Pramāṇa, Vol. 3, No. 1, 1974, pp 56-60. © Printed in India.

Chiral symmetry breaking and *KN* sigma commutator

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MS received 28 January 1974; after revision 4 May 1974

Abstract. *KN* sigma commutator has been calculated in the framework of the $(6, \bar{6}) + (\bar{6}, 6)$ model. It is found that though this model could not be discarded in favour of the $(3, \bar{3}) + (\bar{3}, 3)$ or (8, 8) model, a very large value for *KN* sigma term is required to get a positive value of πN sigma term.

Keywords. Chiral symmetry breaking; Kaon-nucleon sigma commutator.

1. Introduction

Since Gell-Mann *et al* (1968) suggested that the SU(3) × SU(3) symmetry breaking part of the energy density transforms under SU(3) × SU(3), there have been many attempts to calculate both πN and *KN* sigma commutators in the framework of the $(3, \bar{3}) + (\bar{3}, 3)$ model. Von Hippel and Kim (1971) calculated the *KN* sigma term on the assumption that the symmetry breaking term in the Hamiltonian density belongs to the $(3, \bar{3}) + (\bar{3}, 3)$ representation of the SU(3) × SU(3), with one adjustable parameter. Kopp *et al* (1972) used off shell finite energy sum rules to calculate the *KN* sigma term. Reya (1973) obtained various estimations of the *KN* sigma commutator in terms of *s* and *p* wave scattering lengths and an integral over $K^{\pm}N$ cross-section.

Recently Dittner *et al* (1972) have investigated the features of a model where the SU(3) × SU(3) symmetry breaking Hamiltonian transforms according to the $(6, \bar{6}) + (\bar{6}, 6)$ representation. They have also calculated the $\eta \rightarrow 3\pi$ decay width in the $(6, \bar{6}) + (\bar{6}, 6)$ model (Dittner *et al* 1973). Though this model does not give the correct width for the $\eta \rightarrow 3\pi$ decay, it gives at least a better result than that obtained in the $(3, \bar{3}) + (\bar{3}, 3)$ model. Also the result given by this model in case of the πN sigma term is as good as that obtained in the other model. Hence, one cannot discard this model in favour of the $(3, \bar{3}) + (\bar{3}, 3)$ or the (8, 8) model.

In this note we present a calculation of the much discussed *KN* sigma commutator, expressed in terms of the baryon mass in the exact SU(3) × SU(3) symmetry limit. The object of this exercise is to see whether the value of the *KN* sigma commutator, calculated in the framework of the $(6, \bar{6}) + (\bar{6}, 6)$ model is consistent with that of the πN sigma term, calculated by Dittner *et al.* In section 2 we discuss briefly the main features of the $(6, \bar{6}) + (\bar{6}, 6)$ model and in section 3 we present our calculation of the $\bar{K}N$ sigma commutator in this model. In section 4 we have discussed our results and compared them with those of other workers.

2. The $(6,\bar{6})+(\bar{6},6)$ algebra

The $(6,\bar{6})+(\bar{6},6)$ algebra has been discussed extensively by Auvil (1973) and Dittner *et al* (1972). In what follows we make liberal use of their notations and results and show the connection between the two. In calculating the σ-commutator, however, we shall use the notations and results of Dittner *et al* (1972) only. The 6 and $\bar{6}$ representation of SU(3) occur in the outer product of the basic quark representations according to the decompositions $3\times 3 = 6+\bar{3}$ and $\bar{3}\times\bar{3}=\bar{6}+3$ and satisfy the following commutation relations

$$[Q_\alpha, T_i] = \tfrac{1}{2} T_j S_{ji}^{\alpha} \quad \text{for } 6 \tag{1 a}$$

$$[Q_\alpha, W_i] = -\tfrac{1}{2} S_{ij}^{\alpha} W_j \quad \text{for } \bar{6} \tag{1 b}$$

where $\alpha = 1, 2, \ldots, 8$ and $i, j = 1, 2, \ldots, 6$.

The eight 6×6 matrices $\{S^\alpha\}$ are the representations of the SU(3) generators in the 6-dimensional representation of SU(3). The matrices $\{S^\alpha\}$ satisfy the relations

$$S_{ij}^{\alpha *} = S_{ji}^{\alpha} \tag{2 a}$$

$$[S^\alpha, S^\beta] = 2if_{\alpha\beta\gamma}S^\gamma \tag{2 b}$$

where $f_{\alpha\beta\gamma}$ are SU(3) structure constants.

For $(6,\bar{6})$ we have

$$[Q_\alpha^+, T_{ij}] = \tfrac{1}{2} S_{ik}^{\alpha *} T_{kj} \tag{3 a}$$

$$[Q_\alpha^-, T_{ij}] = -\tfrac{1}{2} S_{jk}^{\alpha} T_{ik} \tag{3 b}$$

and for $(\bar{6},6)$

$$[Q_\alpha^+, W_{ij}] = -\tfrac{1}{2} S_{ik}^{\alpha} W_{kj} \tag{4 a}$$

$$[Q_\alpha^-, W_{ij}] = \tfrac{1}{2} S_{jk}^{\alpha *} W_{ik} \tag{4 b}$$

Also

$$PT_{ij}P^{-1} = T_{ji}^{\dagger}$$

where

$$Q_\alpha^{\pm} = \tfrac{1}{2}(Q_\alpha \pm Q_\alpha^5)$$

and P is the parity operator.

From eq. (2) it follows that $T_{ji}^{\dagger}$ transform like $(\bar{6},6)$. The tensors $M_{\alpha\beta}^{\gamma\delta}$ and $N_{\alpha\beta}^{\gamma\delta}$ $(\alpha,\beta,\gamma,\delta = 1, 2, 3)$ of Dittner *et al* are essentially the tensors T_{ij} and W_{ij} of Auvil (1973) (Note that $M_{\alpha\beta}^{\gamma\delta}$ has only 36 independent components because of the symmetry property $M_{\alpha\beta}^{\gamma\delta} = M_{\beta\alpha}^{\gamma\delta}$, $M_{\alpha\beta}^{\gamma\delta} = M_{\alpha\beta}^{\delta\gamma}$). Also the matrices $\{S^\alpha\}$ are six-dimensional representation of the generators of SU(3). The symmetric tensors R_{ij} and S_{ij} defined by the following equations

$$R_{ij} = (\lambda_i)_\gamma{}^{\alpha} (\lambda_j)_\delta{}^{\beta} M_{\alpha\beta}^{\gamma\delta} \tag{6 a}$$

$$S_{ij} = (\lambda_i)_\gamma{}^{\alpha} (\lambda_j)_\delta{}^{\beta} N_{\alpha\beta}^{\gamma\delta} \tag{6 b}$$

are still $(6,\bar{6})$ and $(\bar{6},6)$ tensors under SU(3) $\times$ SU(3). The SU(3) decomposition of $(6,\bar{6})$ is as follows

$$6 \times \bar{6} = 1 + 8 + 27 \tag{7}$$

The even and odd parity tensors defined by

$$T_{ij}{}^{\pm} = R_{ij} \pm S_{ij} \tag{8}$$

have the following decomposition according to (7)

$$T_{ij}{}^{\pm}(1) = \tfrac{1}{8}\,\delta_{ij}\,T_{pp}{}^{\pm} \tag{9 a}$$

$$T_{ij}{}^{\pm}(8) = \tfrac{3}{8}\,d_{ijk}d_{kpq}T_{pq}{}^{\pm} \tag{9 b}$$

$$T_{ij}{}^{\pm}(27) = T_{ij}{}^{\pm} - \tfrac{1}{8}\,\delta_{ij}T_{pp}{}^{\pm} - \tfrac{3}{5}\,d_{ijk}d_{kpq}T_{pq}{}^{\pm} \tag{9 c}$$

and have the following commutation relations

$$[Q_i, T_{jk}{}^{\pm}] = \mathrm{i}f_{ijk}T_{pk}{}^{\pm} + \mathrm{i}f_{ikp}T_{jp}{}^{\pm} \tag{10 a}$$

$$[Q_i{}^5, T_{jk}{}^{\pm}] = -\{d_{ijp}T_{pk}{}^{\mp} + d_{ikp}T_{jp}{}^{\mp} + \delta_{ij}d_{kpq}T_{pq}{}^{\mp} + \delta_{ik}d_{jpq}T_{pq}{}^{\mp}\} \tag{10 b}$$

3. Kaon-nucleon sigma commutator

We take the hadron energy density as follows:

$$H = H_0 + H_{SB} \tag{11}$$

where H_0 is invariant under SU(3) $\times$ SU(3) and H_{SB} is the symmetry breaking part. In the $(6, \bar{6}) + (\bar{6}, 6)$ model H_{SB} is written as

$$H_{SB} = \epsilon_1\bar{S}_0{}^+ + \epsilon_8\,\bar{S}_8{}^+ + \epsilon_{27}\bar{S}_{27}{}^+ \tag{12}$$

where $\bar{S}_0{}^+$, $\bar{S}_8{}^+$ and $\bar{S}_{27}{}^+$ transform under SU (3) as 1, 8 and 27 respectively. Also the S's are defined by:

$$\left.\begin{aligned} \bar{S}_0{}^+ &= T_{88}{}^+(1) \\ \bar{S}_8{}^+ &= T_{88}{}^+(8) \\ \bar{S}_{27}{}^+ &= T_{88}{}^+(27) \end{aligned}\right\} \tag{13}$$

where

$$T_{ij}{}^{\pm} = R_{ij} \pm S_{ij} \qquad i, j = 1, 2, \cdots, 8 \tag{14}$$

R_{ij} and S_{ij} are symmetric tensors defined by equations (6). The KN sigma term is defined by

$$\sigma_{NN}{}^{K^+K^-} = \langle N \mid [F_{K^+}{}^5, [F_{K^-}{}^5, H_{SB}(0)]] \mid N \rangle \tag{15}$$

where F_i and $F_i{}^5$ are SU(3) vector and axial vector generators. Using the commutation relations (10), we get from (12) and (15)

$$\begin{aligned} \sigma_{NN}{}^{K^+K^-} = \langle N \mid [&\tfrac{1}{12}(\epsilon_1 + \epsilon_8)\Big(\sum_{p=1}^{8} T_{pp}{}^+ + 2T_{88}{}^+\Big) \\ &- \tfrac{1}{10}(2\epsilon_8 + 3\epsilon_{27})\sum_{l=1}^{3} T_{ll}{}^+ + \\ &\tfrac{1}{60}(25\epsilon_1 + 7\epsilon_8 - 27\epsilon_{27})(T_{44}{}^+ + T_{55}{}^+) \\ &- \tfrac{1}{10\sqrt{3}}(5\epsilon_1 + 3\epsilon_8 - 3\epsilon_{27})\,T_{38}{}^+] \mid N \rangle \end{aligned} \tag{16}$$

ϵ's are calculated from the following low energy theorem

$$f_i{}^2\,m_i{}^2\,\delta_{ij} = -\langle O \mid [F_i{}^5, [F_j{}^5, H_{SB}(0)]] \mid O \rangle \tag{17}$$

(no sum on i)

where f_i are the meson decay constants defined by

$$f_i^2 m_i^2 \delta_{ij} = \langle O \mid \partial_\mu A_i^\mu \mid \phi_j \rangle \tag{18}$$

(no sum on i)

Considering only the lowest order term in H_{SB} we take $f_\pi = f_k = f_\eta$ and then ϵ's can be expressed in terms of the pseudoscalar meson masses. We just quote the result.

$$\frac{\epsilon_8}{\epsilon_1} = \frac{20\,(-3m_\pi^2 + 2m_k^2 + m_\eta^2)}{7\,(3m_\pi^2 + 4m_k^2 + m_\eta^2)} = 1\cdot567 \tag{19}$$

$$\frac{\epsilon_{27}}{\epsilon_1} = \frac{5\,(m_\pi^2 - 4m_k^2 + 3m_\eta^2)}{9\,(3m_\pi^2 + 4m_k^2 + m_\eta^2)} = -\,0\cdot026 \tag{20}$$

To calculate (16) we must know the matrix element of $T_{pq}{}^+$ between the octet baryon states. Using Wigner-Eckart theorem we get

$$\langle B_i \mid T_{pq}{}^+ \mid B_j \rangle = \alpha\delta_{ij}\delta_{pq} + \beta\,(\delta_{ip}\delta_{jq} + \delta_{iq}\delta_{jp}) + \gamma d_{ijk} d_{kpq} + i\delta f_{ijk} d_{kpq} \tag{21}$$

The parameters α, β, γ and δ can be expressed in terms of the baryon masses, by use of the formula

$$M_B = M_0 + \langle B \mid H_{SB}\,(0) \mid B \rangle \tag{22}$$

where M_0 is the baryon mass in the SU(3) $\times$ SU(3) symmetry limit. Equations (12), (21) and (22) give

$$M_N = M_0 + \epsilon_1\alpha + \tfrac{1}{20}\,(5\epsilon_1 + 4\epsilon_8 - 9\epsilon_{27})\,\beta + \tfrac{1}{6}\,\epsilon_8\gamma - \tfrac{1}{2}\,\epsilon_8\delta \tag{23}$$

$$M_\Xi = M_0 + \epsilon_1\alpha + \tfrac{1}{20}\,(5\epsilon_1 + 4\epsilon_8 - 9\epsilon_{27})\,\beta + \tfrac{1}{6}\,\epsilon_8\gamma + \tfrac{1}{2}\,\epsilon_8\delta \tag{24}$$

$$M_\Sigma = M_0 + \epsilon_1\alpha + \tfrac{1}{20}\,(5\epsilon_1 - 8\epsilon_8 + 3\epsilon_{27})\,\beta - \tfrac{1}{3}\,\epsilon_8\gamma \tag{25}$$

$$M_\Lambda = M_0 + \epsilon_1\alpha + \tfrac{1}{20}\,(5\epsilon_1 + 8\epsilon_8 + 27\,\epsilon_{27})\,\beta + \tfrac{1}{3}\,\epsilon_8\gamma \tag{26}$$

(Equation (25) differs from that of Dittner *et al* (1972) possibly due to some misprints in their paper)

From (16) and (21) we get

$$\sigma_{NN}{}^{K^+K^-} = \tfrac{1}{15}\,(25\epsilon_1 + 7\epsilon_8 - 27\epsilon_{27})\,\alpha + \tfrac{1}{10}\,(10\epsilon_1 + 4\epsilon_8 - 9\epsilon_{27})\,\beta + \tfrac{1}{90}\,(20\epsilon_1 + 14\epsilon_8 - 9\epsilon_{27})\,\gamma - \tfrac{1}{30}\,(5\epsilon_1 + 13\epsilon_8 + 12\epsilon_{27})\,\delta \tag{27}$$

Taking as input the experimental value for the baryon masses we obtain from eqs (19), (20) and (23)–(27)

$$\sigma_{NN}{}^{K^+K^-} \simeq 12\cdot7\,\sigma_{NN}{}^{\pi\pi} + 1108 \quad \text{(MeV)} \tag{28}$$

$$\simeq 2480 - 2\cdot44\,M_0 \quad \text{(MeV)} \tag{29}$$

$$M_0 \simeq 1000 - 0\cdot41\,\sigma_{NN}{}^{K^+K^-} \tag{30}$$

4. Conclusion

The $(3, \bar{3}) + (\bar{3}, 3)$ model as quoted by Dittner *et al* (1972) gives $M_0 \simeq 1300 - 13\sigma_{NN}{}^{\pi\pi}$ (MeV). It is clear that the result given by equations (28–30) obtained from the $(6, \bar{6}) + (\bar{6}, 6)$ model is at least as good as that obtained from other models. From the experimental point of view, however, one cannot choose between these different models. Von Hippel and Kim (1971) give the result

$$\sigma_{NN}^{K^+K^-} = 180 \pm 55\,(\text{MeV})$$

and

$$\sigma_{NN}^{K^+K^-} = 540 \pm 160\,(\text{MeV})$$

while Reya (1971) found the following estimates for $\sigma_{NN}^{K^+K^-}$ terms

$$\sigma_{NN}^{K^+K^-} = 540 \pm 160\,(\text{MeV})$$

from dispersive approach and

$$\sigma_{NN}^{K^+K^-} = 480 \pm 110\,(\text{MeV})$$

from the smoothness hypothesis.

To get a value of $\sigma_{NN}^{K^+K^-}$ compatible with the above results the restriction on the value of M_0 comes out to be

$$720\text{ MeV} < M_0 < 950\text{ MeV}.$$

But this value of M_0 gives a negative value of $\sigma_{NN}^{\pi\pi}$. To get a positive value of $\sigma_{NN}^{\pi\pi}$ the value of $\sigma_{NN}^{K^+K^-}$ must be as large as 1108 MeV. In a very recent work the same conclusion is obtained by Cornwell *et al* (1973) in the case of the (8, 8) model.

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