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Phenomenological effective interaction for ^{40}K

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Abstract. Effective interactions of various forms incorporating central, spin-orbit and tensor dependences of two-nucleon potential are parametrized so as to give a satisfactory description of energy levels of ^{40}K. These parameters are applied to calculate energy levels of ^{38}Cl. Except for the lowest 3^- level, the agreement is satisfactory.

Keywords. Phenomenological effective interaction; ^{40}K; ^{38}Cl.

1. Introduction

The nuclear two-body effective interaction may be either the one generated through infinite perturbation expansion in terms of the basic two-nucleon interaction or it may be a phenomenological effective interaction. In order that a satisfactory determination of the parameters of a phenomenological effective interaction may be possible, one should be able to assign simple, pure configurations to experimental levels. In most cases this is not possible since the levels involve significant configuration mixing. In view of this, the ^{40}K and ^{38}Cl nuclei present a favourable example for parametrizing a phenomenological effective interaction. Goldstein and Talmi (1956) observed that if the same two-body matrix elements of the interaction were assumed for the two nuclei, the energy levels of one could be predicted if those of the other were known. The observation that, even on assuming only two active orbits $0d_{3/2}$ and $0f_{7/2}$—the levels so predicted agreed very well with experimental ones, also resulted in the conclusion that the first four levels of ^{40}K arise from almost pure configurations. It was later shown by Pandya (1956) that the matrix elements of two-body interaction occurring in the spectroscopic study of ^{40}K and ^{38}Cl are related through a transformation involving only Racah coefficients. Bass and Wechsung (1970) have also shown that the low-lying levels of ^{40}K can be represented by almost pure configurations. Thus, if the parameters of a phenomenological effective interaction are determined for ^{40}K, the same can be applied for calculation of energy levels of ^{38}Cl.

While formulating the form of the phenomenological potential, one would like to have the number of parameters as small as possible. However, it may not always be a good approximation and for getting the correct description one may have to include more parameters in the potential representing several qualitative

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features of the two-nucleon potential. In search of a best set of parameters for ^{40}K, we have parametrized phenomenological potentials of the following forms:

(i) central, (ii) central + spin-orbit, (iii) central + tensor,

(iv) central + spin-orbit + tensor.

2. Method of calculation

The ^{40}K nucleus has a proton hole in $0d_{3/2}$ orbit while the ^{38}Cl nucleus has a proton in the same orbit. Both these nuclei have a neutron in $0f_{7/2}$ orbit. The particle-hole matrix elements are calculated using Pandya transformation. The experimental values of single particle (hole) energies (Goode and Zamick 1969) used in the present work are shown in table 1.

Although a model space allowing the hole to be anywhere in sd-shell and the particle to be anywhere in fp-shell would be desirable, yet, in view of available information about wave functions, we feel it would be reasonable enough to choose the smaller model space:

hole states: $0d_{3/2}$, $1s_{1/2}$; particle states: $0f_{7/2}$, $1p_{3/2}$.

In table 2 we summarize the experimental information (Johnson and Kennett 1970, Fink and Schiffer 1974, Endt and Van der Leun 1973) about levels of ^{40}K together with the prospective predominant configurations in the actual wave functions.

The assignment to levels 8 and 10 is not expected to be very satisfactory since the unperturbed energy itself of the $1s_{1/2}^{-1}$ $0f_{7/2}$ configuration is 2·5 MeV (ex-

Table 1. Experimental single particle (hole) energies

Orbit	..	$1s_{1/2}$	$0d_{3/2}$	$0f_{7/2}$	$1p_{3/2}$
Energy (MeV)	..	−9·7	−7·2	0·0	1·9

Table 2. Adopted level energies and prospective dominant configurations in the structures of respective levels of ^{40}K

Level number	J^π	E_x (MeV)	
1	4_1^-	0	$0d_{3/2}^{-1}\,0f_{7/2}$
2	3_1^-	0·0296	$0d_{3/2}^{-1}\,0f_{7/2}$
3	2_1^-	0·8000	$0d_{3/2}^{-1}\,0f_{7/2}$
4	5_1^-	0·8920	$0d_{3/2}^{-1}\,0f_{7/2}$
5	2_2^-	2·0473	$0d_{3/2}^{-1}\,1p_{3/2}$
6	3_2^-	2·0693	$0d_{3/2}^{-1}\,1p_{3/2}$
7	1_1^-	2·1043	$0d_{3/2}^{-1}\,1p_{3/2}$
8	3_3^-	2·4575	$1s_{1/2}^{-1}\,0f_{7/2}$
9	2_3^-	2·4191	,,
10	4_2^-	2·3976	$1s_{1/2}^{-1}\,0f_{7/2}$
11	0_1^-	2·6260	$0d_{3/2}^{-1}\,1p_{3/2}$

cluding the 7.2 MeV separation between the $0d_{3/2}$ and $0f_{7/2}$ orbits which is common to all diagonal matrix elements and, hence, irrelevant for calculation of excitation energies relative to the ground state) while each of the 3_3^- and 4_2^- levels appears at an excitation energy less than 2·5 MeV. This is possible only if the particle-hole interaction in this configuration is strongly attractive so that the unperturbed matrix elements of the Hamiltonian for both these J states fall well lower than 2·5 MeV since configuration mixing will send them up again. The unperturbed positions of the $0d_{3/2}^{-1}\ 1p_{3/2}$ and $1s_{1/2}^{-1}\ 0f_{7/2}$ configurations are quite close—1·9 MeV and 2·5 MeV respectively. As such, we anticipate the approximation of assigning pure configurations to 3_2^- and 3_3^- levels to be somewhat restricted. In comparison to the 2_2^- level energy, the 2_3^- level appears at too low an excitation energy to be accounted for by some simple configuration like $(0d_{3/2}^{-1}\ 1p_{1/2})$, $(1s_{1/2}^{-1}\ 1p_{3/2})$ or $(0d_{5/2}^{-1}\ 0f_{7/2})$. We do not include it in the calculations.

First we choose the residual interaction as the pure central force

$$V_c(r) = \sum_{T,S} {}^{2T+1,\ 2S+1}P A_{TS}\ e^{-(r^2/r_{0S}^2)} \tag{1}$$

where ${}^{2T+1,\ 2S+1}P$ is the projection operator that projects out states of isospin T and spin S, A_{TS} is the strength in the particular (T, S) channel of interaction and r_{0S} is the range in the spin state S, *i.e.*, we assume an isospin-independent range. Matrix elements of V_c (r), as a function of λ $(= r_0/\sqrt{2}b)$, are calculated using Moshinsky brackets (Brody and Moshinsky 1967). We take $b = 2{\cdot}0$ fm for ^{40}K. Thus, after λ is determined, the range can be calculated from

$$r_{0S} = 2\sqrt{2}\,\lambda_S \tag{2}$$

There are six parameters specifying the central force. However, we have been able to reduce the effective number of unknown parameters to four since we have an idea of the upper and lower limits for the nuclear force range r_0. We let λ_S (for both $S = 0$ and $S = 1$) vary independently between 0·3 and 1·5. Thus, for a given (λ_0, λ_1) set, the number of unknown parameters is reduced to four. We assume that in first approximation the excitation energy of a level can be given by the difference between the diagonal matrix element in that particular state and the diagonal matrix element of the Hamiltonian for the ground state 4^- level. For every (λ_0, λ_1) set, we minimise the function

$$\chi^2 = \sum_i \frac{(E_{i,\,\mathrm{calc}} - E_{i,\,\mathrm{exp}})^2}{E_{i,\,\mathrm{exp}}}$$

for five excitation energies. The $E_{i,\,\mathrm{calc}}$'s are the calculated excitation energies corresponding to the respective experimental excitation energies $E_{i,\,\mathrm{exp}}$'s. For a given (λ_0, λ_1) set, $E_{i,\,\mathrm{calc}}$'s are linear functions of all the four A_{TS}'s. The unknown strengths are determined by imposing mathematical restriction of χ^2 being minimum with respect to each A_{TS}. The set of strengths which gives the least value of χ^2 is thus singled out as the best set.

However, true energy levels require an exact diagonalization to be made. A set of parameters may give a least value of χ^2 in case of pure configurations. However, χ^2 obtained on diagonalization with this parameter set may not be the minimum obtainable χ^2. Still, in view of the earlier observation that this nucleus is a case

of pure enough configurations we feel that the parameter set that gives minimum χ^2 on diagonalization would not be very much different from the one that gives minimum χ^2 for pure configurations. Therefore, besides the set which gives minimum χ^2 for pure configurations, we also pick up other few which give a χ^2 value not very much different from $\chi^2_{\min}$. The Hamiltonian matrices are then set up for all J states in the model space $(1s_{1/2}^{-1}\ 0d_{3/2}^{-1}\ 0f_{7/2}\ 1p_{3/2})$ for each parameter set separately and diagonalized. The value of χ^2 is calculated again for the same five excitation energies, this time with calculated perturbed energy eigenvalues, with each parameter set. The set which gives a minimum value of χ^2 is thus taken as the best representative of effective interaction parameters for the five excitation energies. It turns out that the parameter set which gives a least χ^2 does not also describe other excited levels equally well as can be done by sacrificing the value of χ^2 by a small amount. We, finally, select only that set which gives an overall best agreement with levels' excitation energies.

After this we determine the parameters of the two-body effective interaction of the form — central + spin-orbit. The two-body matrix elements of the spin-orbit force

$$V_{LS}(r) = \sum_T {}^{2T+1,3}PS_T\, \vec{l}\,.\,\vec{S}\, e^{-(r^2/r_{LS}^2)} \tag{3}$$

are calculated as prescribed by Brody and Moshinsky (1967). We assume isospin dependence of the spin-orbit force strength S_T. Thus, the inclusion of spin-orbit part increases the number of unknown parameters from four to six and therefore two more experimental energies are needed. We let λ' defined through

$$r_{LS} = 2\sqrt{2}\,\lambda' \tag{4}$$

vary from 1·1 to 1·355. For each set of λ_0, λ_1 and λ' the six strength parameters are determined through the least square fitting procedure described above.

The procedure is repeated for parametrizing the effective potential of the form central + tensor. The form of the tensor force is

$$V_{\text{Ten}} = \sum_T {}^{2T+1,3}PS'_T \left[\frac{3(\vec{\sigma}_1\,.\,\vec{r})(\vec{\sigma}_2\,.\,\vec{r})}{r^2} - \vec{\sigma}_1\,.\,\vec{\sigma}_2\right] e^{-(r^2/r_{\text{Ten}}^2)} \tag{5}$$

where isospin (T) dependence of the strength S'_T is assumed.

We take the 4_1^-, 3_1^-, 2_1^-, 5^-, 1_1^- and 0_1^- levels for determination of pure central force parameters. For inclusion of spin-orbit and tensor parts we consider the 2_2^- and 3_2^- levels also.

Lastly, we parametrize the effective potential in the form centsal + spin-orbit + tensor. Thus, there are eight strength parameters requiring a knowledge of nine excitation energies. However, only eight excitation energies were available (the 4_2^- level was not established at the time these calculations were performed). The 3_3^- level at 2·4575 MeV was the only other, hitherto unused, level which could be used with some reasonable degree of definiteness as regards the main component in the wave function. Therefore, we had to resort to one more approximation. We assumed spin-isospin-dependence of strength and spin-dependence of range of central part (four parameters). We also assumed isospin-dependence of

strength of spin-orbit part (two parameters more). However, we assumed that the isospin-dependence of the strength of the tensor force can be absorbed by taking ranges in the two isospin states to be different and assuming the same strength in both the isospin states. Like this, the calculated energy level becomes a linear function of seven strength parameters which can now be determined from a knowledge of eight excitation energies through the least square fitting procedure. Thus, for this part of the calculation we assume the following form of the tensor force,

$$V_{\text{Ten}} = \sum_{T} {}^{2T+1,\,3}PS'' \left[\frac{3(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r})}{r^2} - (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \right] e^{-(r^2/r^2_{\text{Ten},T})} \tag{6}$$

where each of $r_{\text{Ten},0}$ and $r_{\text{Ten},1}$ is varied independently.

Twelve energy levels for ^{40}K are expected to arise within the chosen model space. Out of these, we have used nine for determination of parameters. As discussed earlier, a serious attempt of comparison of calculated 4_2^- with experimental 4_2^- level should not be made. In the energy region around 4·5 MeV, where we expect the 1_2^- and 2_3^- levels arising from the $(1s_{1/2}^{-1}\,1p_{3/2})$ configuration to lie, the spin assignments are uncertain and there are several levels suspected to be 1^- and 2^-. Hence, a reasonable comparison of predicted energies of these states is not possible.

We calculate the energy levels of ^{38}Cl nucleus in the model space:

proton: $0d_{3/2}$; neutron: $0f_{7/2}$, $1p_{3/2}$.

The experimental energy levels (Endt and Van der Leun 1973) of ^{38}Cl are as shown in table 3.

The lowest four levels are precisely the ones that would be expected from the $0d_{3/2}0f_{7/2}$ configuration. However, appearance of two low-excited 3^- levels makes it much doubtful if the simple two-particle model will work for them as it is expected to be for the lowest four levels. We shall only see how well are the lowest four levels of ^{38}Cl predicted with an effective interaction parametrized for ^{40}K.

The ^{38}Cl nucleus is one with neutron excess. We write a total angular momentum (J) coupled proton-neutron state as $|\,a(p)\,b(n)\,J\rangle$ where the proton is in the single particle configuration a and the neutron is in the single particle configuration b. This state is obtained from states of well defined isospins through

$$|\,a(p)\,b(n)\,J\rangle = \frac{1}{\sqrt{2}}\,[\,|\,abJT\rangle_{T=1} + |\,abJT\rangle_{T=0}].$$

Therefore, a proton-neutron interaction matrix element will be

$$\langle\, a(p)\,b(n)\,J \mid V \mid c(p)\,d(n)\,J\,\rangle$$
$$= \tfrac{1}{2}\,[\langle\, abJT \mid V \mid cdJT\,\rangle_{T=1} + \langle\, abJT \mid V \mid cdJT\,\rangle_{T=0}]$$

Table 3. Experimental energy levels of ^{38}Cl

J^π ..	2^-	5^-	3^-	4^-	3^-	3^-
Energy (MeV) ..	0	0·6713	0·7553	1·3089	1·6172	2·7431

Therefore, the particle-particle matrix elements for $T = 0$ and $T = 1$ states (which have been calculated above), for the appropriate configurations in the model space as considered for ^{38}Cl, are picked up and the Hamiltonian matrices for different J states constructed by taking average of corresponding matrix elements in $T = 0$ and $T = 1$ states which are then diagonalized to get the energy levels of ^{38}Cl.

3. Results and Discussions

As discussed above, six experimental levels were used for parametrization of the pure central force and there remained three more levels to be predicted with the parameters which gave best fit (*i.e.*, minimum χ^2) for the six levels used. Therefore, it was only for the central part that the six levels were best reproduced for the (λ_0, λ_1) set at values (0·39, 1·11) while the overall agreement with the experimental spectrum including the other excited states also was obtained for the values (0·38, 1·14). Otherwise, for the parametrization of the effective potential in other forms, all the available information was utilized in determining the potential parameters. The values of the parameters are presented in table 4. The spectra calculated with these parameters are presented in figure 1.

Table 4. Effective interaction parameters for ^{40}K for different forms of the potential. For explanation of notation refer to eqs (1) through (6). The pure central force parameters are those for the (λ_0, λ_1) set (0·38, 1·14). The strength parameters of the tensor part at S. Nos. 9 and 10 are not in exact conformity with eq. (6). The two strengths are equal in magnitude but opposite in sign. This arises due to the l-dependent phase factor in the expression for the two-body matrix elements of tensor force (Brody and Moshinsky 1967). As such, to differentiate, we have further subscripted S″ of eq. (6) by isospin.

Sl. No.	Parameters \ Interaction		Central	Central + spin-orbit	Central + tensor	Central + spin-orbit + tensor
1	A_{00}	(MeV)	−218·7	−694·9	1337·5	590·8
2	A_{01}	,,	− 20·0	− 30·2	19·4	6·1
3	A_{10}	,,	− 89·7	− 39·0	− 944·5	−342·5
4	A_{11}	,,	− 13·5	− 4·2	−50·8	−35·9
5	S_0	,,		− 1·1		−1·9
6	S_1	,,		− 8·3		11·4
7	S_0'	,,			− 1·5	
8	S_1'	,,			5·5	
9	S_0''	,,				−0·3
10	S_1''	,,				0·3
11	r_{00}	(fm)	1·0748	1·103	0·8485	1·103
12	r_{01}	,,	3·2243	3·1113	3·1113	3·1113
13	r_{LS}	,,		3·4932		3·1962
14	r_{Ten}	,,			3·1537	
15	$r_{Ten,0}$	,,				3·1113
16	$r_{Ten,1}$	,,				3·1962

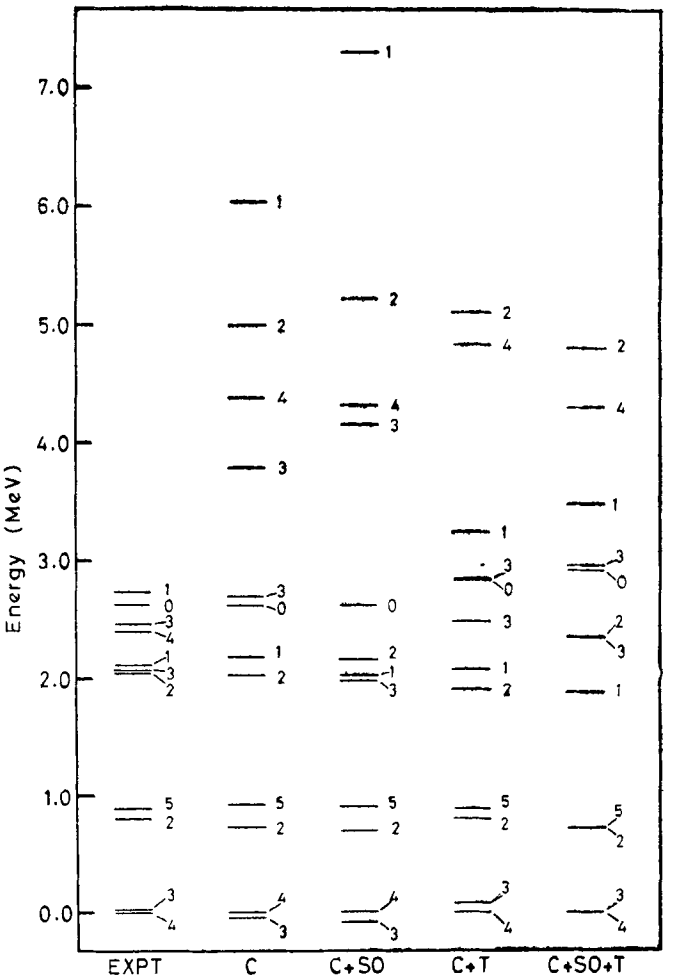

Figure 1. Experimental and the twelve calculated energy levels of ^{40}K as arising in the model space considered-predicted for different forms of the effective potential. Levels fitted with a pure central force are denoted by C, those with central + spin-orbit form by C + SO, those with central + tensor form by C + T and those with central + spin-orbit + tensor form by C + SO + T.

It is observed that the addition of tensor force substantially worsens the value of χ^2 for levels predicted under the assumption of pure configurations. The energy of 3_1^- state is 0·0296 MeV and, therefore, even small deviations in this level energy contribute largely to χ^2. The largest effect that configuration mixing does to this nucleus is to reverse the ordering of 4_1^- and 3_1^- levels, thus predicting 3_1^- to be the ground state J^π. It is because of this factor that χ^2 becomes much worse on diagonalization. It is only on the addition of the tensor force that the correct ordering of the 4_1^- and 3_1^- levels is achieved and the value of χ^2 for configuration mixed levels is also the least for the central + tensor form. In view of this it can be said that presence of the tensor force is a must as a part of effective interaction for this nucleus, to get the correct ordering of the $0d_{3/2}^{-1}\ 0f_{7/2}$ configuration levels at least, although correct excitation energy of the 3_1^- level is not reproducible.

It can be seen that the spin-orbit and tensor forces act in opposite directions for the 3_1^- level. Addition of the spin-orbit term to the central part further lowers the 3_1^- level. On the other hand, addition of the tensor part to the central part pushes up this level to the extent that the correct ordering of the 4_1^- and 3_1^- levels is achieved with the 3_1^- level appearing at 0.0738 MeV. Addition of both the spin-orbit and the tensor parts to the central part causes an elevation in the 3_1^- level position, from a value obtainable otherwise with the pure central force, which is much smaller than that obtained with the central + tensor form. Thus, with the central + spin-orbit + tensor form, the 3_1^- level is predicted only at 0·0013 MeV and the overall agreement with experimental spectrum is also worst for this form. The discrepancy may be largely attributed to the fact that the 3_3^- level had to be considered for determination of parameters.

In table 5 we present the wave functions for 3^- states as obtained with central + tensor and central + spin-orbit + tensor forms of the effective potential. With the central + tensor form of potential, the 3_1^- level is relatively pure but the 3_2^- and 3_3^- levels are heavily mixed with each other. Such large configuration mixing is not observed in any of the other levels. Since the effective potential of the form central + spin-orbit + tensor has been parametrized under the assumption of pure configuration for the 3_3^- level, the wave function for this level is more pure in respect of the $1s_{1/2}^{-1}\ 0f_{7/2}$ configuration than the corresponding structure for the effective potential of the form central + tensor. However, the 3_3^- level for the central + spin-orbit + tensor effective potential is in worse agreement with experiment than the 3_3^- level obtained with the central + tensor effective potential and we have seen that the spectrum is also overall worst reproduced in former case.

Energy levels of ^{38}Cl as calculated with different effective potentials as determined for ^{40}K are presented in table 6. It is clearly seen that the 3^- level position is in worst agreement—the effective potential of any form predicts 3^- as the ground state J^π instead of 2^- as experimentally observed. The 3^- predicted level is closest to zero for the effective potential of the form central + spin-orbit. However, we may recall that this potential is also the one which gives the worst agreement for the 3_1^- level in ^{40}K. The central + tensor potential predicts the correct ordering of 4_1^- and 3_1^- levels in ^{40}K but for ^{38}Cl, this form predicts a deepest 3^- level. These observations regarding the behaviour of 3^- level are in conformity with those of

Table 5. Wave functions for 3^- levels of ^{40}K

Interaction	Predicted excitation energy (MeV)	$0d_{3/2}^{-1}\ 0f_{7/2}$	$0d_{3/2}^{-1}\ 1p_{3/2}$	$1s_{1/2}^{-1}\ 0f_{7/2}$
Central + tensor ..	0·0738	0·9833	−0·1291	0·1286
	2·4789	0·0324	0·8183	0·5739
	2·8378	0·1793	0·5601	−0·8088
Central + spin-orbit + tensor ..	0·0013	0·9835	−0·0341	0·1776
	2·3569	0·0485	0·9958	−0·0774
	2·9606	0·1742	−0·0847	−0·9811

Table 6. Level excitation energies of ^{38}Cl predicted for different forms of the effective potential with parameters as those given in table 4. To facilitate comparison with experiment, all excitation energies have been calculated relative to the 2^- level.

J^π	Experimental (Mev)	Predicted energies (in MeV)			
		Central	Central + spin-orbit	Central + tensor	Central + spin-orbit + tensor
2^-	0	0	0	0	0
5^-	0·6713	0·7147	0·7294	0·5432	0·3797
3^-	0·7553	−2·7804	−0·8181	−7·0509	−6·3300
4^-	1·3089	1·3423	1·3560	1·1639	1·0188

Fink and Schiffer (1974) who find that an improved value of $(0d_{3/2}\ 0f_{7/2})$ two-particle matrix element in ^{38}Cl predicts a worse agreement for 3^- level in ^{40}K. However, except for the 3^- level, the ordering of the lowest 2^-, 5^- and 4^- levels is correctly reproduced and the excitation energies of 5^- and 4^- levels are in impressive agreement with corresponding experimental values. Nothing much should be expected of the form central + spin-orbit + tensor for reasons discussed above. However, the form central + tensor also does not reproduce the excitation energy of the 5^- level as well as is done by central and central + spin-orbit forms. Best agreement, excepting the 3^- level, with experimental spectrum is achieved with the central effective potential.

4. Conclusions

In the present work, we have parametrized phenomenological effective interactions of different forms for ^{40}K nucleus. For forms other than the central form of the interaction, it has not been possible to check the parameters of the interaction by applying them for prediction of higher excited states of ^{40}K and, thus, see their agreement with experimental ones. However, the reasonable agreement of the predicted ^{38}Cl levels (other than the 3^-) puts a check on the validity of these parameters. We could not perform a true χ^2 test with perturbed eigenvalues. We had to resort to approximating the diagonalized eigenvalues by diagonal matrix elements. As was anticipated (also, Bass and Wechsung 1970), and as it is clear from table 5 also, this approximation is valid for the 3_1^- level but not for the 3_2^- and 3_3^- levels. The position of the 3_1^- level is sensitive to presence, or otherwise, of the tensor force as a component of effective interaction. It is observed that the spectra of ^{40}K and ^{38}Cl are correlated in such a way that an improvement in levels of ^{40}K causes those of ^{38}Cl to become worse. None of the interactions considered could predict the correct position of the 3^- level in ^{38}Cl.

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