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(π^+, p) Reaction in ^{12}C

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Abstract. Hartree-Fock wave functions obtained from realistic nucleon-nucleon interaction are employed to calculate cross-sections for the reaction $^{12}C\,(\pi^+, p)\,^{11}C$ (g.s.). These wave functions take into account central correlations between nucleons inside the nucleus. This itself is found to change the cross-section by more than an order of magnitude. The incoming pion is represented by a plane wave while proton-distortion is taken into account in the high-energy or semi-classical approximation, thereby determining the proton optical well parameters. These values agree well with those obtained by more conventional methods. Variation of the cross-section with the oscillator well parameter is also studied. Calculations have been made using the one-nucleon mechanism for the pion-absorption process.

Keywords. Cross section $^{12}C\,(\pi^+, p)^{11}C$ (g.s.); Hartree-Fock wave functions; high energy scattering approximation; proton square optical well parameters.

1. Introduction

The last few years have seen many advances in our understanding of the two related reactions (π^+, p) and (p, π^+) on a variety of nuclei and at different energies. Their importance in studying nuclear structure is well known. The first attempt to calculate cross-section for the reaction $^{12}C\,(\pi^+, p)^{11}C$ was made by LeTourneux *et al* (1966). In their calculation the pion and proton were described by plane waves and the bound neutron was assumed to be moving in a harmonic oscillator (HO) well. The resulting cross-sections were almost an order of magnitude smaller than the experimental values (Witten *et al* 1968) and were extremely sensitive to the HO well parameter. The first calculation that took the pion- and proton distortion into account was done by Jones *et al* (1970). They employed optical model wave functions for the pion and proton whereas the bound neutron wave functions were generated by Woods-Saxon potential. Their results, however, were about two orders of magnitude higher than the experimental values. Similar results were obtained by Keating *et al* (1973) for (p, π^+). In the DWBA calculation of Rost *et al* (1973) the π-N coupling constant f was treated as an adjustable parameter and it was varied to fit the measurements of the reaction $^{12}C\ (p, \pi^+)\ ^{13}C$. The relative (π^+, p) and (p, π^+) cross-sections were explained when the normalization factor was 80 or 160 depending on the diffuseness parameter of the Woods-Saxon well. The pion-nucleus optical potentials used by Jones *et al* (1970) and Keating *et al* (1973) were of Kisslinger or Kroll-Kisslinger type. Recently Miller *et al* (1974) have used π-nucleus optical

potential obtained from π-nucleon collision matrices with non-divergent off-shell behaviour and they find this to be reasonably consistent with both the elastic π-nucleus scattering and the π-absorption data.

It is to be noted that none of these calculations takes into account nucleon-nucleon (N-N) correlations which were found to increase the (π^+, p) cross-section by more than an order of magnitude (Kaushal *et al* 1970 *b*). Here we present the results of a calculation which takes into account N-N correlations in the Hartree-Fock (HF) theory as well as distortion of the outgoing proton in the high-energy approximation.

2 Theory

In the first order time-dependent perturbation theory the differential cross-section for the (π^+, p) reaction is given by

$$\frac{d\sigma}{d\Omega} = \frac{M}{(2\pi)^2} E\pi \frac{k}{q} \sum_f |\langle f | H' | i \rangle|^2, \tag{1}$$

where the symbols i and f stand for the initial and final states of the system and H' is the usual non-relativistic *ps-pv* interaction vertex:

$$H' = \frac{f}{\mu} \sum_{i=1}^{A} \left[\left(1 - \frac{\mu}{2M}\right) \vec{\sigma}\cdot\vec{\Delta}_\pi \, \vec{\tau}\cdot\vec{\phi} - \frac{\mu}{M} \vec{\tau}\cdot\vec{\phi}\,\vec{\sigma}\cdot\vec{\Delta}_N \right]_i \tag{2}$$

and k (q) is the momentum of the proton (pion), E_π the pion energy and M the nucleon mass. In eq. (2) f is the pion-nucleon coupling constant, μ is the pion mass and rest of the symbols have their usual meanings (*see e.g.*, Jones *et al* 1970). For the initial state we assume the nucleons to be moving in spherical *HF* orbits:

$$\begin{aligned} |i\rangle &= |qn'ljm\rangle_{HF} = \sum_n C^l_{n'n} |qnljm\rangle_{HO} \\ &= \sum_n \sum_{\lambda\Lambda} C^l_{n'n} C^{l\frac{1}{2}j}_{\lambda\Lambda m} |qnl\lambda\rangle \chi_{\frac{1}{2}\Lambda} \end{aligned} \tag{3}$$

where $C^l_{n'n}$ are mixing coefficients (Kakkar 1969) which introduce central correlations between nucleons in the nucleus, $C^{l\frac{1}{2}j}_{\lambda\Lambda m}$ is the Clebsch-Gordan coefficient and $\chi_{\frac{1}{2}\Lambda}$ the spin-part of the nucleon wave function. The final state $|f\rangle = |k\Lambda'\rangle$ is just $e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \chi_{\frac{1}{2}\Lambda'}$ if proton-nucleus scattering is neglected. We, however, include this effect as follows. Taking into consideration the high energy ($\sim$ 160 meV in the present case) of the outgoing proton we treat its distortion in the high-energy or semi-classical approximation (Schiff 1956, Glauber 1959) using square well optical potential of radius R_0 and strength $V = -(U_0 + iW_0)$. It has been shown (Yu 1966) that this in effect modifies the momentum of the proton as

$$\boldsymbol{k} \to \boldsymbol{k}' \equiv \boldsymbol{k}\left(1 + \frac{U_0 M}{k^2} - i\frac{W_0 M}{k^2}\right),$$

and reduces the cross-section by a factor of $\exp(-2W_0 M\bar{R}/k)$ where $\bar{R} = \langle (R_0^2 - r^2 \sin^2\theta)^{\frac{1}{2}} \rangle = 3R_0/4$, θ being the angle between the vectors $\boldsymbol{k}$ and $\boldsymbol{r}$.

With this modification the matrix element in eq. (1) can be shown to be

$$\langle f \mid H^{\pi N} \mid i \rangle = \frac{if}{\mu \sqrt{E_\pi}} \frac{\sqrt{3}}{2} \sum_{n\lambda\Lambda\nu} (-)^\nu Q_{-\nu} C^l_{n',n} C^{l\frac{1}{2}j}_{\lambda\Lambda m} C^{\frac{1}{2}1\frac{1}{2}}_{\Lambda\nu\Lambda'} \times \langle e^{i\boldsymbol{k}'\cdot\boldsymbol{r}} \mid e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \mid nl\lambda \rangle, \tag{4}$$

where we have assumed plane wave function for the incoming pion and

$$\boldsymbol{Q} \equiv \left(1 + \frac{\mu}{2M}\right)\boldsymbol{q} - \frac{\mu}{M}\boldsymbol{k}'^*$$

is a complex vector. In eq. (4) $C^{l\frac{1}{2}j}_{\lambda\Lambda m}$ and $C^{\frac{1}{2}1\frac{1}{2}}_{\Lambda\nu\Lambda'}$ are the Clebsch-Gordan coefficients. Defining

$$\boldsymbol{x} \equiv \boldsymbol{q} - \boldsymbol{k}\left(1 + \frac{U_0 M}{k^2}\right) \quad \text{and} \quad \boldsymbol{k}'' \equiv \boldsymbol{k}\frac{W_0 M}{k^2},$$

we write

$$e^{i(\boldsymbol{q} - \boldsymbol{k}'^*)\cdot\boldsymbol{r}} = e^{i\boldsymbol{x}\cdot\boldsymbol{r} + \boldsymbol{k}''\cdot\boldsymbol{r}} = 4\pi \sum_{lm} i^l f_l(xr)\, Y^*_{lm}(\Omega_x)\, Y_{lm}(\Omega_r), \tag{5}$$

where

$$f_l(xr) = \frac{1}{4\pi i^l} \int\int d\Omega_r d\Omega_x e^{i\boldsymbol{x}\cdot\boldsymbol{r} + \boldsymbol{k}''\cdot\boldsymbol{r}}\, Y^*_{lm}(\Omega_r)\, Y_{lm}(\Omega_x) \simeq e^{k''r/2} j_l(xr). \tag{6}$$

From eqs (5) and (6) we have

$$\langle e^{i\boldsymbol{k}'\cdot\boldsymbol{r}} \mid e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \mid nl\lambda \rangle = 4\pi i^l\, Y^*_{l\lambda}(\Omega_x)\, I_{nl} \tag{7}$$

where

$$I_{nl} = \int e^{k''r/2} j_l(xr)\, R_{nl}(r)\, r^2\, dr. \tag{8}$$

Substituting this in eq. (4), squaring the latter and summing over m and Λ' we obtain

$$d\sigma/d\Omega = [M \mid k' \mid \mid Q \mid^2 (2j+1)/\pi q] \left(\frac{f}{\mu}\right)^2 \left[\sum_n C^l_{n',n} I_{nl}\right]^2 e^{-3W_0 M R_0/2\hbar} \tag{9}$$

3. Calculations and results

Assuming pion-absorption by a $0p_{3/2}$ neutron, cross-sections have been calculated for $^{12}C(\pi^+, p)\,^{11}C$ (g.s.) for various values of the parameters U_0 and W_0 taking $R_0 = 1{\cdot}25\ A^{1/3}$ fm and allowing the oscillator well parameter $\alpha = 1/b = \sqrt{m\omega/\hbar}$ to vary over its allowable range. (Electron scattering data give $\alpha \simeq 120{\cdot}7$ MeV while proton scattering data give a larger value $\alpha \simeq 134{\cdot}9$ MeV). The mixing coefficients $C^l_{n'n}$ appearing in eq. (9) are given in table 1.

Tables 2 and 3 give the calculated differential cross-sections as a function of

Table 1. Expansion coefficients $C^l_{n',n}$ for a $0p_{3/2}$ neutron in ^{12}C, obtained with Sussex interaction.

n'/n	0	1	2
0	0·9647	0·2140	0·1532

Table 2. Differential cross-sections (*mb/sr*) for the reaction $^{12}C(\pi^+, p)^{11}C$ (g.s) as a function of θ_L for various proton optical well parameters. $\alpha = 120$ MeV, $R_0 = 1{\cdot}25\,A^{1/3}$ fm

U_0 MeV	W_0 MeV	θ_L deg				
		0	10	20	30	40
9	12	·397	·365	·277	·163	·0688
9	27	·184	·164	·113	·0541	·0139
15	12	·253	·232	·173	·0978	·0384
15	27	·107	·0947	·0625	·0267	·00498

Table 3. Differential cross-sections (*mb/sr*) for the reaction $^{12}C(\pi^+, p)^{11}C$ (g.s.) as a function of θ_L for various proton optical well parameters. $\alpha = 125$ MeV, $R_0 = 1{\cdot}25\,A^{1/3}$ fm.

U_0 MeV	W_0 MeV	θ_L deg				
		0	10	20	30	40
9	12	·732	·685	·549	·359	·181
9	27	·385	·354	·267	·154	·0613
15	12	·493	·461	·365	·233	·114
15	27	·245	·223	·165	·0906	·0327

the proton emission angle (in the laboratory coordinate system) for various values of the parameters U_0 and W_0 and for $\alpha = 120$ and 125 MeV respectively.

Figure 1 shows the cross-sections obtained with HF wave functions taking into consideration the distortion of the outgoing proton and assuming that the proton originates from a neutron in the $0p_{3/2}$ shell. The proton optical well parameters used in this calculation are $U_0 = 15$ MeV, $W_0 = 27$ MeV and $R_0 = 1{\cdot}25\,A^{1/3}$ fm. The experimental points at 0° and 11° (Witten *et al* 1968) are for the π-absorption by a $0p_{3/2}$ neutron, the pion kinetic energy in the laboratory frame being 68 MeV. With their energy resolution it was impossible for them to determine whether ^{11}C nucleus is left in the ground state or in one of its low-energy excited states at 2·0 and 4·3 MeV. The points at 20° and 32° (Amato *et al* 1974) are measured with energy resolution which corresponds to ^{11}C excitation energies from 0 to 8 MeV. The pion kinetic energy in this case was 70 MeV. This figure also serves to bring out the relative importance of the two effects namely N-N correlations and proton distortion.

In figure 2 we compare our results with existing theoretical calculations. The curve labelled *W* is the result of a field theoretical calculation of Wienke (1973). In this approach Feynman graphs representing both the direct pion-nucleon interaction (eq. 2) and pion-rescattering terms were used. Plane waves were used for both the pion and proton, and the nuclei were described by harmonic

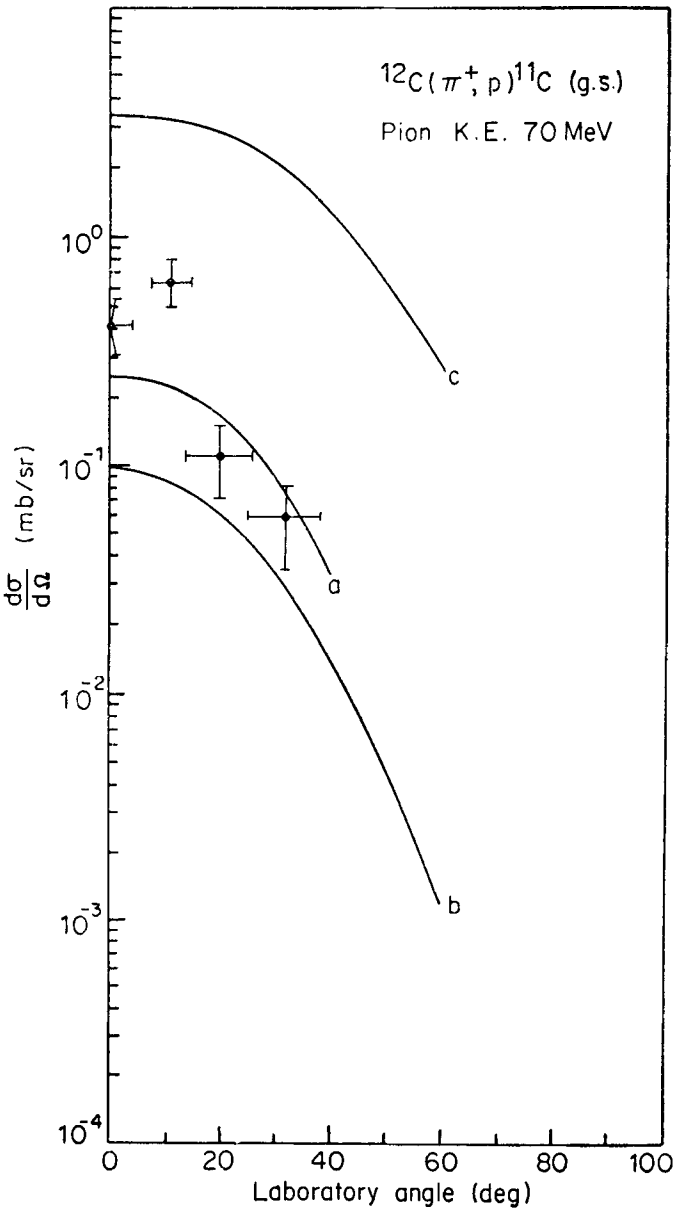

Figure 1. Differential cross-sections for the reaction $^{12}C(\pi^+, p)^{11}C$ (g.s.). Curve *a*: with HF wave functions and proton distortion. *b*: with HO wave functions. without proton distortion. *c*: with HF wave functions without proton distortion. Experimental points are from Amato *et al* (1974) (20° and 32°) and Witten *et al* (1968) (0° and 11°). Oscillator well parameter $\alpha = 125$ MeV.

oscillator wave functions. The curves labelled *JE*, *RK* and *LE* represent respectively the results of the calculations by Jones *et al* (1970), Rost *et al* (1973) and LeTourneux *et al* (1966).

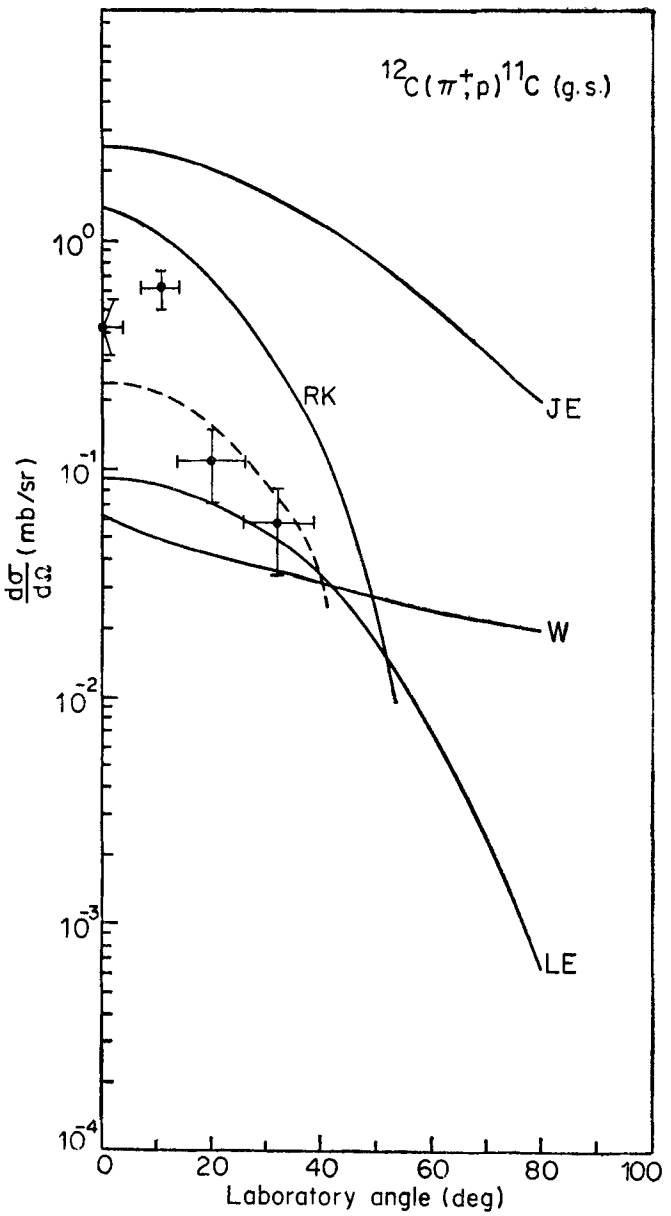

Figure 2. Differential cross-sections for $^{12}C(\pi^+, p)^{11}C$ (g.s.) The curves labelled *JE*, *RK*, *LE* and *W* represent respectively the results of the calculations by Jones *et al* (1970), Rost *et al* (1973), LeTourneux *et al* (1966) and Wienke (1973). Experimental points as in figure 1. The dotted curve represents the present calculation and it is not renormalized. The curve *RK* has a renormalization factor 80 or 160 for the Woods-Saxon well diffuseness parameter $a = 0{\cdot}5$ or $0{\cdot}7$ fm respectively. The curve *JE* has a renormalization factor of 10. Other curves have not been renormalized.

4. Discussion

We have presented above the results of our calculation of the differential cross-section for the reaction $^{12}C(\pi^+, p)\ ^{11}C$ (g.s.) taking into consideration *N-N* corre-

lations in the Hartree-Fock theory and the proton-nucleus scattering in high-energy approximation but omitting the distortion of the incoming pion.

The Hartree-Fock theory has provided satisfactory results for closed shell nuclei (Shakin *et al* 1967). Also the HF wave functions which incorporate central correlations between nucleons have previously been tested for processes like two-nucleon emission following bound-pion absorption (Kaushal *et al* 1970 *a*) and electron scattering with large momentum transfer (Gunye 1968, Kaushal *et al* 1969). In the present case an important thing to notice is that the cross-sections increase by more than an order of magnitude (figure 1) as the *N-N* correlations (*HF*) are introduced. Such a change is expected because of the high momentum transfer ($\gtrsim$ 500 MeV/c) in the (π^+, p) reactions for which harmonic oscillator (HO) wave functions are not appropriate. Another important effect is that when HF wave functions are used the cross-sections are not as sensitive to the variation in the well parameter α as they were in the case of HO wave functions. For example, at $\theta_L = 0°$, as α is varied over its allowable range (120 to 135 MeV), the fractional change in the cross-section is found to be 1·7 as against 7·1 for the results obtained with HO wave functions. The cross-sections are stabilized further (*i.e.*, become more insensitive to variation in α) if the proton distortion is also taken into consideration.

We have used square well optical potential to treat the proton-nucleus scattering. Strictly, the potential should not plunge to zero abruptly at $r = R_0$. However, the details of the nuclear surface are expected to play an important role only for large Z where Coulomb repulsion is most effective. Also the proton energy ($\sim$ 160 MeV) is large enough for the absorption to be more or less uniform throughout the volume than to be enhanced at the nuclear surface. It is only at low energies that the exclusion principle severely inhibits *N-N* collisions inside the nucleus. Moreover, the spin-orbit term is just about 2–3 MeV at the proton energy considered. Thus the error introduced is likely to be small.

The best fit values of the parameters U_0 and W_0 compare well with the values obtained from the analysis of proton-nucleus scattering data (Passatore 1968). Nevertheless the actual values of U_0 and W_0 will be somewhat higher (*i.e.*, potential more attractive) than the values given here because we have neglected the pion distortion and the contribution of the excited states of the residual nucleus ^{11}C, both of which tend to raise the differential cross-section. Calculations taking into account the pion-distortion and the contribution of the low-lying excited states of ^{11}C are in progress and will be reported later.

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