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Scission-point distributions in LRA fission

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Abstract. Trajectory calculations were performed using the three-point-charge model. The input parameters took values distributed over a wide range. Using experimental distributions, each trajectory was assigned a 'weight'; and the trajectories and their statistical 'weights' were used in obtaining initial distributions which in turn reproduced other experimental distributions, such as the angular distribution of α-particles, very well. Some information about the initial distributions has been obtained. The effect of the anticorrelation, between the final α and fragment energies, on the initial distributions has been examined.

Keywords. LRA fission; trajectory calculations; initial distributions.

1. Introduction

It is believed that quantitative information on the scission configuration can be obtained by comparing the experimentally determined results of the long-range-alpha (LRA) fission process with the asymptotic solutions of trajectory calculations. A large amount of experimental results have become available from various elaborate experiments in the last few years. Several trajectory calculations have been carried out but as yet there is no satisfactory agreement on the set of initial dynamical variables describing the scission-point configuration.

In trajectory calculations one assumes an initial scission configuration in terms of some dynamical variables, proceeds to calculate the trajectories, compares the asymptotic results with experimental ones and changes the values of these variables until a satisfactory set of initial values are obtained. [For references see Halpern (1971).]

In the derivation of certain observed relationships between the asymptotic values of these parameters, most trajectory calculations are performed by fixing all but one initial parameter. Since all the initial parameters do have a distribution in values, however small, an assumption of this kind could lead to misleading conclusions. We have formulated a procedure by which we have attempted to avoid such a severe limitation and arrive at the scission-point distributions of the dynamical variables which reproduce experimentally observed distributions and relationships between different variables.

2. Details of calculations

Trajectory calculations were performed in the two-dimensional approximation using the three-point-charge model (Halpern 1963). The input parameters were varied over a range of values covering almost all regions studied so far. The input parameters and their ranges are given in table 1.

Initial calculations were performed by fixing all but one initial parameter, to check the sensitivity of the asymptotic values of E_α, E_F, θ_L, etc, to that parameter. It was seen that variations in Y_0 and θ_L^0 do not effect these values significantly and hence in the calculations we fixed $Y_0 = 0$ and θ_L^0 was allowed an isotropic variation between 45° and 135°. The other parameters, namely D, E_α^0, E_F^0 and X_0 take values in the ranges given in table 1. Each range was divided into ten intervals uniformly distributed, thus allowing each parameter to take any of the ten values. The calculations were performed for 7 different values of mass ratio. Each trajectory was computed using a set of initial values for the parameters, characterized by a five digit number—corresponding to the five varying initial parameters. It took 0·44 sec to calculate a trajectory on an IBM 7044 Computer and 70,000 trajectories were calculated (Krishnarajulu *et al* 1973). The time interval was the same as that used by Boneh *et al* (1967) and the total number of time intervals was 81. The charges of the fragments were determined from their masses using the relations (Mukherji 1969):

$$Z_H = M_H/2\cdot 587; \quad Z_L = Z_{tot} - Z_H - Z_\alpha \tag{1}$$

The calculations were performed for ^{252}Cf. The maximum percentage error in energy conservation was 0·15%.

Each trajectory was assigned a 'weight'. This was done by using the experimental distributions in E_α and E_F for different values of mass ratio (Mehta *et al* 1973). The experimental distributions were assumed to be Gaussian and were defined by specifying the mean value and the variance of the distributions for

Table 1. The input parameters and their range of values

	Input parameter	Symbol	Range of values	Unit
1.	Mass ratio	R	1·0 to 1·8	
2.	Interfragment distance	D	17 to 28	10^{-13} cm
3.	Initial distance of α-particle from zero field point. (Zero field point taken as origin)	X_0	−7·0 to 6·5	10^{-13} cm
4.	Initial distance of α-particle from the fission axis	Y_0	0	10^{-13} cm
5.	Initial α-energy	E_α^0	0·4 to 4·0	MeV
6.	Initial total fragment energy	E_F^0	0·5 to 40·0	MeV
7.	Initial angle between the direction of motion of the α-particle and the direction of motion of the light fragment	θ_L^0	45° to 135°	

various mass ratios. The 'weight' was a product of the terms of the type

$$P_x = \exp(-|X - \bar{X}|^2/2\sigma^2) \tag{2}$$

where $\bar{X}$ and σ^2 are the mean value and the variance of the experimental distribution being compared with the calculated value X, of the variable (E_α or E_f in the present case). All trajectories with $P \geqslant 0 \cdot 1$, where $P = P_x P_Y$, were used in subsequent analysis. The initial values of the parameters, characterizing such trajectories, were plotted as a function of their weights to obtain the initial distributions.

3. Results and discussions

3.1. *Variation of θ_L with R and the dependence of $\bar{E}_\alpha$ on θ_L*

The angular distributions of α-particles for different values of mass ratio, were obtained and are compared with the experimental results of Fluss *et al* (1973). These results, for a few values of R, are shown in figure 1 *a*. The variation of

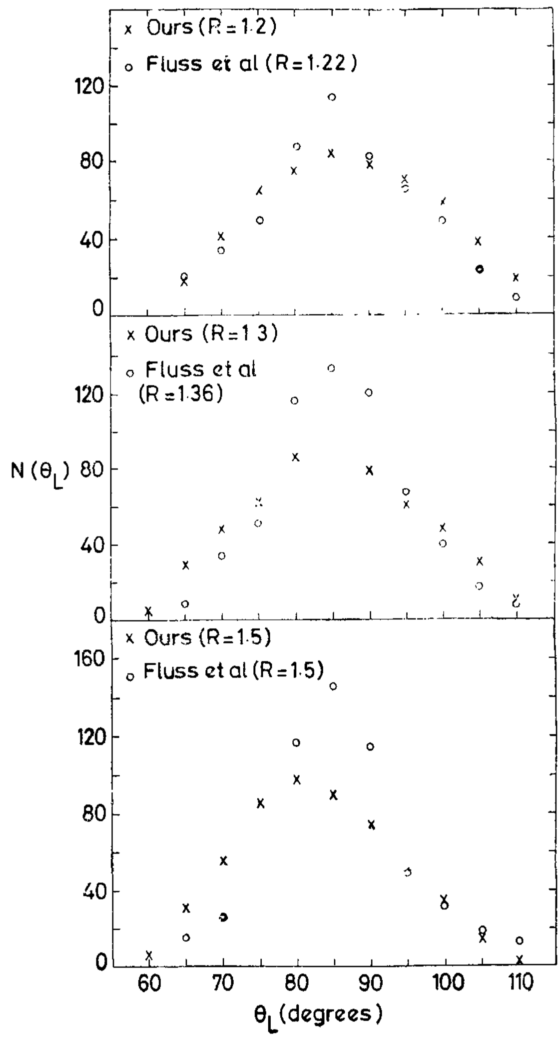

Figure 1 *a*. The calculated final angular distributions for a few values of mass ratio. Our results are compared with the experimental results of Fluss *et al* (1973). The ordinate is in arbitrary units.

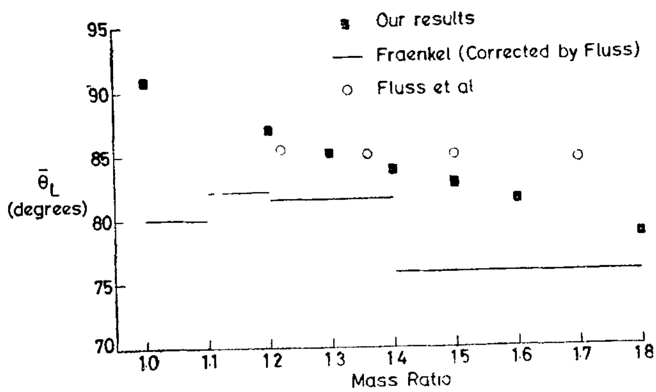

Figure 1 *b*. The variation of the average angle of emission of the α-particle with mass ratio. Our results are compared with experimental results (Fraenkel 1967. Fluss *et al* 1973). Fraenkel's results are after the correction for α-recoil effects (Fluss *et al* 1973).

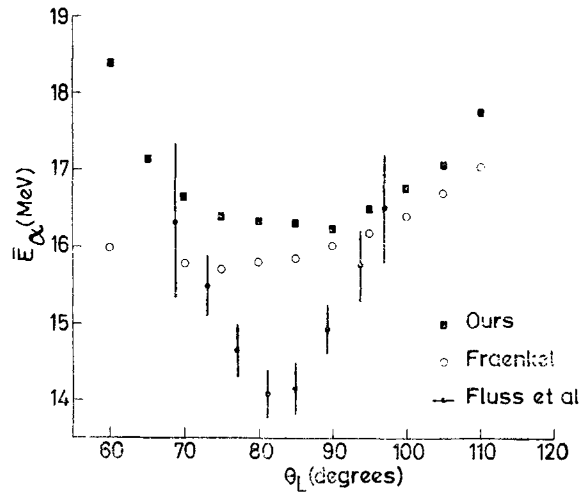

Figure 2. The variation of the average α-energy with angle of emission. Our results are compared with experimental results (Fraenkel 1967 and Fluss *et al* 1973).

the average angle of emission, $\bar{\theta}_L$, of the α-particle with mass ratio R is shown in figure 1 *b*. Our results are compared with those of Fluss *et al* (1973) and Fraenkel (1967). The mean angle of alpha emission decreases with increasing mass ratio. This trend of variation is in agreement with experimental results and for $R = 1 \cdot 3$ and $1 \cdot 4$ the actual values are very nearly equal to the observed ones. However, there seems to be disagreement between the calculated and experimental results in the interval $2 > R > 1 \cdot 5$ which may be due to inherent experimental inaccuracies in these regions of mass ratio. In the case of large values of R, Fraenkel's results corrected for α-recoil by Fluss *et al* show an appreciable decrease in $\bar{\theta}_L$ whereas the results of Fluss *et al* indicate a very small decrease. Our results lie in between these two experimental results.

The variation of the average α-energy, $\bar{E}_\alpha$ with the angle of emission θ_L is shown in figure 2. The trend of our results compares well with the experimental results of Fluss *et al* (1973). The average value of α-energy is minimum at the most probable angle of emission and increases as θ_L changes in either direction. However, the calculated values of $\bar{E}_\alpha$ are, in general, higher. This is understandable because the α-energy averaged over all angles, obtained by Fluss *et al* (1973) is about 1 MeV less than the accepted value of 16 MeV used in assigning weights in the present calculations.

We have demonstrated that all these experimental distributions can be reproduced with the initial parameters having distributions obtained by our procedure of assigning statistical weights to the trajectories using a few experimental distributions. It may be pointed out that the point charge approximation, characteristic of all trajectory calculations, implies that the deformation energy of the fragments is inherently not taken into account. Moreover, the average total fragment kinetic energy would be independent of mass ratio if deformation energy were neglected. We have, in a crude way, taken this energy into account by using the experimental E_F distributions in assigning weights.

3.2. *Anticorrelation between $\bar{E}_F$ and E_α*

The total kinetic energy of the three-particle system is a conserved quantity in the framework of such trajectory calculations. It would thus seem that the total kinetic energy of the fragments and the α-energy would be negatively correlated. That is, a loss of a certain amount of kinetic energy of the fragments would appear as a gain in kinetic energy of the α-particle, giving $\delta \bar{E}_F/\delta E_\alpha = -1$. As pointed out by Halpern (1971), this would be the case if the scission-point configuration were always the same. In reality all the initial variables specifying the scission configuration have distributions and not fixed values. The main aim of the present calculations is to arrive at these distributions. The anticorrelation between $\bar{E}_F$ and E_α observed in experiments (Fraenkel 1967, Mehta *et al* 1973) is $-0{\cdot}44$. An understanding of this anticorrelation is expected to provide some insight into the distributions of initial variables specifying the scission-point configuration.

There has been only one search for scission conditions which lead to the calculated anticorrelation, close to the experimental one, that by Boneh *et al* (1967). They obtained the correct anticorrelation by keeping all initial variables fixed except E_α^0, which was varied between $1{\cdot}0$ and 5 MeV. With lower values for initial energies they obtained a stronger anticorrelation. This led them to conclude that the fission fragments are moving with about 20% of their final kinetic energy (40 MeV) at the moment of scission and the α-energy is about 3 MeV.

It is necessary to understand quantitatively the effect of distributions in the initial variables, on the final anticorrelation between $\bar{E}_F$ and E_α, before arriving at any conclusions about the scission conditions. We tried to calculate the anticorrelation allowing distributions in initial parameters. When all parameters are allowed to have distributed values a very small anticorrelation is obtained as shown by curve A in figure 3. However, the correct anticorrelation is obtained when the interfragment separation, D, is held fixed (curve B). It may be pointed out that in this case too all other initial parameters are allowed to have their normal statistical distribution demanded by the experimental E_F and E_α distributions used in assigning weights. In particular, no regard has been given to the initial

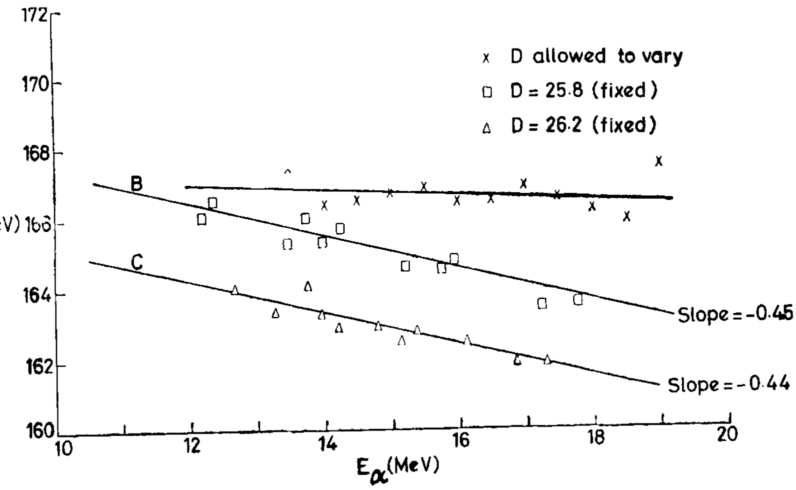

Figure 3. The anticorrelation between the average fission fragment kinetic energy E_F, and α-particle energy, E_α; for fixed values of D and for D having a distribution of values.

energies being high or low. This result indicates that when the interfragment distance D is held fixed a negative correlation exists between $\bar{E}_F$ and E_α with no restriction on the E_α^0 distribution. Thus, it is not reasonable to say that the observed anticorrelation demands that the particles are moving with an appreciable fraction (about 20%) of their final kinetic energy at the moment of scission.

The result that the anticorrelation almost disappears when all the parameters, including D, are allowed to have distributed values could be due to the initial positive correlation between E_α^0 and E_F^0 (discussed in section 3.4). This positive correlation between initial energies could in turn be due to the distribution in the values of D, as was also pointed out by Boneh *et al* (1967). However, they had conjectured that this positive correlation would be partially compensated by the negative correlation due to the distribution in the initial heavy-fragment velocity. Our results do not seem to support this argument.

Assuming that the initial correlation between the α and fragment energies is largely responsible for the final anticorrelation; it would, in the light of trajectory calculations, be necessary to negatively correlate the initial energies. The fact that the anticorrelation can be obtained by fixing D could then mean that fixing D implies a negative correlation between E_α^0 and E_F^0.

As mentioned in section 3.1, deformation effects have not been taken into account in any of the trajectory calculations so far. However, a consideration of deformation effects could throw some light on the initial negative correlation as well as substantiate the argument for a fixed value of D.

The motion of the fragments just after scission and the change in deformation in this period could, in an extreme case, result in zero nett displacement of the charge centres of the fragments, thereby implying a fixed D. A large part of the potential energy could be transformed into deformation energy of the fragments and kinetic energy of the α-particle. The resulting effect, in this period, could be a negative correlation between E_α^0 and E_F^0.

3.3. *Initial distributions*

The initial distributions in α-particle energy, E_α^0 when the experimental distri-

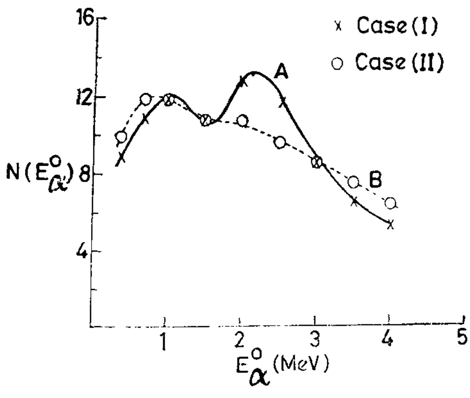

Figure 4. The initial α-energy distribution for fixed mass ratio ($R = 1 \cdot 4$). The ordinate is in arbitrary units. Case (I) is when E_α, E_F distributions are used in assigning weights. Case (II) when E_α, E_F distributions and the anticorrelation condition are used inassigning weights.

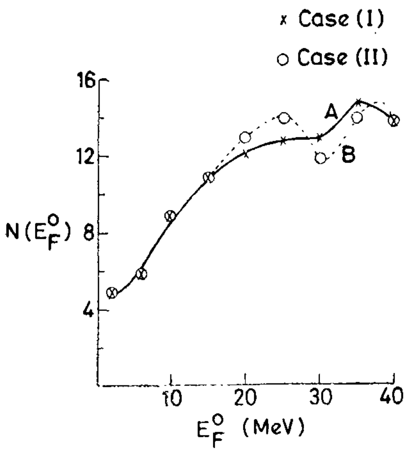

Figure 5. The initial fragment energy distribution for fixed mass ratio ($R = 1 \cdot 4$).
The ordinate is in arbitrary units. Case (I) and Case (II) same as for figure 4.

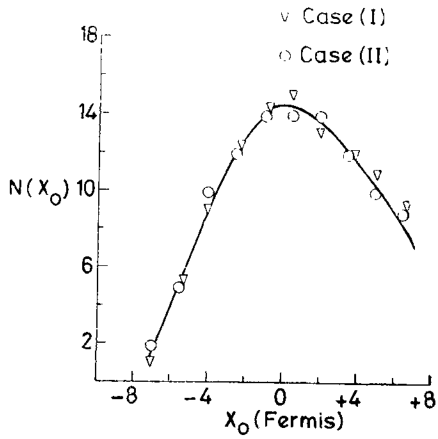

Figure 6. The X_0 distribution for fixed mass ratio ($R = 1 \cdot 4$). The ordinate is in arbitrary units. Case (I) and Case (II) same as for figure 4.

butions of E_α and E_F are used in assigning weights, is shown in curve A in figure 4. It is clear that this distribution does not give the much-sought-for information on whether the initial energy is low or is an appreciable fraction of the final value. The indication is that the E^0_α distribution is broad with possible peaking in the 1 and 2 MeV regions (figure 4).

The initial E^0_F distributions, shown in figure 5 (curve A), is again quite broad and shows possible multiple peaking. It indicates that we should have extended our initial E^0_F variation to include higher values. Work on this is in progress.

The initial distributions naturally reproduce the final distributions of E_α and E_F as these were used in assigning 'weights' to the trajectories. These initial distributions also reproduce angular ditributions, as elaborated in section 3.1. However, the observed anticorrelation between $\bar{E}_F$ and E_α cannot be reproduced by these distributions as explained in the last section. It was thought that if this anticorrelation was used as additional data in assigning 'weights', besides the E_F and E_α distributions, we would be able to provide an answer to the problem of initial energy distributions. The initial distributions, obtained when the anticorrelation condition was also used, are shown by curves B in figures 4 and 5. Here again there is an indication of two peaks in both the E_α^0 and E_F^0 distributions with some changes in their positions and relative amplitudes. But by and large there is no significant change in the distributions. This would imply that the anticorrelation condition has no significant effect on the E_α^0 and E_F^0 distributions.

The distribution in the point of emission of the α-particle along the interfragment axis, $N(X_0)$, obtained in our trajectory calculations is shown in figure 6. We observe that the most probable initial starting position for the α-particle is close to the zero-field point, which has been assumed to be so in several previous trajectory calculations, for example in Raisbeck and Thomas (1968). Fong (1970) has remarked that the main difference which accounts for different values for E_α^0 in his and other calculations is the initial position of the α-particle. The zero field position, as the most probable emission point, amounts to a position closer to the light fragment, whereas Fong requires it to be nearer the heavy fragment. The X_0 distribution obtained by Boneh *et al* (1967) using the α-particle angular distribution for different intervals of the fission fragment mass ratio, is reasonably close to the distribution obtained in our trajectory calculations. It is interesting to note that the X_0 distribution also does not change when the anticorrelation condition is included in assigning weights to trajectories.

3.4. *Correlations between initial parameters*

It is necessary to know the correlations that might exist between the initial parameters to be able to interpret the final correlation between $\bar{E}_F$ and E_α. With this in mind the distribution in E_F^0 when $E_\alpha^0 \leqslant 1$ MeV and when $E_\alpha^0 > 1$ MeV were obtained and are shown in figure 7. It is seen that there exists a positive

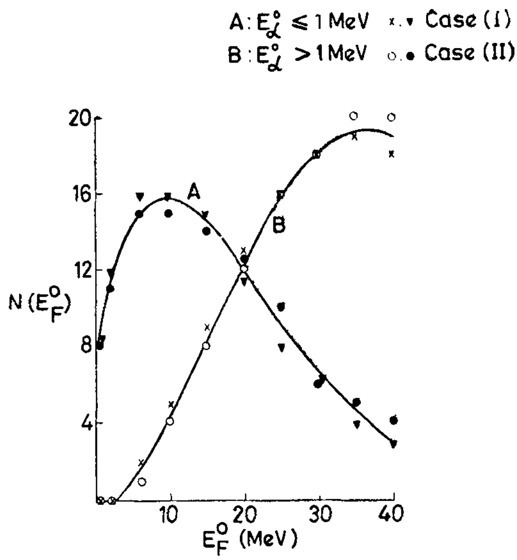

Figure 7. The initial fragment energy distributions for $E_\alpha^0 \leqslant 1$ MeV (A) and $E_\alpha^0 > 1$ MeV (B) for fixed mass ratio ($R = 1 \cdot 4$). The ordinate is in arbitrary units. Case (I) and Case (II) same as for figure 4.

correlation between E_α^0 and E_F^0. The mean values of E_F^0 from these distributions are very close to the ones predicted by previous workers (Boneh *et al* 1967, Raisbeck and Thomas 1968, Fong 1971 and others). There seems to be no *a priori* reason to choose either low or high values for the initial energies since both sets can reproduce asymptotic distributions. Moreover, we find that if we do not put an artificial restriction on any of the initial parameters, the initial distributions are rather broad but reproduce experimental distributions reasonably well.

There does not seem to be any evidence that the initial α-energy is dependent on X_0 as suggested by Boneh *et al* (1967). We have seen that the X_0 distribution is not affected by a choice of either low or high initial energies.

4. Summary of results

The initial distributions of the α-energy, E_α^0, the fragment energy, E_F^0, and the point of emission of the α-particle, X_0, have been obtained by comparing the results of the present trajectory calculations with the experimental E_α and E_F distributions. The initial energy distributions are broad indicating that one can neither assume the initial energies to be low (Fong 1971) nor to be an appreciable fraction of their final values (Boneh *et al* 1967). The X_0 distribution indicates that the mean position of emission of the α-particle is the zero field point. These initial distributions reproduce experimental distributions well. However, the anticorrelation between $\bar{E}_F$ and E_α cannot be reproduced unless the variation in the initial value of the interfragment distance is severely restricted. It is also seen that the initial distributions do not change when only the trajectories which reproduce the anticorrelation are considered. This fact suggests that to be able to understand the anticorrelation, it is necessary to introduce some correlations in the initial parameters.

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