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Chiral constraints in dual models

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Abstract. An attempt is made to see how much of chiral symmetry is contained in dual models for pseudoscalar mesons. The chiral constraints are found to be contained in dual models as either dynamical restriction or kinematical factors. At the phenomenological level there is no serious inconsistency.

Keywords. Chiral constraints; dual models.

Introduction

The aim of this paper is to critically re-examine how far chiral symmetry is incorporated in the dual models for pseudoscalar mesons. Lovelace (1968) was the first to observe the link between chiral symmetry and soft meson properties together with the absence of resonances in the exotic channels on the one hand and the Veneziano model without satellite terms on the other. While Lovelace demonstrated in the dual model for $\pi\pi$ scattering the results obtained by Weinberg (1966, 1967, 1968) by the use of current algebra and the isoscalar σ terms, Kawarabayashi *et al* (1968) showed similar results for $K\pi$ and KK scattering by relating the dual models with the current algebra treatment of Cronin (1967) and Griffith (1968). The chiral content demonstrated in both these papers is a direct consequence of the presence of Adler zero in the amplitude. It is natural to ask whether there are other results typical of chiral symmetry contained in the dual models. To answer this, it is necessary to see the correspondence between the parameters in the models of chiral symmetry and the symmetry breaking mechanism with a typical dual model and the coupling constants and the trajectory parameters in them.

We shall confine ourselves to the case involving only pseudoscalar mesons; the problem is sufficiently realistic and at the same time both chiral model and dual model are free from any serious anomalies. First, we recall all the effects of the presence of Adler zeros in the various dual amplitudes. The dual amplitudes may contain the Adler zeros either by suitable choice of the Regge trajectory that enter the amplitude or by the behaviour of the coupling constants. This differentiates two kinds of Adler zeros. While the former may be said to have more dynamical content the latter is essentially a consequence of kinematical factors.

For the comparison between the chiral symmetric models and the dual models, we then write down two identities, which make use of the soft meson limits, PCAC and current algebra. Gellmann, Oakes, Renner (1968) model of symmetry breaking and the parameters of such model are used to quantify the consequences of chiral symmetry breaking. The identities are repeatedly used in conjunction with a sufficiently general dual model. The analysis provides the chiral parameters of the conventional dual model and is further checked for internal consistency.

We have taken a sufficiently general dual model, which in particular carries in its coupling constant the dependence of the amplitude on the mass of the external lines. It was Cronin and Kang (1969) who first noticed that a conventional dual model for Kπ amplitude without such factors will lead to inconsistencies. Osborn (1970) and Csikor (1970) employed the off mass shell extrapolation factors to remove the internal inconsistencies noted by Cronin and Kang. However, in the process, we now have a good deal more of freedom in the choice of the dual amplitude. We make use of this to find the maximum correspondence between the chiral models and dual models.

Adler zero and constraint on the Regge intercepts

Dynamical zero

The most direct manifestation of the features that result from chiral symmetry is the existence of Adler zeros in the amplitudes. These arise in amplitudes involving pions, whenever one of the pions has its four-momentum equal to zero. Lovelace observed the presence of such zeros in the Lovelace-Veneziano $\pi\pi$ amplitude principally as a consequence of the condition.

$$\alpha_\rho (m_\pi^2) = \tfrac{1}{2} \tag{1}$$

The amplitude for $\pi^+\pi^- \rightarrow \pi^+\pi^-$ is given by

$$A_{\pi\pi}(s, t) = \beta \frac{\Gamma\left(1 - \alpha_\rho(s)\right) \Gamma\left(1 - \alpha_\rho(t)\right)}{\Gamma\left(1 - \alpha_\rho(s) - \alpha_\rho(t)\right)} \tag{2}$$

where β could be a function of the mass q_i^2 of the pions. Since the Adler point corresponds to $s = t = u = m_\pi^2$, Adler zero is assured in eq. (2) if the ρ-trajectory satisfies the condition given in eq. (1). It may be observed that the zero is obtained without making use of the 'mass' dependence of the coupling constant β. We shall refer to this as a dynamical Adler zero.

The dynamical Adler zero implies constraints on the ingredients that enter into a dual amplitude. Since the only ingredients of a dual model are straight line Regge trajectories, the constraints are in terms of them. More conditions similar to eq. (1) have been derived by Ademello *et al* (1969). Consider the dual amplitude for $\pi + \mathrm{A} \rightarrow \mathrm{B} + \mathrm{C}$;

$$M_{\pi\mathrm{A};\,\mathrm{BC}} = \beta \frac{\Gamma\left(k + J_\mathrm{A} - \alpha_\mathrm{X}(s)\right) \Gamma\left(l + J_\mathrm{B} - \alpha_\mathrm{Y}(t)\right)}{\Gamma\left(n + J_\mathrm{A} + J_\mathrm{B} - \alpha_\mathrm{X}(s) - \alpha_\mathrm{Y}(t)\right)} \tag{3}$$

where the lowest resonance that contributes to the s-channel has spin $k + J_\mathrm{A}$ and similarly $l + J_\mathrm{B}$ is the spin of the lowest resonance in the t-channel X and Y are the Regge trajectories that contribute in the s and t channel respectively. The asymptotic behaviour of the amplitude as $s \rightarrow \infty$ will be given $s^{\alpha_\mathrm{Y}(t) - \Delta}$, where $\Delta = J_\mathrm{B} + n - k$ is the number of helicity flips. Thus the integers n, k and l are fixed for each helicity amplitude once we know what A, B and C are. The dynamical Adler zero at $s = m_\mathrm{A}^2$, $t = m_\mathrm{B}^2$ and $u = m_\mathrm{C}^2$, in the above amplitude leads to constraints similar to eq. (1).

If the slopes of all trajectories are the same, we can derive

$$\alpha_X(0) - \alpha_A(0) = 0{\cdot}5,\ 1{\cdot}5,\ 2{\cdot}5,\ \text{etc} \tag{4}$$

where α_X refers to the Regge trajectory exchanged in the channel $\pi + A$ and α_A to the trajectory to which the particle A belongs. The principal consequence of a dynamical Adler zero is that whenever particles on a trajectory decays into a particle of opposite normality and a pion, then the two trajectories (with the same slope) have intercepts that differ by a half odd integer. This is borne out in several examples. It is possible to find Regge fits, so that $\alpha_\rho(0) - \alpha_\pi(0)$, $\alpha_\Delta - \alpha_N$, $\alpha_{K^*} - \alpha_K$, $\alpha_{Y_1^*} - \alpha_\Lambda$ and $\alpha_{Y_1^*} - \alpha_\Lambda$ are all nearly $\frac{1}{2}$.

As a further consequence of constraints implied by eq. (8), we have several hybrid mass formula, such as

$$m_{K^*}^2 - m_\rho^2 = m_K^2 - m_\pi^2 \tag{5}$$

Kinematical zero

There are amplitudes for which the dynamical constraint implied as above cannot be satisfied. One such amplitude is for the first reaction for which the dual amplitude was written by Veneziano (1968). In the case $\omega \to \pi^+ \pi^- \pi^\circ$, if we use the dynamical Adler zero condition we should get

$$\alpha_\omega(0) - \alpha_\rho(0) = \tfrac{1}{2} \tag{6}$$

which cannot be satisfied experimentally. In fact ω and ρ trajectories are degenerate, if we use some exoticity conditions. The Adler zero in this reaction is produced by the vanishing of the factor β at the Adler point, when taken with other purely kinematic factors. For example the amplitude for $\omega \to \pi^+ + \pi^- + \pi^\circ$ has the form

$$\begin{aligned} M &= \epsilon^{\mu\nu\lambda\sigma}\, \epsilon_\mu(p)\, p_{1\nu}\, p_{2\lambda}\, p_{3\sigma}\, \beta'\, V(s, t) \\ V(s, t) &= \frac{\Gamma\big(1 - \alpha_\rho(s)\big)\, \Gamma\big(1 - \alpha_\rho(t)\big)}{\Gamma\big(1 - \alpha_\rho(s) - \alpha_\rho(t)\big)} \end{aligned} \tag{7}$$

where $\epsilon_\mu(p)$ denotes the polarisation of ω and p_1, p_2 and p_3 are the momenta of pions and $V(s, t)$ is the Lorentz invariant part. The vanishing of this amplitude at the Adler point is ensured by the kinematical factors present in eq. (7). We shall refer to such a zero in the amplitude as kinematical Adler zero.

We thus see that the chiral constraint in the form of Adler zero can arise in two essentially different mechanisms. In the remainder of the paper, we shall sketch how other chiral constraints may lead to either dynamical constraints or appropriate kinematical factors. In either case it is necessary to test the internal consistency of the resultant consequences.

The Adler condition related to the kaon four-momentum set to zero in any amplitude can similarly be ensured by either a dynamical or kinematical constraint. We get typically conditions such as†

$$\alpha_{K^*}(m_K^2) = \tfrac{1}{2} \tag{8}$$

† The analysis of Ademello, Veneziano and Weinberg, if extended to processes $K + A \to B + C$, shall give the difference in the intercepts of K^* and π trajectory also to be $\frac{1}{2}$ unit. This will then imply, when taken along with the soft pion constraints, degeneracy of pion and kaon mass. We should therefore expect that SU(3) breaking will seriously affect chiral constraints that involve kaons.

Reduction of four point function and chiral identities

In this section, we shall collect together all identities and chiral constraints that need be satisfied by a four point function made up of pseudoscalar mesons as external legs.

Let $M_{jk;\,ln}$ refer to the four point meson amplitude where j, k, l and n are the SU(3) indices. We may reduce the amplitude to give, for the process $\pi_j + \pi_k \rightarrow \pi_l + \pi_n$

$$\mathrm{i}M_{jk;\,ln} = \left(\frac{q_1^2 - \mu_j^2}{f_j\mu_j^2}\right)\left(\frac{q_2^2 - \mu_l^2}{f_l\mu_l^2}\right)\Big[q_{1\mu}q_{2\lambda}\int \mathrm{d}x \exp(\mathrm{i}q_1\cdot x)\sqrt{4p_{10}p_{20}}$$
$$\langle \pi_n(p_2) \mid \mathrm{T}\{A_\mu^j(\mathrm{x})\, A_\lambda^l(0)\} \mid \pi_k(p_1)\rangle$$
$$-\, \mathrm{i}q_{2\lambda}\sqrt{4p_{10}p_{20}}\langle \pi_n(p_2) \mid [Q_5^j(0),\, A_\lambda^l(0)] \mid \pi_k(p_1)\rangle$$
$$-\, \sqrt{4p_{10}p_{20}}\,\langle \pi_n(p_2) \mid [Q_5^l(0),\, \partial_\mu A_\mu^j(0)] \mid \pi_k(p_1)\rangle\Big] \tag{9}$$

where $Q_5^j(t)$ are the 'axial charges', being the space integral of time like component of the axial current density. We have used PCAC (Partially Conserved Axial Vector Current) to express meson field operators through

$$\phi_j(x) = \frac{1}{f_j\mu_j^2}\,\partial_\mu A_\mu^j(x) \tag{10}$$

From the chiral algebra, we have, at equal times

$$[Q_5^j(0),\, A_\lambda^l(0)] = \mathrm{i}f_{jlr}\, V_\lambda^r(0) \tag{11}$$

$$[Q_5^l(0),\, \partial_\mu A_\mu^j(0)] = \mathrm{i}\sigma_{lj}(0) \tag{12}$$

where V_λ^{*} is the vector current density and σ_{lj} is the σ term, which can be calculated if the specific symmetry breaking mechanism is known. We may now write the two identities valid in the soft meson limit. We have, for $M(s,\, t,\, u;\, p_1^2,\, q_1^2;\, p_2^2,\, q_2^2)$

$$M(s = m_k^2,\ t,\ u = m_n^2;\ m_k^2,\ t;\ m_n^2,\ 0)$$
$$= \sqrt{4p_{10}p_{20}}\,\langle n \mid \sigma_{lj}(0) \mid k\rangle\,\frac{t - \mu_j^2}{f_j\mu_j^2} \times \frac{1}{f_l} \tag{13}$$

where s, t and u are the customary variables. $s = (p_1 + q_1)^2$, $t = (p_1 - p_2)^2$ and $u = (p_1 - q_2)^2$.

$$\frac{\partial M_{jk;ln}}{\partial s}\, ln\,(m_k^2,\, t,\, m_n^2;\ m_k^2,\, t,\, m_n^2\ 0) = f_{jlr}f_{nkr}\,\frac{t - \mu_j^2}{f_j\mu_j^2}\,\frac{1}{f_l}\,F_+(t) \tag{14}$$

where

$$\sqrt{4p_{10}p_{20}}\langle n \mid V_\lambda^r(0) \mid k\rangle = \mathrm{i}f_{nkr}\,[F_+(t)(p_1 + p_2)_\lambda + F_-(t)(p_1 - p_2)_\lambda] \tag{15}$$

We shall make repeated use of these two identities in evaluating the chiral content of the dual four point functions.

In order to identify the chiral content in the dual models, it is necessary to specify a model of symmetry breaking. While the experimental situation may warrant several different mechanisms of symmetry breaking to be equally applicable, we shall take up the most popular one introduced by Gellmann, Oakes and Renner (1968). The SU (3) $\times$ SU (3) symmetry breaking part is given by

$$\epsilon H' = u_0 + c u_8 \tag{16}$$

where u_0 and u_8 are scalar densities transforming like (3, 3*) + (3*, 3) representation of SU (3) $\times$ SU (3) symmetry. It is easily seen that, for such a Hamiltonian, the divergence of Axial and Vector currents can be calculated. We shall have

$$\partial_\mu A_\pi^\mu = - \frac{\sqrt{2} + c}{\sqrt{3}} v_\pi$$

$$\partial_\mu A_K^\mu = - \frac{\sqrt{2} - c/2}{\sqrt{3}} v_K$$

and

$$\partial_\mu v_K^\mu = - \frac{\sqrt{3}}{2} c u_{K^-} \tag{17}$$

where v_π, and v_K are pseudoscalar densities and u_{K^-} is a scalar density, all transforming according to the same (3, 3*) + (3*, 3) representation. Using these in eq. (12), we get

$$\sigma_{\pi^+\pi^-} = \frac{\sqrt{2} + c}{3} (\sqrt{2}\, u_0 + u_8)$$

$$\sigma_{\pi^- K^+} = \frac{\sqrt{2} - c/2}{\sqrt{6}} u_{K^0}$$

$$\sigma_{\pi^- K^0} = \frac{\sqrt{2} - c/2}{\sqrt{6}} u_{K^-}$$

$$\sigma_{K^+\pi^-} = \frac{\sqrt{2} + c}{\sqrt{6}} u_{K^0}$$

$$\sigma_{\bar{K}^0\pi^-} = \frac{\sqrt{2} + c}{\sqrt{6}} {}_{K^-}u$$

and

$$\sigma_{\bar{K}K} = \frac{\sqrt{2} - c/2}{3} [\sqrt{2} u_0 - \tfrac{1}{2} u_8] \tag{18}$$

The various identities implied by eqs (13) and (14) can now be expressed in terms of the parameters of the symmetry breaking.

Dual amplitude for $\pi\pi$ scattering

We require the off mass shell amplitude for $\pi\pi$ scattering for use in the identities. The dual amplitude of Lovelace (1968) can be expected to have the off mass shell extrapolations contained in the coupling constant β of eq. (2). We may explicitly write the amplitude for $\pi^- + \pi^+ \to \pi^- + \pi^+$ in the form: (u-channel is exotic)

$$M = \beta_{\pi\pi\gamma}(q_i^2) \frac{\Gamma(1 - \alpha_\rho(s))\, \Gamma(1 - \alpha_\rho(t))}{\Gamma(1 - \alpha_\rho(s) - \alpha_\rho(t))} \tag{19}$$

We assume straight line trajectories and the dynamical Adler zero, with the result

$$\alpha_\rho (m_\pi^2) = \frac{1}{2} \text{ and } \alpha' = \frac{1}{2(m_\rho^2 - m_\pi^2)}.$$

The identities, relevant for this amplitude, are (*see* notation given earlier)

$$M(m_\pi^2, 0, m_\pi^2; m_\pi^2, 0, m_\pi^2, 0)$$

$$= \frac{1}{f_\pi^2}\sqrt{4p_{10}p_{20}}\,\langle \pi^+(p_2) \,|\, \sigma_{\pi^+\pi^-} \,|\, \pi^+(p_1)\rangle \tag{20}$$

and for the same argument

$$\frac{\partial M}{\partial s}\bigg|_{s=m_\pi^2} = f_{\pi^+\pi^-\rho} f_{\pi^-\rho\pi^+} \frac{1}{f_\pi^2} = \frac{1}{f_\pi^2} \tag{21}$$

Using eqs (20), (21) and (19)*

$$\left[\frac{\sqrt{2}+c}{3}\langle \pi^+ \,|\, \sqrt{2}u_0 + u_8 \,|\, \pi^+\rangle\right]^{-1} = -\frac{\partial}{\partial s}\ln M\bigg|_{s=m_\pi^2}$$

$$= 51\cdot 12\ \text{GeV}^{-2} \tag{22}$$

Further, if we assume the off mass shell factor to be separable such that

$$\gamma(q_1^2, q_2^2; p_1^2, p_2^2) = \prod_{i=1}^{4} \gamma_i(q_i^2);\ \gamma_i(m_i^2) = 1 \tag{23}$$

we have from eq. (21)

$$\frac{1}{f_\pi^2} = \gamma_\pi^2(0)\,\beta_{\pi\pi}\,(2\cdot 67\ \text{GeV}^{-2}) \tag{24}$$

Dual amplitude for πK scattering

The basic dual ingradient of any $\pi K \to \pi K$ scattering is expressible in the form

$$V_{\pi K}(s,t) = \beta_{K\pi}\,\gamma(q_i^2)\,\frac{\Gamma(1-\alpha_{K^*}(s))\ \Gamma(1-\alpha_\rho(t))}{\Gamma(1-\alpha_{K^*}(s)-\alpha_\rho(t))} \tag{25}$$

where $\alpha_{K^*}(s)$ is a straight line Regge trajectory with the same universal slope as the ρ-trajectory. The dynamical Adler zero requires $\alpha_{K^*}(m_K^2) = \frac{1}{2}$. The amplitude displays resonances corresponding to K* trajectory in the s-channel and also ρ-trajectory resonances in the t-channel. The Regge asymptotic behaviour is also built in this form. However, the amplitudes that display evenness or oddness under s, u crossing are

$$A^\pm(s,t) = V_{\pi K}(s,t) \pm V_{\pi K}(u,t) \tag{26}$$

The amplitude for $\pi_\alpha + K \to \pi_\beta + K$ is then given by the various terms of

$$(\bar{K}^+\,\bar{K}^0)\,T_{\alpha\beta}\begin{pmatrix}K^+\\K^0\end{pmatrix};\qquad \alpha,\beta = 1,2,3 \tag{27}$$

where

$$T_{\alpha\beta} = \delta_{\alpha\beta}A^+(s,t) + \tfrac{1}{2}[\tau_\alpha, \tau_\beta]\,A^-(s,t)$$

and $\tau_{\alpha\beta}$ are (2×2) Pauli matrices operating on the spinor space.

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* The covariant normalisation factor $\sqrt{2p_0}$ is absorbed into the state vector.

We may write several different identities of the kind given in eq. (19) by taking off the mass shell any two of the four mesons. Let us begin by considering the elastic scattering $K^- + \pi^+ \to K^- + \pi^+$ and take either the two pions or kaons off shell. Using eqs (25), (26) and (27), we find that the amplitude has the following Veneziano form:

$$A(s, t) = V_{\pi K}(s, t) \tag{28}$$

The amplitude displays the resonances in the s-channel and the t-channel and that the u-channel being exotic is automatically incorporated. Reducing the two pions off mass shell in this amplitude, we get, similar to eq. (21), an identity:

$$\frac{\partial}{\partial s} V_{\pi K}(s, 0) \bigg|_{\substack{s = m_K^2 \\ q_\pi^2 = 0}} = \frac{1}{2f_\pi^2} \tag{29}$$

We notice that the left hand side differs from the left hand side of eq. (21) by the factor $\beta_{K\pi}/\beta_{\pi\pi}$. Thus, for consistency between eqs (21) and (29), we need

$$\frac{\beta_{K\pi}}{\beta_{\pi\pi}} = \frac{1}{2} \tag{30}$$

Now by taking the kaons off the mass shell instead, we get

$$A(m_\pi^2, 0, m_\pi^2;\ m_\pi^2, 0;\ m_\pi^2, 0) = -\frac{1}{f_K^2}\sqrt{4p_{10}p_{20}}\ \langle \pi^+ \mid \sigma_{\bar{K}K} \mid \pi^+ \rangle$$

$$= -\frac{1}{f_K^2}\,\frac{\sqrt{2}-\frac{1}{2}c}{3}\,\langle \pi^+ \mid \sqrt{2}\,u_0 - \tfrac{1}{2}\,u_8 \mid \pi^+ \rangle \tag{31}$$

and for the same argument

$$\frac{\partial A}{\partial s}\bigg|_{s = m_\pi^2} = \frac{1}{2f_K^2} \tag{32}$$

From these two equations, we may derive another equation independent of the off mass shell extrapolation factors analogous to eq. (22):

$$\frac{1}{2}\left[\frac{\sqrt{2} - c/2}{3}\,\langle \pi^+ \mid \sqrt{2}u_0 - \frac{1}{2}\,u_8 \mid \pi^+ \rangle\right]^{-1} = -\frac{\partial}{\partial s}\ln A(m_\pi^2, 0)$$

$$= 3{\cdot}23\ \text{GeV}^{-2} \tag{33}$$

Further, analogous to eq. (24), we have

$$\frac{1}{2f_K^2} = -\gamma_K^2(0)\,\beta_{K\pi}\,(1{\cdot}70\ \text{GeV}^{-2}) \tag{34}$$

From eqs (24) and (34), after setting $\beta_{K\pi} = \frac{1}{2}\beta_{\pi\pi}$, we obtain

$$\frac{f_K}{f_\pi} = 1{\cdot}25\,\frac{\gamma_\pi(0)}{\gamma_K(0)} \tag{35}$$

Equations (22) and (33) relate the matrix elements of $u_0(0)$ and $u_8(0)$ and the chiral symmetry breaking parameter c to those features of the dual amplitude in which the dependence on the off mass shell extrapolation does not occur. In other words, apart from the straight line trajectory functions very little has gone into these equations. Now if we supplement these by one more equation, which indicates that the symmetry breaking Hamiltonian is responsible for the mass of the pions, we have

$$\langle \pi^+ \mid u_0(0) + cu_8(0) \mid \pi^+ \rangle = m_\pi^2 \tag{36}$$

Solving eqs (22), (33) and (36), we arrive at

$$c = -1\cdot 27. \tag{37}$$

$$\langle \pi^+ \mid u_0 \mid \pi^+ \rangle = 0\cdot 216 \text{ GeV}^2 \tag{38a}$$

$$(\pi^+ \mid u_8 \mid \pi^+) = 0\cdot 154 \text{ GeV}^2 \tag{38b}$$

It is significant that this value of c is very close to the one obtained in a phenomenological analysis using the experimental information. This should be, therefore, regarded as the chiral symmetry breaking parameter implied by dual models.*

We shall now turn to the identities which arise as a consequence of taking one pion and one kaon off the mass shell in some typical Kπ scattering. These will yield information regarding the form factors of the strangeness changing current.

Let us consider the reaction $K^+ + \pi^- \to K^\circ + \pi^\circ$. The amplitude $T(s, t)$ for this reaction in terms of the Veneziano factors is given by

$$T(s, t) = -\frac{1}{\sqrt{2}}\left[V_{\pi K}(s, t) - V_{\pi K}(u, t)\right] \tag{39}$$

Let us keep K^+ in the initial state and π° in the final state to be on the mass shell and let either π^- momentum or K° momentum go to zero. We then have

$$T(m_K^2, m_\pi^2, u;\ m_K^2, 0;\ m_\pi^2, u) = \frac{u - m_K^2}{f_\pi f_K m_K^2}\sqrt{4p_{10}p_{20}}\,\langle \pi^\circ(p_2) \mid \sigma_{\pi^-\bar{K}^0} \mid K^+(p_1)\rangle \tag{40}$$

and

$$T(m_\pi^2, m_K^2, u;\ m_K^2, u;\ m_\pi^2, 0) = \frac{u - m_\pi^2}{f_\pi f_K m_\pi^2}\sqrt{4p_{10}p_{20}}\,\langle \pi^\circ(p_2) \mid \sigma_{\bar{K}^0\pi^-} \mid K^+(p_1)\rangle \tag{41}$$

Using eqs (17) and (18), we may write, suppressing q_1^2 variables

$$T(m_K^2, m_\pi^2, u)$$

$$= -\frac{u - m_K^2}{f_\pi f_K m_K^2}\left(\frac{2\sqrt{2} - c}{3c}\right)\sqrt{2}\,\sqrt{4p_{10}p_{20}}\,\langle \pi^\circ(p_2) \mid \partial_\mu V^\mu_{K^-}(0) \mid K^+(p_1)\rangle$$

$$= -\frac{u - m_K^2}{f_\pi f_K m_K^2}\left(\frac{2\sqrt{2} - c}{6c}\right)\frac{1}{\sqrt{2}}\left[f_+(u)(m_K^2 - m_\pi^2) + uf_-(u)\right] \tag{42}$$

where $f_+(u)$ and $f_-(u)$ are the form factors of strangeness charging vector current. Similarly starting from eq. (41) we would get

* The value $c = -\sqrt{2}$ corresponds to exact SU(2) $\times$ SU (2) symmetry and the small deviation from this value is generally regarded as the reason for the small pion mass.

$$T(m_\pi^2, m_K^2, u) = -\frac{u - m_\pi^2}{f_\pi f_K m_\pi^2}\frac{\sqrt{2}+c}{3c}\frac{1}{\sqrt{2}}[(f_+(u)(m_K^2 - m_\pi^2) + uf_-(u)] \tag{43}$$

The use of dual amplitude for T will give two identities that relate the parameters of dual model and the chiral model. We have from eq. (42), using eqs (25) and (39),

$$\begin{aligned} -\frac{u - m_K^2}{f_\pi f_K m_K^2}\frac{2\sqrt{2}-c}{6c}\frac{1}{\sqrt{2}}[f_+(u)(m_K^2 - m_\pi^2) + uf_-(u)] \\ = -\frac{1}{\sqrt{2}}[V_{\pi K}(m_K^2, m_\pi^2) - V_{\pi K}(u, m_\pi^2)] \\ = \frac{1}{\sqrt{2}}\beta_{K\pi}\gamma_\pi(0)\gamma_K(u)\frac{\Gamma\big(1 - \alpha_{K^*}(u)\big)\Gamma(\frac{1}{2})}{\Gamma\big(\frac{1}{2} - \alpha_{K^*}(u)\big)} \end{aligned} \tag{44}$$

We have used the fact that $V_{\pi K}(m_K^2, m_\pi^2)$ is zero because of the pole in the denominator. The zero at $u = m_K^2$ is ensured by the constraint $\alpha_{K^*}(m_K^2) = \frac{1}{2}$. This is in fact the Adler zero of the amplitude. The equation relates the dependence of the form factor to the off mass shell extrapolation factor $\gamma_K(u)$ in the dual model. In particular if we let $u \to 0$, we get

$$\frac{2\sqrt{2}-c}{6c}f_+(0) = \beta_{K\pi}\gamma_\pi(0)\gamma_K(0)f_\pi f_K(2\cdot 34) \tag{45}$$

Similarly starting from eq. (43), we would get the other identity which relates the form factor to the extrapolation factor. In the limit $u \to 0$ in such an identity, we recover

$$\frac{\sqrt{2}+c}{3c}f_+(0) = \beta_{K\pi}\gamma_\pi(0)\gamma_K(0)f_\pi f_K(0\cdot 277) \tag{46}$$

If both eqs (45) and (46) can be treated as equally reliable, then we get the value of c as $-1\cdot 17$, which is not far from the number $-1\cdot 27$, we have obtained as a consequence of the identities due to two pions or two kaons off the mass shell in the Kπ scattering amplitudes.

Can both eqs (45) and (46) be equally reliable? While both the identities refer to the point when both pion and kaon have zero mass, the first identity lets the pion four-momenta go to zero first and make smooth extrapolation in kaon mass and in the second identity the role of pion and kaon are interchanged. It is also observed the latter extrapolation is much sharper and perhaps some descrepancy could be introduced in the process of continuation.

The identity corresponding to eq. (44) may also be verified at the point when $u = m_k^2$. With $u = m_K^2$ in eq. (44), we get

$$\frac{2\sqrt{2}-c}{6c}[f_+(m_K^2) + f_-(m_K^2)] \approx \beta_{K\pi}f_\pi f_K\gamma_\pi(0)\pi\alpha' \tag{47}$$

where we have neglected the square of the mass of pion as compared to that of kaon. The form factor at this point can be calculated again by appeal to the soft pion approximation and the algebra of currents. We have (Callan and Treiman 1966, Mathur *et al* 1966, Suzuki 1966)

$$f_+(m_K^2) + f_-(m_K^2) = \frac{f_K}{f_\pi} \tag{48}$$

We shall now examine the internal consistency of the various relations between chiral parameters and dual model parameters and analyse which of them contain a dynamical input and the ones that are of a kinematical nature.

We have already noted the approximate internal consistency between eqs (45) and (46). From now on we shall drop from our consideration the implication of eq. (46) since it appears to incorporate rather steep extrapolations. Now substituting for the product of parameters $\gamma_\pi(0) f_\pi \gamma_K(0) f_K \beta_{K\pi}$ the value obtained from the identities given in eqs (24) and (34), we obtain

$$f_+(0) \approx 1\cdot 00 \tag{49}$$

if we set $c = -1\cdot 27$. Further the ratio of eqs (47) and (45) yields

$$\frac{f_+(m_K^2) + f_-(m_K^2)}{f_+(0)} = \frac{1\cdot 17}{\gamma_K(0)} \tag{50}$$

We have already noted that the ratio f_K/f_π is related to $\gamma_\pi(0)/\gamma_K(0)$. Thus the implication of CT-MOP-S relation (eq. 48) is then merely that

$$\gamma_\pi(0) = 0\cdot 94 \tag{51}$$

It appears as though the dual model can accommodate a chiral constraint such as CT-MOP-S relation through the choice of our essentially kinematical factor $\gamma_\pi(0)$. This still leaves other dual parameters such as $\gamma_K(0)$ unspecified.

We may now compare eq. (50) with the experimental number. The left hand side is in fact the ratio of the tangents of Cabibbo angle (Gaillard and Chounet 1970) for the axial and vector currents, since

$$\frac{1}{f_+(0)} \frac{f_K}{f_\pi} = \frac{\tan\theta_A}{\tan\theta_V} \tag{52}$$

This ratio is determined to be $1\cdot 28$ (Brene *et al* 1968) and if utilised in eq. (50), we derive a value for $\gamma_K(0)$. The kaon off mass shell extrapolation factor is then seen to be rather smooth, and slow since

$$\gamma_K(0) = 0\cdot 91 \tag{53}$$

It is significant to observe that the use of various identities do not give rise to any inconsistency; but on the other hand leads to fairly reasonable values to all the parameters that might be identifiable. Further owing to the off mass shell extrapolation factors most of the identities are essentially satisfied by the freedom we possess in the choice of these factors. We should therefore take it that chiral symmetry together with the symmetry breaking is being incorporated through what may be termed as kinematical factors.

Conclusion

We have not considered all possible techniques of chiral symmetry in our analysis. Instead of using CT-MOP-S relation as constraint, it is possible to use dispersion relations (for example Mathur *et al* 1966) and carry out extrapolation from the soft pion or soft kaon point to the mass shell. Alternatively one may use asymptotic SU (3) constraints on the form factors. We have limited ourselves to the CT-MOP-S constraint mainly since it uses only the charge algebra and an extrapolation from $q^2 = 0$ to $q^2 = m_\pi^2$ and both the steps are well established at least on phenomenological grounds. Among dual models for the four point functions for pseudoscalar mesons, there are no major ambiguities, apart from the ghosts that occur at the second daughter level. Indeed there are unsolved problems connected with tachyons and ghosts, particularly when the model is extended to general n-point amplitude. While we are ignoring these difficulties, we are principally concerned with the phenomenological consequences of the four-point function only, which has fairly unique form with minimum of the formal difficulties. What appears evident from this analysis is that the principal architect of chiral symmetry in dual models is the presence of Adler zeros in the amplitude. The Adler zero is either of a consequence of a dynamical constraint on the trajectory parameters or simply of kinematical origin. Both possibilities do not seem to cause any basic inconsistency. There is enough room in dual model off mass shell extrapolation factors to accommodate the various consequences of chiral symmetry and their breaking mechanisms.

There are theorems incorporating chiral symmetry for processes involving several pions.

Since they involve the same ingredients as Adler zero, it is expected that many pion dual amplitude will have no difficulty in incorporating them. In fact the dual amplitude for six pions due to Brower and Chu (1973) contains in it both the factorisation into two four pion amplitudes as well as two $\omega \to \pi^+ \pi^- \pi^0$ amplitudes. Thus it is capable of exhibiting both the dynamical and kinematical Adler zeros.

We have checked the consistency between the operational or phenomenological aspect of the chiral symmetry with the freedom permitted under a dual model. This does not preclude the possibility that the various extrapolations assumed within the framework of the chiral symmetry are not valid due to non-analytic behaviour of the theories. The small discrepancy between two different ways for calculating the symmetry breaking parameter c may be due to such a reason. However, in such a situation, it is the basic consistency of the chiral symmetry itself that is being questioned. The question of evaluating the extent of chiral symmetry in dual models is then rather ambiguous.

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