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Model-independent analysis of the neutral-current interaction in the inclusive neutrino reactions

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Abstract. On the basis of some general assumptions on the deep inelastic structure functions, such as scaling and chiral symmetry we determine the values of $\{(H_V^3)^2 + (H_A^3)^2\} + \eta\{(H_V^0)^2 + (H_A^0)^2\}$ and $H_V^3 H_A^3 + \eta H_V^0 H_A^0$ where $H_V^{3,0}$ and $H_A^{3,0}$ are the four coupling constants characterizing the hadronic neutral current and η is the ratio of the isoscalar to isovector structure functions. General expressions are given for the kinematical averages $\langle\nu\rangle$ and $\langle Q^2\rangle$ for the neutral-current reactions in terms of the coupling constants. This analysis does not depend on the validity of the quark-parton model.

Keywords. Neutral current; weak interaction; chiral symmetry; inclusive neutrino reactions; phenomenological analysis; deep inelastic structure functions.

1. Introduction

A large number of ν_μ-induced inelastic interactions without outgoing muons have been observed by Hasert *et al* (1973). These events have been tentatively ascribed to neutral-current weak interaction, although other explanations are not yet completely ruled out. There have been a number of calculations for these neutrino-induced inclusive neutral-current processes (Riazuddin and Fayyazuddin 1972, Pais and Treiman 1972, Budny and Scharbach 1972, Paschos and Wolfenstein 1973, Sehgal 1974, Palmer 1973, Albright 1973 and Pakvasa and Tuan 1973). However, all these calculations have assumed some specific model for the neutral-current weak interaction.

Our aim is to analyse the experimental results within a sufficiently general theoretical framework rather than interpret the data using a particular model of the neutral-current weak interaction. We attempted such a general analysis in our earlier paper (Rajasekaran and Sarma 1973 to be hereafter called paper I) where we determined the strength of the neutral current as well as the amount of the *VA* interference term on the basis of the quark-parton model. However in that paper, we did not investigate to what extent our results depended on the specific details of the quark-parton model. Also we restricted ourselves to the total cross sections and did not consider differential cross sections.

The present paper is devoted to a study of both these aspects. We have tried to see how far one can go without assuming a specific quark-parton model. We still have to make some assumptions which are of a general nature. Apart from the scaling of the deep inelastic structure functions, we assume the equality between the VV and AA structure functions (which is chiral symmetry) and also assume a relation between the VA structure function and the VV structure function. In addition, we identify the isoscalar vector part of the neutral current with the isoscalar part of the electromagnetic current. Because of this, we will need the experimental information on electromagnetic structure functions also. On the basis of these assumptions, we obtain expressions for the neutral-current coupling constants in terms of the total cross sections for the neutrino-induced reactions. Further the more general framework which we are presently adopting allows us to go beyond the total cross sections. We give formulae for double-differential cross sections as well as the kinematical averages $\langle \nu \rangle$ and $\langle Q^2 \rangle$ for the neutral-current processes.

In the next section, we introduce the various currents and the corresponding structure functions. In section 3, all the cross sections are calculated and the equations determining the neutral-current coupling constants are obtained. The calculation of $\langle \nu \rangle$ and $\langle Q^2 \rangle$ is given in section 4. In the last section, our assumptions are discussed and the results are summarized.

2. Currents and structure functions

The effective Lagrangian relevant for the ν_μ-induced reactions can be written as

$$\mathcal{L}_{\text{int}} = \frac{G}{\sqrt{2}}\{\bar{\mu}\gamma_\lambda (1 + \gamma_5)\, \nu\, (V_\lambda^+ + A_\lambda^+) + h\cdot c\}$$
$$+ \frac{1}{\sqrt{2}}\{\bar{\nu}\gamma_\lambda (1 + \gamma_5)\, \nu\, (H_V^3\, V_\lambda^3 + H_V^0 V_\lambda^0 + H_A^3\, A_\lambda^3 + H_A^0\, A_\lambda^0)\} \tag{1}$$

where G is the Fermi coupling constant and ν stands for ν_μ. The first term is the charged-current interaction while the second term is the neutral-current interaction. We ignore the strangeness-changing part of the charged-current interaction since its contribution to neutrino-reactions is known to be very small. Of course the strangeness-changing part of the neutral current can be ignored completely. The vector and axial vector currents are denoted by V_λ and A_λ respectively. The superscripts $\pm$ ($= 1 \pm i2$) and 3 refer to the isospin index of the isovector currents and the superscript 0 refers to isoscalar currents. The neutral current involves four coupling constants $H_V^{3,0}$ and $H_A^{3,0}$ which we shall take as real, and our aim is to try to determine these from experimental data on inclusive neutrino reactions.

We assume that the isoscalar current V_μ^0 occurring in the neutral current is the same as the isoscalar part of the electromagnetic current, apart from a possible numerical factor, which can be absorbed into the coupling constant H_V^0. So the electromagnetic current can be written as:

$$j_\mu^{\text{em}} = V_\mu^3 + V_\mu^0 \tag{2}$$

The inelastic structure functions of the proton are defined by the following equations:

$$\frac{1}{2\pi} \int d^4x e^{iq\cdot x} \langle p \mid [V_\mu^i (x), V_\nu^j (0)] \mid p \rangle = \delta_{\mu\nu} W_1^{ij} + \frac{p_\mu p_\nu}{m_p^2} W_2^{ij} \tag{3}$$

$$\frac{1}{2\pi}\int d^4x e^{iq\cdot x}\langle p \mid [A_\mu^i(x), A_\nu^j(0)] \mid p\rangle = \delta_{\mu\nu} U_1^{ij} + \frac{p_\mu p_\nu}{m_p^2} U_2^{ij} \tag{4}$$

$$\frac{1}{2\pi}\int d^4x e^{iq\cdot x}\langle p \mid [V_\mu^i(x), A_\nu^j(0)] \mid p\rangle = \mathscr{E}_{\mu\nu\alpha\beta}\frac{q_\alpha p_\beta}{m_p^2} W_3^{i,5j} \tag{5}$$

$$\frac{1}{2\pi}\int d^4x e^{iq\cdot x}\langle p \mid [A_\mu^i(x), V_\nu^j(0)] \mid p\rangle = \mathscr{E}_{\mu\nu\alpha\beta}\frac{q_\alpha p_\beta}{m_p^2} W_3^{5i,j} \tag{6}$$

where i and j go over $\pm$, 3 and 0, $\mid p\rangle$ denotes a spin-averaged proton state with momentum p and m_p is the mass of the proton. We have omitted the terms involving q_μ or q_ν. The structure functions W and U are functions of the invariants $Q^2 \equiv -q^2$ and $\nu = p\cdot q/m_p$. We may also note here the consequence of isospin invariance

$$W^{+-} + W^{-+} = 4W^{33}, \tag{7}$$

where W stands for any one of the functions $W_{1,2,3}$ and $U_{1,2}$.

We shall assume scaling (Bjorken 1969) so that for large values of ν and Q^2 such that $x = Q^2/2m_p\nu$ is finite, we have

$$\left.\begin{array}{ll} m_p W_1^{ij}(Q^2,\nu) \to F_1^{ij}(x) \;; & m_p U_1^{ij}(Q^2,\nu) \to D_1^{ij}(x) \\ \nu W_2^{ij}(Q^2,\nu) \to F_2^{ij}(x) \;; & \nu U_2^{ij}(Q^2,\nu) \to D_2^{ij}(x) \\ \nu W_3^{i,5j}(Q^2,\nu) \to F_3^{i,5j}(x) \;; & \nu W_3^{5i,j}(Q^2,\nu) \to F_3^{5i,j}(x) \end{array}\right\} \tag{8}$$

A basis assumption of our analysis is the equality of the vector and axial vector matrix elements in the scaling limit; *i.e.*, we assume

$$D_a^{ij} = F_a^{ij} \qquad (a = 1, 2) \tag{9}$$

$$F_3^{i,5j} = F_3^{5i,j} \equiv F_3^{ij} \tag{10}$$

Both these equations can be regarded as consequences of chiral symmetry for the isovector and isoscalar currents in the scaling limit, or, equivalently, on the light cone. Specifically, eqs (9) and (10) result, respectively, from the following two relations on the light-cone:

$$[V_\mu^i(x), V_\nu^j(0)] = [A_\mu^i(x), A_\nu^j(0)] \quad \text{for} \quad x^2 \to 0 \tag{11}$$

$$[V_\mu^i(x), A_\nu^j(0)] = [A_\mu^i(x), V_\nu^j(0)] \quad \text{for} \quad x^2 \to 0 \tag{12}$$

An example where such light-cone relations hold is the quark-model SU(3) $\otimes$ SU(3) light-cone algebra of Fritzsch and Gell-Mann (1971).

3. Cross sections

The processes of interest to us and our notation for the corresponding cross sections are as follows:

$\nu^\pm + \mathrm{p} \to \nu^\pm +$ hadrons	$\sigma_N(\nu^\pm p)$
$\nu^\pm + \mathrm{n} \to \nu^\pm +$ do.	$\sigma_N(\nu^\pm \mathrm{n})$
$\nu^\pm + \mathrm{p} \to \mu^\mp +$ do.	$\sigma_C(\nu^\pm \mathrm{p})$
$\nu^\pm + \mathrm{n} \to \mu^\mp +$ do.	$\sigma_C(\nu^\pm n)$.

We use ν^+ and ν^- to denote ν_μ and $\bar{\nu}_\mu$, respectively.

The double-differential cross sections for these inclusive neutrino reactions in the limit of very high neutrino energy E in the laboratory system can be expressed in terms of the scaled structure functions introduced in the last section. The variable Q^2 and ν are to be interpreted as the square of the four-momentum transfer (≥ 0) and the energy loss, respectively, of the incident lepton and we also define $y = \nu/E$. We get, in the limit of large values of E,

$$\frac{d^2\sigma_N}{dxdy}(\nu^{\pm}p) = \frac{m_p E}{\pi}\Big((1-y)\,[\{(H_V{}^3)^2+(H_A{}^3)^2\}\,F_2{}^{33}+\{(H_V{}^0)^2+(H_A{}^0)^2\}\,F_2{}^{00} + \{H_V{}^3H_V{}^0+H_A{}^3\,H_A{}^0\}\,(F_2{}^{30}+F_2{}^{03})] + y^2x\,[\{(H_V{}^3)^2+(H_A{}^3)^2\}F_1{}^{33} + \{(H_V{}^0)^2+(H_A{}^0)^2\}\,F_1{}^{00}+\{H_V{}^3\,H_V{}^0+H_A{}^3\,H_A{}^0\}\,(F_1{}^{30}+F_1{}^{03})] \pm y\,(1-\tfrac{1}{2}y)\,x\,[2H_V{}^3H_A{}^3\,F_3{}^{33}+2H_V{}^0\,H_A{}^0\,F_3{}^{00}+\{H_V{}^3\,H_A{}^0 + H_V{}^0\,H_A{}^3\}\,(F_3{}^{30}+F_3{}^{03})]\Big) \qquad (13)$$

$$\frac{d^2\sigma_N}{dxdy}(\nu^{\pm}n) = \frac{m_p E}{\pi}\Big((1-y)\,[\{(H_V{}^3)^2+(H_A{}^3)^2\}\,F_2{}^{33}+\{(H_V{}^0)^2 + (H_A{}^0)^2\}\,F_2{}^{00}-\{H_V{}^3\,H_V{}^0+H_A{}^3\,H_A{}^0\}\,(F_2{}^{30}+F_2{}^{03})] + y^2x\,[\{(H_V{}^3)^2+(H_A{}^3)^2\}\,F_1{}^{33}+\{(H_V{}^0)^2+(H_A{}^0)^2\}\,F_1{}^{00} - \{H_V{}^3H_V{}^0+H_A{}^3\,H_A{}^0\}\,(F_1{}^{30}+F_1{}^{03})] \pm y\,(1-\tfrac{1}{2}y)\,x\,[2H_V{}^3H_A{}^3\,F_3{}^{33} + 2H_V{}^0H_A{}^0\,F_3{}^{00}-\{H_V{}^3\,H_A{}^0+H_V{}^0\,H_A{}^3\}\,(F_3{}^{30}+F_3{}^{03})]\Big) \qquad (14)$$

$$\frac{d^2\,\sigma_C}{dxdy}(\nu^{\pm}p) = \frac{2m_p EG^2}{\pi}\,[(1-y)\,F_2{}^{\mp\pm}+y^2xF_1{}^{\mp\pm}\pm y\,(1-\tfrac{1}{2}y)\,xF_3{}^{\mp\pm}] \qquad (15)$$

$$\frac{d^2\sigma_C}{dxdy}(\nu^{\pm}n) = \frac{2m_p EG^2}{\pi}\,[(1-y)\,F_2{}^{\pm\mp}+y^2xF_1{}^{\pm\mp}\pm y\,(1-\tfrac{1}{2}y)\,xF_3{}^{\pm\mp}] \qquad (16)$$

In writing these equations, in addition to scaling, we have used the chiral symmetry assumption, namely, eqs (9) and (10).

Integrating over both x and y from 0 to 1, we get the total cross sections:

$$\sigma_N(\nu^{\pm}p) = \frac{2m_p E}{3\pi}\int_0^1 dx\,[\{(H_V{}^3)^2+(H_A{}^3)^2\}\,F^{33}+\{(H_V{}^0)^2+(H_A{}^0)^2\}\,F^{00} + \{H_V{}^3\,H_V{}^0+H_A{}^3\,H_A{}^0\}\,(F^{30}+F^{03})\pm\{H_V{}^3\,H_A{}^3\,xF_3{}^{33} + H_V{}^0\,H_A{}^0\,xF_3{}^{00}+\tfrac{1}{2}\,(H_V{}^3\,H_A{}^0+H_V{}^0\,H_A{}^3)\,x\,(F_3{}^{30}+F_3{}^{03})\}] \qquad (17)$$

$$\sigma_N(\nu^{\pm}n) = \frac{2m_p E}{3\pi}\int_0^1 dx\,[\{(H_V{}^3)^2+(H_A{}^3)^2\}\,F^{33}+\{(H_V{}^0)^2+(H_A{}^0)^2\}\,F^{00} - \{H_V{}^3\,H_V{}^0+H_A{}^3\,H_A{}^0\}\,(F^{30}+F^{03})\pm\{H_V{}^3\,H_A{}^3\,xF_3{}^{33} + H_V{}^0\,H_A{}^0\,xF_3{}^{00}-\tfrac{1}{2}\,(H_V{}^3\,H_A{}^0+H_V{}^0\,H_A{}^3)\,x\,(F_3{}^{30}+F_3{}^{03})\}] \qquad (18)$$

$$\sigma_C(\nu^{\pm}p) = \frac{2m_p EG^2}{3\pi}\int_0^1 dx\,[2F^{\mp\pm}\pm xF_3{}^{\mp\pm}] \qquad (19)$$

$$\sigma_C(\nu^{\pm}n) = \frac{2m_p\,EG^2}{3\pi}\int_0^1 dx\,[2F^{\pm\mp}\pm xF_3{}^{\pm\mp}] \qquad (20)$$

where we have defined

$$F^{ij} \equiv \tfrac{1}{4}(3F_2^{ij} + 2xF_1^{ij}) \tag{21}$$

At present, experimental data for neutrino reactions are available only for nuclear targets containing approximately equal number of protons and neutrons. So, it is more appropriate to consider the cross sections on the isospin-averaged nucleon target

$$\sigma_{N,C}(\nu^{\pm}\mathscr{N}) \equiv \tfrac{1}{2}\{\sigma_{N,C}(\nu^{\pm}\,p) + \sigma_{N,C}(\nu^{\pm}\,n)\}. \tag{22}$$

We obtain

$$\sigma_N(\nu^{\pm}\mathscr{N}) = \frac{2m_pE}{3\pi}\int_0^1 dx\,[\{(H_V^3)^2 + (H_A^3)^2\}F^{33} + \{(H_V^0)^2 + (H_A^0)^2\}F^{00} \pm \{H_V^3H_A^3\,xF_3^{33} + H_V^0H_A^0\,xF_3^{00}\}] \tag{23}$$

$$\sigma_C(\nu^{\pm}\mathscr{N}) = \frac{2m_pE}{3\pi}\int_0^1 dx\,[4G^2F^{33} \pm 2G^2\,xF_3^{33}] \tag{24}$$

where isospin invariance (eq. (7)) has been used in eq. (24).

Before proceeding further, we shall make our second assumption on the structure functions:

$$xF_3^{ij} = F^{ij} \tag{25}$$

This assumption which concerns the VA interference term is motivated by the following fact. By substituting eq. (25) in eq. (24), one gets

$$R_c \equiv \frac{\sigma_C(\bar{\nu}\mathscr{N})}{\sigma_C(\nu\mathscr{N})} = \frac{1}{3} \tag{26}$$

which is in remarkable agreement with the experimental values $0\cdot38 \pm 0\cdot02$ (Perkins 1972) and $0\cdot34 \pm 0\cdot03$ (Benvenuti *et al* 1974). Although this consideration motivates the assumption (25) only for isovector currents we shall assume it to be valid for isoscalar currents also. From the theoretical point of view, eq. (25) is valid in a large class of parton models—all models with no antifermion constituents inside the nucleon—and hence we may abstract it and use it as a general relation ignoring other results which depend on the details of specific parton models.

Using eq. (25) in eqs (23) and (24) and taking sum and difference of ν and $\bar{\nu}$ cross-sections, we get the desired relations for the coupling constants:

$$\frac{1}{4G^2}[\{(H_V^3)^2 + (H_A^3)^2\} + \eta\{(H_V^0)^2 + (H_A^0)^2\}] = \frac{\sigma_N(\nu\mathscr{N}) + \sigma_N(\bar{\nu}\mathscr{N})}{\sigma_C(\nu\mathscr{N}) + \sigma_C(\bar{\nu}\mathscr{N})} \tag{27}$$

$$\frac{1}{2G^2}[H_V^3H_A^3 + \eta H_V^0H_A^0] = \frac{\sigma_N(\nu\mathscr{N}) - \sigma_N(\bar{\nu}\mathscr{N})}{\sigma_C(\nu\mathscr{N}) - \sigma_C(\bar{\nu}\mathscr{N})} \tag{28}$$

where

$$\eta \equiv \int_0^1 F^{00}\,dx \Big/ \int_0^1 F^{33}\,dx \tag{29}$$

It should be noted that only scaling and chiral symmetry have been used in eq. (27) whereas the additional assumption of eq. (25) is needed for obtaining eq. (28).

Furthermore, in the derivation of both eq. (27) and eq. (28) actually we need only a milder version of the assumptions stated in eqs (9), (10) and (25)—namely these relations for the relevant integrals over x.

The ratio η can be determined from the data on deep inelastic electron scattering and charged-current neutrino interactions. In view of eq. (2) the electromagnetic structure functions measured in deep inelastic ep and en scattering can be written as*

$$F^{\text{ep}} = F^{33} + F^{00} + F^{30} + F^{03} \tag{30}$$

$$F^{\text{en}} = F^{33} + F^{00} - F^{30} - F^{03} \tag{31}$$

$$F^{\text{eN}} \equiv \tfrac{1}{2}(F^{\text{ep}} + F^{\text{en}}) = F^{33} + F^{00}. \tag{32}$$

From the inelastic ep and en scattering experiments of the SLAC-MIT group (*see* Poucher *et al* 1974 and references quoted therein; actually we have taken the numbers from Llewelyn Smith 1972 and Paschos 1973) one obtains

$$\int_0^1 dx F^{\text{ep}} = 0\cdot 16 \pm 0\cdot 02 \tag{33}$$

$$\int_0^1 dx F^{\text{en}} = 0\cdot 12 \pm 0\cdot 02 \tag{34}$$

$$\int_0^1 dx F^{\text{eN}} = 0\cdot 14 \pm 0\cdot 02 \tag{35}$$

On the other hand, using eq. (24), $\int_0^1 dx F^{33}$ can be obtained from the experimental value of $\{\sigma_{\text{C}}(\nu\mathscr{N}) + \sigma_{\text{C}}(\bar{\nu}\mathscr{N})\}$ (Perkins 1972) and the result is

$$\int_0^1 dx F^{33} = 0\cdot 12 \pm 0\cdot 02. \tag{36}$$

Hence we get

$$\eta = \frac{\int_0^1 dx F^{\text{eN}}}{\int_0^1 dx F^{33}} - 1 = 0\cdot 17 \pm 0\cdot 26. \tag{37}$$

Thus the value of η is very poorly known, ranging from zero (since it cannot be negative by definition) up to about 0·5. Because of the near equality of $\int_0^1 dx F^{\text{eN}}$ and $\int_0^1 dx F^{33}$, the numerical value of η has this large uncertainty within which the prediction of the quark-parton model

$$\eta_{q-p} = 1/9 \tag{38}$$

is included.

The experimental results of Hasert *et al* (1973) on the neutral-current processes are

$$r \equiv \frac{\sigma_{\text{N}}(\nu\mathscr{N})}{\sigma_{\text{c}}(\nu\mathscr{N})} = 0\cdot 21 \pm 0\cdot 03 \tag{39}$$

$$\bar{r} \equiv \frac{\sigma_{\text{N}}(\bar{\nu}\mathscr{N})}{\sigma_{\text{c}}(\bar{\nu}\mathscr{N})} = 0\cdot 45 \pm 0\cdot 09 \tag{40}$$

Here we have ignored the fact that the experimental ratios of Hasert *et al* pertain to only those events for which the energy-transfer to the target is greater than

* Superscript $^{\text{N}}$ denotes $\mathscr{N}$

1 GeV. By virtue of the expected Bjorken scaling, the correction due to this cut would become negligible as the energy E of the incident neutrino increases. This may not be the case for the above data which are for E in the range 1–10 GeV. Hence our numerical results based on the above values of r and $\bar{r}$ should be regarded as only illustrative, and could easily be recalculated when higher energy data become available.

We can now use the numerical results of eqs (26), (39) and (40) to calculate the right hand sides of eqs (27) and (28) and thus get*

$$\frac{1}{G^2}[\{(H_V^3)^2+(H_A^3)^2\}+\eta\{(H_V^0)^2+(H_A^0)^2\}]=1\cdot08\pm0\cdot12 \tag{41}$$

$$\frac{1}{G^2}[H_V^3 H_A^3+\eta H_V^0 H_A^0]=0\cdot18\pm0\cdot13. \tag{42}$$

We can get two more equations for the determination of the coupling constants if we go back to the formulae for $\sigma_N(\nu^\pm p)$ and $\sigma_N(\nu^\pm n)$ (eqs (17) and (18)). From these we get the following equations for the determination of the products of the isoscalar and isovector coupling constants:

$$\frac{\xi}{G^2}\{H_V^3 H_V^0+H_A^3 H_A^0\}=\frac{\{\sigma_N(\nu p)+\sigma_N(\bar{\nu}p)\}-\{\sigma_N(\nu n)+\sigma_N(\bar{\nu}n)\}}{\sigma_C(\nu\mathscr{N})+\sigma_C(\bar{\nu}\mathscr{N})} \tag{43}$$

$$\frac{\xi}{2G^2}\{H_V^3 H_A^0+H_V^0 H_A^3\}=\frac{\{\sigma_N(\nu p)-\sigma_N(\bar{\nu}p)\}-\{\sigma_N(\nu n)-\sigma_N(\bar{\nu}n)\}}{\sigma_C(\nu\mathscr{N})+\sigma_C(\bar{\nu}\mathscr{N})} \tag{44}$$

where we have defined

$$\xi=\frac{\int_0^1 dx\,(F^{30}+F^{03})}{2\int_0^1 dx F^{33}} \tag{45}$$

As before, only scaling and chiral symmetry have been used in deriving eq. (43) whereas eq. (44) requires the additional assumption of eq. (25). From eqs (45), (30) and (31) and using the experimental data in eqs (33) and (34), we get

$$\xi=\frac{1}{4}\frac{\int_0^1 dx\,(F^{ep}-F^{en})}{\int_0^1 dx F^{33}}=0\cdot08\pm0\cdot06 \tag{46}$$

which can again be compared with the prediction of the quark-parton model:

$$\xi_{q-p}=1/9 \tag{47}$$

Thus, we have obtained four equations (27), (28), (43) and (44) for the four coupling constants $H_{V,A}^{3,0}$. For the determination of these four coupling constants, it may be more convenient to cast the above four equations into the following alternative form:

* In comparing eqs (27) and (28) or eqs (41) and (42) with the corresponding results of paper I, a difference in the normalisation of the isoscalar currents of the present paper as compared to that of paper I should be noted. In view of eq. (2), the isoscalar currents V_μ^0 and A_μ^0 in the present paper are a factor $\frac{1}{2}$ smaller than those in paper I so that, effectively $\frac{1}{2}H_{V,A}^0$ of the present paper equals $H_{V,A}^0$ appearing in paper I.

$$\frac{\left(1+\frac{\xi}{\sqrt{\eta}}\right)\sigma_N(\bar{\nu}p)-\left(1-\frac{\xi}{\sqrt{\eta}}\right)\sigma_N(\bar{\nu}n)}{\left(1+\frac{\xi}{\sqrt{\eta}}\right)\sigma_N(\nu p)-\left(1-\frac{\xi}{\sqrt{\eta}}\right)\sigma_N(\nu n)}=\frac{1+\beta_+^2-\beta_+}{1+\beta_+^2+\beta_+} \tag{48}$$

$$\frac{\left(1+\frac{\xi}{\sqrt{\eta}}\right)\sigma_N(\bar{\nu}n)-\left(1-\frac{\xi}{\sqrt{\eta}}\right)\sigma_N(\bar{\nu}p)}{\left(1+\frac{\xi}{\sqrt{\eta}}\right)\sigma_N(\nu n)-\left(1-\frac{\xi}{\sqrt{\eta}}\right)\sigma_N(\nu p)}=\frac{1+\beta_-^2-\beta_-}{1+\beta_-^2+\beta_-} \tag{49}$$

$$\left(1+\frac{\xi}{\sqrt{\eta}}\right)\frac{\sigma_N(\nu p)}{\sigma_C(\nu\mathscr{N})}-\left(1-\frac{\xi}{\sqrt{\eta}}\right)\frac{\sigma_N(\nu n)}{\sigma_C(\nu\mathscr{N})}=\frac{4}{3}\frac{\xi}{\sqrt{\eta}}\left(\frac{H_V^+}{G}\right)^2(1+\beta_+^2+\beta_+) \tag{50}$$

$$\left(1+\frac{\xi}{\sqrt{\eta}}\right)\frac{\sigma_N(\nu n)}{\sigma_C(\nu\mathscr{N})}-\left(1-\frac{\xi}{\sqrt{\eta}}\right)\frac{\sigma_N(\nu p)}{\sigma_C(\nu\mathscr{N})}=\frac{4}{3}\frac{\xi}{\sqrt{\eta}}\left(\frac{H_V^-}{G}\right)^2(1+\beta_-^2+\beta_-) \tag{51}$$

where we have defined

$$\left.\begin{aligned}H_{V,A}^{\pm} &\equiv \tfrac{1}{2}(\sqrt{\eta}\,H_{V,A}^{0}\pm H_{V,A}^{3})\\ \beta_\pm &\equiv H_A^{\pm}/H_V^{\pm}\end{aligned}\right\} \tag{52}$$

These four equations are equivalent to eqs (14)–(17) of paper I if the quark-parton model values $\eta_{q-p}=\xi_{q-p}=\frac{1}{9}$ are used. Thus the important feature of the present analysis is that instead of assuming the validity of the quark-parton model we need to introduce two new parameters η and ξ whose values can, however, be obtained directly from experimental data on deep inelastic electron scattering. It may be useful to note that the parameter $\xi/\sqrt{\eta}$ occurring in the above equations is constrained by the Schwarz inequality:

$$|\xi|/\sqrt{\eta}\leqslant 1 \tag{53}$$

From the four eqs (48)–(51), the four parameters β_+, β_-, H_V^+ and H_V^- and hence the parameters $H_{V,A}^{3,0}$ can be determined, apart from the quadratic ambiguities of the type already enumerated in paper I. However, such a programme has to await the measurement of $\sigma_N(\nu^\pm p)$.

Finally we may remark that, in view of the large uncertainties in the values of the parameters η and ξ, it may be worthwhile to eliminate these parameters. We can eliminate η between the two eqs (41) and (42) and ξ between eqs (43) and (44) and thus get

$$\frac{H_V^3H_A^3-(0\cdot18\pm0\cdot13)G^2}{(H_V^3)^2+(H_A^3)^2-(1\cdot08\pm0\cdot12)G^2}=\frac{H_V^0H_A^0}{(H_V^0)^2+(H_A^0)^2}$$

$$\frac{H_V^3H_A^0+H_A^3H_V^0}{H_V^3H_V^0+H_A^3H_A^0}=\frac{\{\sigma_N(\nu p)-\sigma_N(\bar{\nu}p)\}-\{\sigma_N(\nu n)-\sigma_N(\bar{\nu}n)\}}{\{\sigma_N(\nu p)+\sigma_N(\bar{\nu}p)\}-\{\sigma_N(\nu n)+\sigma_N(\bar{\nu}n)\}}$$

The first equation provides a constraint on the coupling constants whereas the second equation can give the value of a particular combination of the coupling constants. Both these equations are independent of the validity of the assumption that the isoscalar vector current occurring in the neutral current is the same as the isoscalar part of the electromagnetic current.

4. Calculation of $\langle\nu\rangle$ and $\langle Q^2\rangle$

The average values $\langle\nu\rangle$ and $\langle Q^2\rangle$ are defined by the formulae:

$$\langle\nu\rangle = E\langle y\rangle = \frac{E}{\sigma}\int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y\, y\, \frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}y} \tag{54}$$

$$\langle Q^2\rangle = 2m_p E\langle xy\rangle = \frac{2m_p E}{\sigma}\int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y\, xy\, \frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}y} \tag{55}$$

where σ is the total cross section. It is straightforward to calculate these quantities from eqs (13)–(16). In order to obtain results in a form which are usable in practice we shall assume the Callan-Gross relation (Callan and Gross 1969):

$$F_2^{ij} = 2xF_1^{ij} \equiv F^{ij} \tag{56}$$

using the same symbol F^{ij} defined in eq. (21). This relation, it may be recalled, follows if the currents are constructed out of spin $\frac{1}{2}$ fields only.

Thus we get for the neutral-current reactions on the isospin-averaged target $\mathscr{N}$,

$$\langle\nu\rangle_N(\nu^{\pm}\mathscr{N}) = \frac{E}{16}\left(\frac{7 \pm 10J}{1 \pm J}\right) \tag{57}$$

where

$$J = \frac{H_V^3 H_A^3 + H_V^0 H_A^0 \eta}{\{(H_V^3)^2 + (H_A^3)^2\} + \{(H_V^0)^2 + (H_A^0)^2\}\eta}\,. \tag{58}$$

This ratio J is determined by eqs (41) and (42) to be $0{\cdot}17 \pm 0{\cdot}12$ and hence we predict

$$\langle\nu\rangle_N(\nu\mathscr{N}) = E \times (0{\cdot}46 \pm 0{\cdot}02);\ \langle\nu\rangle_N(\bar{\nu}\mathscr{N}) = E \times (0{\cdot}40 \pm 0{\cdot}03). \tag{59}$$

An experimental verification of these values for $\langle\nu\rangle_N$ will provide a test of the assumptions we have made.

For $\langle Q^2\rangle$, we get

$$\langle Q^2\rangle_N(\nu^{\pm}\mathscr{N}) = \frac{m_p E}{8}\rho\, \frac{7\{(H_V^3)^2 + (H_A^3)^2\} + 7\{(H_V^0)^2 + (H_A^0)^2\}\eta' \pm 10\{H_V^3 H_A^3 + H_V^0 H_A^0 \eta'\}}{\{(H_V^3)^2 + (H_A^3)^2\} + \{(H_V^0)^2 + (H_A^0)^2\}\eta \pm \{H_V^3 H_A^3 + H_V^0 H_A^0 \eta\}} \tag{60}$$

where

$$\eta' = \int_0^1 \mathrm{d}x\, x\, F^{00} \Big/ \int_0^1 \mathrm{d}x\, x\, F^{33} \tag{61}$$

and

$$\rho = \int_0^1 \mathrm{d}x\, x\, F^{33} \Big/ \int_0^1 \mathrm{d}x\, F^{33} \tag{62}$$

Both η' and ρ can be determined from experimental data as follows. Let us first calculate $\langle\nu\rangle$ and $\langle Q^2\rangle$ for the charged-current processes. We find from eqs (15) and (16), [or by setting $H_V^0 = H_A^0 = 0$ and $H_V^3 = H_A^3$ in eqs (57) and (60)],

$$\langle\nu\rangle_C(\nu\mathscr{N}) = E/2;\ \langle\nu\rangle_C(\bar{\nu}\mathscr{N}) = E/4 \tag{63}$$

$$\langle Q^2\rangle_C(\nu\mathscr{N}) = m_p E\rho;\ \langle Q^2\rangle_C(\bar{\nu}\mathscr{N}) = m_p E\rho/2 \tag{64}$$

Hence, ρ can be determined from the experimental value of $\langle Q^2\rangle_C$ which is (Myatt and Perkins 1971)

$$\langle Q^2\rangle_C (\nu\mathscr{N}) \approx \tfrac{1}{9}\, 2m_p E \tag{65}$$

and therefore

$$\rho \approx 0{\cdot}22 \tag{66}$$

Using this value of ρ and the experimental information on the electro-magnetic structure function

$$\int_0^1 \mathrm{d}x\, x\, F^{eN} \approx 0{\cdot}03 \tag{67}$$

we can determine η'. We get

$$\eta' = \frac{\int_0^1 xF^{eN}\,\mathrm{d}x}{\rho \int_0^1 F^{33}\,\mathrm{d}x} - 1 \approx 0{\cdot}14 \tag{68}$$

where we have also used eq. (36). This value of η' is again consistent with the quark-parton model prediction

$$\eta'_{Q-P} = \tfrac{1}{9} \tag{69}$$

Whatever be the value of η', if $\eta' = \eta$, then we can predict the value of $\langle Q^2\rangle_N$ also:

$$\langle Q^2\rangle_N (\nu\mathscr{N}) \approx m_p E \times 0{\cdot}20; \quad \langle Q^2\rangle_N (\bar{\nu}\mathscr{N}) \approx m_p E \times 0{\cdot}18 \tag{70}$$

On the other hand, if $\eta' \neq \eta$, the values of $\langle Q^2\rangle_N$ $(\nu\mathscr{N})$ and $\langle Q^2\rangle_N$ $(\bar{\nu}\mathscr{N})$ provide information on two combinations of coupling constants not encountered in the formulae for total cross sections for the isospin-averaged target and so eq. (60) along with eqs (27) and (28) may help to determine all the four coupling constants.

If experimental data are available only for a limited range of the variables x and y, kinematical averages over these can also be calculated by trivial modification of the above procedure. In addition, from eqs (13) and (14) which give the complete distributions in x and y for the neutral-current processes, one can calculate many other quantities of interest—x or y distributions, higher moments of these distributions, etc. Experimental information on these can be very useful, particularly because, just as in the case of $\langle Q^2\rangle_N$ already pointed out, such information even for isospin-averaged targets may provide a way of determining all the four coupling constants of the neutral current.

5. Discussion and Summary

First we shall briefly discuss the assumptions on which the present analysis is based. The general form of the hadronic neutral current consists of four parts V_μ^3, A_μ^3, V_μ^0 and A_μ^0. We have assumed that the isovector parts V_μ^3 and A_μ^3 are members of the same isotriplet to which $V_\mu^\pm$ and $A_\mu^\pm$ of the charged current belong. As far as the isoscalar parts of the neutral current are concerned, we identify V_μ^0 with the isoscalar electro-magnetic current and, then, by means of the chiral symmetry assumption we relate A_μ^0 to V_μ^0. Thus, through our assumptions, we have provided sources of empirical information on all the four parts of the neutral current.

Our identification of the isoscalar current V_μ^0 occurring in the neutral current with the isoscalar part of the electromagnetic current means that we have assumed

$V_\mu{}^0$ to be the hypercharge current. A natural question arises as to why one should not identify $V_\mu{}^0$ with the baryonic current (Sakurai 1973), or more generally a linear combination of the hypercharge and baryonic currents. However, in view of the fact that strange constituents seem to make very little contribution to the structure of the nucleon, there is little difference between the hypercharge current and the baryonic current as far as experiments on targets made of nucleons are concerned. Hence our identification of the isoscalar part seems to be a reasonable one, particularly from the point of view of a phenomenological analysis since this enables one to determine the isoscalar part from the electromagnetic data.

Our assumption on the *VA* structure function, namely, eq. (25) is well-supported for the isovector currents by the experimental values of the ratio R_c [*see* eq. (26)] already mentioned. For the isoscalar currents, eq. (25) has to be regarded as a pure hypothesis for the present.

Next we come to the assumption of chiral symmetry, *i.e.*, eqs (9) and (10). As pointed out by Paschos and Zacharov (1973), the experimental value of R_c can also be used to test eq. (9) for the isovector currents. For this purpose, one can use the following Schwarz inequality which can be obtained by considering the total absorption cross sections for the right-handed and left-handed currents (*see* Lee and Yang 1962 and Bjorken and Paschos 1970 for the definitions of these cross sections):

$$\left(\int_0^1 F^{33}\,\mathrm{d}x\right)\left(\int_0^1 D^{33}\,\mathrm{d}x\right) \geqslant \left(\int_0^1 xF_3{}^{33}\,\mathrm{d}x\right)^2 \tag{71}$$

On the other hand, without the chiral symmetry assumption, eq. (24) should be replaced by

$$\sigma_C\,(\nu^\pm\,\mathscr{N}) = \frac{4m_p\,EG^2}{3\pi}\int_0^1 \mathrm{d}x\,[F^{33} + D^{33} \pm x\,F_3{}^{33}] \tag{72}$$

Using eqs (71) and (72), we get the following bound on chiral-symmetry breaking:

$$\frac{\{\int_0^1 \mathrm{d}x\,(F^{33} - D^{33})\}^2}{\{\int_0^1 \mathrm{d}x\,(F^{33} + D^{33})\}^2} = 1 - \frac{4\left(\int_0^1 \mathrm{d}x\,F^{33}\right)\left(\int_0^1 \mathrm{d}x\,D^{33}\right)}{\{\int_0^1 \mathrm{d}x\,(F^{33} + D^{33})\}^2}$$

$$\leqslant 1 - \frac{4\,(1 - R_c)^2}{(1 + R_c)^2} \tag{73}$$

The right-hand side of this inequality is exactly zero for $R_c = 1/3$ or 3. This shows that chiral symmetry for the isovector currents would be exact if R_c were exactly equal to either of these values. For the experimental value $R_c = 0{\cdot}34 \pm 0{\cdot}03$ given by Benvenuti *et al* the above right-hand side is $0{\cdot}03 \pm 0{\cdot}27$ (the negative values are to be ignored however). So, the chiral symmetry assumption (*i.e. eq.* (9)) for the isovector currents is in agreement with the present experimental data although the errors are too large. Unfortunately, no such test is available for the isoscalar currents.

Finally, we have to mention that in the calculation of $\langle\nu\rangle$ and $\langle Q^2\rangle$ we assumed Callan-Gross relation (eq (56)). The validity of this relation for isovector currents again follows from the experimental value of R_c being close to 1/3 (*see* for instance Musset 1973). The ratio of the total absorption cross section for scalar virtual photon to the transverse virtual photon appears to be small (about 18%) both for proton targets as well as for neutron targets (Poucher *et al* 1974). This suggests that Callan-Gross relation is perhaps valid for both the isovector and isoscalar parts of the electromagnetic current.

Let us now sum up the main results of our analysis. The hadronic neutral current involves four coupling constants $H_V^{3,0}$ and $H_A^{3,0}$ to be determined from experimental data. On the basis of certain very general assumptions, we have been able to determine the values of $\{(H_V^3)^2 + (H_A^3)^2\} + \eta\,\{(H_V^0)^2 + (H_A^0)^2\}$ and $H_V^3 H_A^3 + \eta H_V^0 H_A^0$, where η is the ratio of the isoscalar to isovector structure functions. This ratio η, is unfortunately not well-determined by the present data. Separate determination of the isoscalar and isovector coupling constants has to await experimental data on the neutral-current cross sections for proton targets.

We have also written down the double-differential cross sections and the kinematical averages $\langle \nu \rangle$ and $\langle Q^2 \rangle$ for the neutral-current processes. These can be used for pinning down the values of the coupling constants further.

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