
Chaudhary B S, Gupta V and Narayan D S 1974 *Pramāṇa* **2** 73
Chow T T and Yang C N 1973 *Phys. Rev.* **D7** 1425
Cohen D 1973 Rochester University Preprint UR-451
Derrick M 1974 Private Communication of data on charge transfer in *pp* collisions at 205 GeV/C
Fialkowski K 1972 *Phys. Lett.* **41B** 379
Fujiwara K and Kitazoe T 1970 *Prog. Theor. Phys.* **43** 1244
Harari H and Rabinovici E 1973 *Phys. Lett.* **43B** 49
Lach J and Malamud E 1973 *Phys. Lett.* **44B** 474
Narayan D S 1971 *Nucl. Phys.* **B34** 386
Pokorski S and Van Hove L 1974 *Acta Phys. Pol.* **B5** 229
Quigg C and Jackson J D 1972 NAL-THY-93
Quigg C and Thomas G 1973 *Phys. Rev.* **D7** 2752
Van Hove L 1973 *Phys. Lett.* **43B** 65
Wilson K G 1970 Cornell Preprint CLNS-131