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Creation-field theory from dimensional analysis

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Abstract. A Lagrangian is obtained which is invariant under space-time dependent changes in the units of mass, length and time. It contains two scalar fields, one of which is effectively the Brans-Dicke scalar (varying gravitational constant), while the other can be interpreted as a creation-field.

Keywords. Creation-field; dimensional analysis; Brans-Dicke scalar; conformal invariance.

1. Introduction

The earliest physical theory with the property of covariance under conformal mappings (space-time dependent redefinition of the unit of length) was Weyl's geometry (Weyl 1918, 1919). Its significance in this respect has been unfortunately somewhat obscured by the fact that Weyl was attempting to interpret the group of conformal mappings as the gauge group of electromagnetism. The possibility of incorporating conformal invariance in physical theory has been considered more recently by Hoyle and Narlikar (1971, 1972) and by Omote (1971). In a previous paper (Lord 1972) we presented a conformally-covariant scalar-vector-tensor theory in which the vector was essentially the Weyl vector (which is the Yang-Mills field associated with the group of conformal mappings) and the scalar was essentially the Brans-Dicke scalar field (Dicke and Brans 1961) and also the 'mass field' of Hoyle and Narlikar. A special choice of the adjustable parameter in the Lagrangian density of the scalar-vector-tensor theory gives Omote's Lagrangian density. Dicke (1962) has shown how conformal mappings can be used to obtain alternative formulations of a physical theory not possessing conformal covariance.

The conformal mappings discussed in the above papers are space-time dependent changes in the unit of *length*. The changes in the units of mass and time are linked to those of length by the requirement that $\hbar$ and c be invariant (and constant). To be completely general we should try to formulate physical laws in a manner that is invariant under three kinds of transformation corresponding to space-time dependent changes in the units of mass, length and time, independently. The motivation here comes from the fact that observations depend on the comparison of two quantities at the same space-time point. We should be able to choose our imposed 'absolute' standards of measurement (units of mass, length and time) arbitrarily and the formulation of physical laws should be capable of expression in a manner that is independent of this choice, reflecting the fact that the actual physical phenomena we are attempting to describe are independent of the choice of units. The choice of a standard of measurement is then a matter of *convenience* only, in that the description of particular physical phenomena will in general be simpler for some choices than for others. Moreover, there is no reason to suppose that a single universal convention will be appropriate for all physical phenomena.

To clarify the above statements, note that the standard of measurement is usually chosen so that $\hbar$ and c are constants, so that only a choice of length unit remains. Two particular choices are: (i) Newton's gravitational parameter G is a constant, and (ii) the rest-mass of an electron is a constant. These two choices of gauge for the group of conformal mappings may not be mutually consistent.

In the following sections we shall present a formulation of physical law that is fully covariant under independent (space-time dependent) redefinition of the units of mass, length and time, and which looks like a creation-field (Hoyle and Narlikar 1963) if we impose the convention (i) and like Brans-Dicke theory if we choose the convention (ii).

The crucial point is that the question of whether or not a physical quantity such as c, $\hbar$ or G is or is not a constant is devoid of meaning, since the ' standard of measurement ' is subject to arbitrary choice and is not inherent in the actual physical world. We are free to let *any* physical quantity that is not dimensionless be a constant, by definition. However, when several such physical quantities can be combined into a *dimensionless* number, then the question of whether this number is a constant is a meaningful question that can be answered by reference to the real physical world.

From these considerations, it is apparent that we should be able to require of a physical theory that it be indifferent to the standard of measurement (choice of units, or conventions concerning which quantities shall be regarded as constants).

The behaviour of known physical theories under space-time dependent changes of the units of mass, length and time was investigated by Nariai and Ueno (1960), who showed that by variation of the length unit independently along three spatial axes at each space-time point, as well as space-time dependent variation of the time unit, any Riemannian geometry (with metric signature $+++-$) can be regarded as flat. The generality involved in the possibility of defining the length unit independently along three axes can be restricted in a natural manner by noting that any metric space (with positive definite metric) is Euclidean in infinitesimal regions, so the system of length measurement in any such region can be chosen so as to conform to the natural Euclidean metric. This amounts to an isotropy condition on the length unit at a point, so that we have a single length unit at each space-time point. This isotropy of length measurement has an absolute meaning since it is related to the Euclidean nature of infinitesimal regions, whereas a homogeneity of length measurement (length unit independent of position in space-time) is only a relative concept. In a similar way, space and time measurement in a Riemannian space-time can be linked by invoking a similar argument involving the natural Minkowskian metric of infinitesimal regions. This is tantamount to choosing the physical quantity c to be equal to unity. The linking of transformations of mass to transformations of length by defining $\hbar$ to be a constant has no such geometrical basis (at least in the theoretical structures that are available at the present time).

In section 3 we shall investigate the consequences of regarding both $\hbar$ and c as space-time dependent quantities.

2. Scalar tensor theories

Let a Lagrangian density be constructed out of metric, a scalar field σ, and a set of 'matter' fields, ψ. Then under arbitrary variations of the fields, write

$$\delta\mathcal{L} \sim \tfrac{1}{2}\mathcal{T}^{\mu\nu}\,\delta g_{\mu\nu} + \mathcal{S}\delta\sigma + \mathcal{F}.\delta\psi \tag{2.1}$$

where $\sim$ is used to indicate that two quantities differ only by the divergence of a four-vector density of weight 1. If $\mathcal{L}$ is the total Lagrangian then

$$\mathcal{T}^{\mu\nu} = 0, \qquad \mathcal{S} = 0, \qquad \mathcal{F} = 0 \tag{2.2}$$

are the Euler-Lagrange equations for the three fields $g_{\mu\nu}$, σ and ψ respectively. General covariance of these equations is ensured by requiring $\mathcal{L}$ to be a scalar-density of weight 1. Then under an infinitesimal co-ordinate transformation $x^{\mu} \to x^{\mu} + \xi^{\mu}$, the variations are

$$\left.\begin{aligned} \delta\mathcal{L} &= -(\xi^{\mu}\mathcal{L})_{|\mu}, \\ \delta g_{\mu\nu} &= -(\xi_{\mu|\nu} + \xi_{\nu|\mu}), \\ \delta\sigma &= -\xi^{\mu}\sigma_{|\mu}, \\ \delta\psi &= \xi_{|\nu}{}^{\mu}\, G_{\mu}{}^{\nu}\psi - \xi^{\nu}\psi_{|\nu}, \end{aligned}\right\} \tag{2.3}$$

where ${}_{|\mu}$ denotes covariant differentiation constructed out of the Christoffel symbols formed from $g_{\mu\nu}$ (ordinary partial differentiation $\partial/\partial x^{\mu}$ will be denoted by $\cdot_{\mu}$). The $G_{\mu\nu}$ are a set of constant matrices associated with the tensor index structure of ψ. Substituting (2.3) in (2.1) gives

$$0 \sim \mathcal{T}^{\mu\nu}{}_{|\nu}\xi_{\mu} - \mathcal{S}\sigma^{\cdot\mu}\,\xi_{\mu} - \xi^{\mu}\,[(\mathcal{F}.G_{\mu}{}^{\nu}\psi)_{|\nu} + \mathcal{F}.\psi_{|\mu}] \tag{2.4}$$

Integrating over a region of four-space on the boundary of which ξ_{μ} vanishes, and noting that the ξ_{μ} are otherwise arbitrary, we obtain the *identity*

$$\mathcal{T}^{\nu}{}_{\mu|\nu} = \sigma_{\cdot\mu}\mathcal{S} + (\mathcal{F}.G_{\mu}{}^{\nu}\psi)_{|\nu} + \mathcal{F}.\psi_{|\mu} \tag{2.5}$$

Hence, for *any* generally covariant scalar-tensor theory, the field equation for the scalar is *implied* by the field equations for the metric and the field equations for matter. The field equation for the scalar is therefore redundant—it does not carry any extra information. This property of scalar-tensor theories was discovered by Horndeski and Lovelock (1972). It is true for example, for the usual Einstein theory where the energy-momentum tensor is that of a scalar meson (Yilmaz 1958), for the Hoyle-Narlikar creation field-theory and for the Jordan theories (Jordan 1955) (including Brans-Dicke theory as a special case).

If $\mathcal{L}$ in (2.1) is taken to be the matter Lagrangian density only (*i.e.* the part that contains ψ), then $\mathcal{T}^{\mu\nu}$ and $\mathcal{S}$ are interpreted as the *sources* of the metric field and the scalar field ($\mathcal{T}^{\mu\nu}$ is the symmetric energy-momentum tensor (density) for ψ), and $\mathcal{F} = 0$ is still the set of field equations for ψ. The above argument then leads to a relationship

$$\mathcal{T}^{\nu}{}_{\mu|\nu} = \sigma_{\cdot\mu}\mathcal{S} \tag{2.6}$$

between the sources, which is valid for any scalar-tensor theory that is generally covariant.

If the matter Lagrangian is also *conformally invariant*, that is, invariant under the transformations

$$\left.\begin{aligned} \delta g_{\mu\nu} &= 2\lambda g_{\mu\nu} \\ \delta\sigma &= -\lambda\sigma \\ \delta\psi &= N\lambda\psi \end{aligned}\right\} \tag{2.7}$$

for space-time dependent λ, we can substitute (2·7) into (2·1) to obtain the identity

$$\mathcal{J} = \mathcal{S} \tag{2.8}$$

The energy-momentum tensor associated with any *conformally invariant* matter Lagrangian constructed from ψ, $g_{\mu\nu}$ and a scalar field σ therefore satisfies the (conformally-covariant) relation

$$\mathcal{J}^{\nu}{}_{\mu|\nu} = \sigma_{\cdot\mu}\mathcal{J} \tag{2.9}$$

3. Mass and time transformations

The units in which the Lagrangian density

$$(-g)^{\frac{1}{2}}(c^4/16\pi G)\,R + \mathcal{L}(\Psi) \tag{3.1}$$

are expressed have the structure ML^3T^{-2} (*i.e.* the same as $\hbar c$). The units of various physical quantities appearing in it are

$$g_{\mu\nu}\colon\ L^2,\ (c^4/16\pi G)\colon\ MLT^{-2},\ (mc/\hbar)\colon L^{-1} \tag{3.2}$$

We assume that $\mathcal{L}$ is bilinear in Ψ. It then follows that the units of Ψ are

$$\Psi\colon\ M^{\frac{1}{2}}\,L^{\frac{1}{2}-s}\,T^{-1} \tag{3.3}$$

for a spin-s field, if Ψ is expressed in spinor formalism. Hence mass and time only occur in the combination MT^{-2}, in any quantity occurring in the Lagrangian density. The requirement that any interaction term involving two or more matter fields ψ shall have units ML^3T^{-2}, to make the overall Lagrangian density dimensionally consistent, then implies that any extra coupling constant shall have units in which mass and time occur only in the combination MT^{-2}. Thus all the usual Lagrangian densities in physics are unchanged if we make an arbitrary space-time dependent change of the units of mass and time according to $T \to \lambda T$, $M \to \lambda^2 T$. Dividing (3.1) throughout by $\hbar c$ gives a *dimensionless* Lagrangian density

$$(-g)^{\frac{1}{2}}(2\kappa)^{-1}R + \mathcal{L}(\psi) \tag{3.4}$$

where $\kappa = 8\pi G\hbar/c^4$, which has dimensions L^2 and ψ is the field $(\hbar c)^{-\frac{1}{2}}\Psi$ which has dimensions L^{-1-s}. Now *only the unit of length* appears in any quantity in (3.4) so that in the new formulation, we have complete invariance under arbitrary changes in the units of mass and time. Clearly, any interaction term in this formulation will have a coupling constant whose units are just a power of a length. For example, the minimal electromagnetic interaction is obtained by replacing ∂_μ by $\partial_\mu + (ie/\hbar c)A_\mu$ where A_μ has dimensions $M^{\frac{1}{2}}L^{3/2}T^{-1}$ [the spinor associated with the electromagnetic field is $\partial_\mu A_\nu \sigma^{\mu\nu}$ and $\sigma^{\mu\nu}$ has dimensions L^{-2}, so this is in agreement with (3.3) for spin 1]. In terms of the field $a_\mu = (\hbar c)^{-\frac{1}{2}}A_\mu$ the derivative operator is $\partial_\mu + ia_\mu\sqrt{\alpha}$, where α is the *dimensionless* coupling constant $e^2/\hbar c$. As a second example, the weak interaction involves four spin-½ fields ψ (dimensions $L^{-3/2}$). Omitting the detailed V–A structure it is just

$$(-g)^{\frac{1}{2}}\, G_{w}\, (\psi)^{4} \tag{3.5}$$

which is required to be dimensionless so G_w has dimensions L^2. To obtain the usual formulation we multiply by $\hbar c$ and rewrite in terms of $\Psi = (\hbar c)^{\frac{1}{2}}\psi$. The usual weak coupling constant is therefore $G_w/\hbar c$, with units $(ML)^{-1}\,T^2$.

Thus there is a very simple prescription for expressing any Lagrangian field theory in a form in which the units of mass and time do not appear. The Lagrangian densities (3.4) with this property are still not covariant under *length* transformations (conformal mappings). The curvature scalar does not transform in a conformally covariant manner because the derivatives of the length unit appear through the derivatives of $g_{\mu\nu}$, and the derivatives of ψ are also not conformally covariant (the fact that $(mc/\hbar)$ and the coupling 'constants' can no longer be regarded as constants when a length transformation is carried out is of course *not* a drawback from the viewpoint we are adopting). However, eq. (3.4) can easily be generalised to a conformally covariant form by means of a scalar field σ.

4. Length transformations

From any scalar field σ we can construct a Weyl vector

$$\phi_{\mu} = -\,\sigma^{-1}\,\sigma_{\cdot\mu} \tag{4.1}$$

which under conformal mappings (2.7) transforms according to $\phi \rightarrow \phi_{\mu} + \partial_{\mu}\lambda$. This vector can be used to construct conformally covariant derivatives of any fielde according to the prescription of Lord (1972). But now there is a considerabl, simplification because the Weyl vector is now taken to be simply the derivative of a scalar. The conformally-covariant derivative will be denoted by a *semi-colon* and for a Weyl vector of the form (4.1) is just

$$\psi_{;\,\mu} = \sigma^{-N}\,(\sigma^{N}\psi)_{,\mu}$$

The comma denotes covariant differentiation constructed out of the Christoffel symbols formed from the 'dimensionless metric' $\mathring{g}_{\mu\nu} = \sigma^2 g_{\mu\nu}$. Then by definition

$$\sigma_{;\mu} = 0, \qquad g_{\mu\nu;\,\rho} = 0 \tag{4.2}$$

The prescription for constructing the conformally covariant Lagrangian now reduces to the following: In any Lagrangian (3.4) replace all the fields throughout by the *dimensionless* fields

$$\mathring{g}_{\mu\nu} = \sigma^2 g_{\mu\nu}, \qquad \mathring{\psi} = \sigma^{N}\,\psi \tag{4.3}$$

and replace all parameters such as coupling 'constants' and Compton wavelengths by dimensionless parameters.

$$(-\mathring{g})^{\frac{1}{2}}\,(2\kappa)^{-1}\,\mathring{R} + \mathcal{L}\,(\mathring{\psi}) \tag{4.4}$$

(where κ is a dimensionless constant) is now conformally covariant. The conformal covariance is trivial in the sense that σ (which effectively defines a length unit) does not appear. However, we can restore it by rewriting (4.4) in terms of $g_{\mu\nu}$, σ and ψ instead of $\mathring{g}_{\mu\nu}$ and $\mathring{\psi}$. We have

$$\left.\begin{aligned}(-\overset{\circ}{g})^{\frac{1}{2}}\overset{\circ}{R} &= (-g)^{\frac{1}{2}}\sigma^2(R-6\sigma^{-1}\Box\sigma)\\ &\sim(-g)^{\frac{1}{2}}(\sigma^2R+6\sigma_{\cdot\mu}\sigma^{\cdot\mu})\end{aligned}\right\} \tag{4.5}$$

where

$$\Box\sigma = (-g)^{-\frac{1}{2}}\,[(-g)^{\frac{1}{2}}\,\sigma^{\cdot\mu}]_{\cdot\mu} \tag{4.6}$$

Thus a Lagrangian density for the free σ field is actually implicit in the formulation and does not have to be added as an extra term. The fact that σ does not appear in the formulation (4.4) gives rise to no paradox, since we have already observed that the Euler-Lagrange equation of the scalar in a scalar-tensor theory contain no additional information. The expression (4.5) is just the Brans-Dicke Lagrangian for $\omega = -3/2$. This value of ω is of course hopelessly unphysical. In order to change the value of ω we would have to add on an *additional* Lagrangian density for σ. But this cannot be done without violating the conformal symmetry, since the conformally-covariant derivative of σ is by definition zero. This difficulty was not encountered in Lord (1972) because ϕ_μ was treated as an independent vector field, it was not constructed according to (4.1). Note also that the matter Lagrangian of (4.4) expressed in terms of $g_{\mu\nu}$ and ψ contains σ, whereas in the Brans-Dicke theory the scalar field is not contained in the 'matter' Lagrangian.

It is possible to obtain the Brans-Dicke theory precisely, with arbitrary ω, from a conformally-invariant Lagrangian density. This is done by introducing a second scalar field C, which is *dimensionless.*

5. Two-scalar theory

Let C be a dimensionless (conformally invariant) field and modify the Lagrangian density (4.4) to

$$(-\overset{\circ}{g})^{\frac{1}{2}}\,(2k)^{-1}\,(\overset{\circ}{R} - \beta\overset{\circ}{g}^{\mu\nu}\,C_{\cdot\mu}C_{\cdot\nu}) + \mathcal{L}(q,\overset{\bullet}{\psi}) \tag{5.1}$$

where $\mathcal{L}(q,\overset{\circ}{\psi})$ is constructed from $q^2\,\overset{\circ}{g}_{\mu\nu}$ and $q^N\,\overset{\circ}{\psi}$ in the same way that the $\mathcal{L}(\overset{\circ}{\psi})$ of (4.4) was constructed from $\overset{\circ}{g}_{\mu\nu}$ and $\overset{\circ}{\psi}$. The constant β is dimensionless and q is some function of C. We now have two scalar fields σ and $\sigma' = \sigma e^C$. In terms of these two fields, the Lagrangian density is

$$(-g)^{\frac{1}{2}}\,(2\kappa)^{-1}\big(\sigma^2\,R + 6\sigma_{\cdot\mu}\sigma^{\cdot\mu} - \beta\,(\sigma^4/\sigma'^2)\,\partial_\mu\,(\sigma'/\sigma)\,\partial^\mu\,(\sigma'/\sigma)\big) + \mathcal{L}(q\sigma,\psi) \tag{5.2}$$

In the *special gauge* in which σ' is constant (without loss of generality, $\sigma' = 1$) the term in brackets becomes the Brans-Dicke Lagrangian with $\omega = \frac{1}{4}(\beta-6)$ (Brans-Dicke field ϕ is σ^2). Thus if we choose

$$q = e^C \tag{5.3}$$

the matter Lagrangian will be *independent* of σ in this gauge, and the theory will be precisely Brans-Dicke theory.

Alternatively, choose the special gauge in which $\sigma = 1$. We obtain

$$(-g)^{\frac{1}{2}}\,(2\kappa)^{-1}\,(R - \beta C_{\cdot\mu}C^{\cdot\mu}) + \mathcal{L}(e^C,\psi), \qquad (C = \log\sigma') \tag{5.4}$$

This is a creation-field theory in which the coupling of the C-field and the matter field ψ comes from the fact that the inverse Compton wavelength $(mc/\hbar)$ is a constant multiple of e^C (the mass terms have become interaction terms, as in Dicke 1962) and coupling 'constants' are constant multiples of appropriate powers of e^C. Additional coupling comes from the kinetic terms because derivatives of ψ occur in the combination

$$\psi_{;\mu} = \psi_{,\mu} + NC._{\mu}\psi \tag{5.5}$$

where the comma denotes covariant differentiation using the affine connection

$$\left\{ {\rho \atop \mu\nu} \right\} + \delta_\mu{}^\rho C._\nu + \delta_\nu{}^\rho C_{\,\mu} - g_{\mu\nu}C^{\cdot\rho} \tag{5.6}$$

It is a curious fact that the affine connection does not actually enter into the Lagrangian densities of integral-spin fields, so the comma in (5.5) could in certain cases be replaced by a dot. To deal with half-integral spin we would have to enter into the complexities of a conformally-covariant generalisation of Kibble-Sciama (Kibble 1960; Sciama 1964) theory. This we shall not do; we shall only point out that the discussion of spinors in (Lord 1972) shows that the Dirac field will couple to the C-field only through its mass term.]

The field equations that follow from (5.4) are, for $g_{\mu\nu}$,

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R = -\kappa\,[T_{\mu\nu} - f(C._{\mu}C._{\nu} - \tfrac{1}{2}g_{\mu\nu}C._{\rho}C^{\cdot\rho})] \tag{5.7}$$

(where $f = \beta/k$), and for the C-field we can make use of the identity (2.8) for the scalar σ' of (5.3) to obtain

$$f\Box\, C = e^C\, T \tag{5.8}$$

The identity which follows from the divergence of (5.7), *i.e.*

$$T^{\mu\nu}{}_{|\nu} = fC^{\cdot\mu}\,\Box\, C \tag{5.9}$$

is seen to be just a particular case of (2.9).

The identification of the source of the C-field as the trace of the energy momentum tensor makes this theory different from the usual C-field theory in several respects. If we take the matter to be a fluid with zero pressure ($T_{\mu\nu} = \rho u_\mu u_\nu$) satisfying (5·7–9) (A naive procedure, considering the manner in which these equations were obtained), then

$$(\rho u^\mu u^\nu)_{|\nu} = fC^{\cdot\mu}\,\Box\, C = C^{\cdot\mu}\,\rho e^C$$

gives

$$u^\mu\,(\rho u^\nu)_{|\nu} + \rho u^\mu{}_{|\nu}u^\nu = C^{\cdot\mu}\rho e^C \tag{5.10}$$

Multiplying by u_μ, we get

$$(\rho u^\nu)_{|\nu} = \rho C^{\cdot\mu}\,u_\mu e^C \tag{5.11}$$

which is an explicit expression for the source term that modifies the continuity equation for rest-mass.

Substituting back into (5.10) gives

$$u^\mu{}_{|\nu}u^\nu = e^C\,(\delta_\nu{}^\mu - u^\mu\,u_\nu)\,C^{\cdot\nu} \tag{5.12}$$

Hence in this theory the world-lines are *not* geodesics. The quantity in brackets in (5.12) is the operator that projects vectors on to the hypersurfaces perpendicular to the world-lines. The world-lines of the fluid are geodesics only if C is a constant over each such hypersurface.

6. Cosmological solutions

We now look for cosmological solutions of the system of equations (5.7), (5.8). That is, solutions with C and ρ dependent only on time, in a co-moving coordinate system in which the metric is of Robertson-Walker form

$$\left.\begin{aligned} &dt^2 - (1 + \kappa r^2/4)^{-2} S^2(t) \left(dr^2 + r^2 (d\theta^2 + \sin^2\theta\, d\theta^2)\right) \\ &\qquad\qquad\qquad\qquad (k = \pm 1 \quad \text{or} \quad \text{zero}) \end{aligned}\right\} \tag{6.1}$$

A remarkable feature of the theory is that it admits *only* universes in which the density remains constant.

Equations (5.8) and (5.12) are

$$f\ddot{C} = \rho e^{c}, \quad \dot{\rho} = \rho e^{C}\dot{C} \tag{6.2}$$

which lead to

$$\dot{\rho} = f\dot{C}\ddot{C}$$

Therefore

$$\rho = \rho_0 + \tfrac{1}{2} f\dot{C}^2$$

With this expression for the density, the Einstein equations (5.7) are simply

$$\kappa(4\rho_0 - 3\rho) = -6\ddot{S}/S \tag{6.3}$$

$$\kappa(2\rho_0 - \rho) = 2k/S^2 - 2\frac{d}{dt}(\dot{S}/S) \tag{6.4}$$

The (44)-equation is

$$\kappa\rho_0 = \frac{3k}{S^2} + 3(\dot{S}/S)^2,$$

which, when differentiated, gives

$$0 = -\kappa\dot{S}/S^3 + (\dot{S}/S)\frac{d}{dt}(\dot{S}/S). \tag{6.5}$$

Eliminating $d/dt\,(\dot{S}/S)$ between (6.4) and (6.5) leads to $\dot{S} = 0$ (*static* universes) *or* $(\rho_0 - \rho/2) = 0$. Thus the only cosmological solutions have constant density

$$\rho = 2\rho_0 \tag{6.6}$$

Substituting this value of ρ in (6.3) and (6.4) gives equations identical to the usual cosmological equations for empty universes with a cosmological constant

$$\Lambda = \kappa\rho_0 = \tfrac{1}{2}\kappa\rho \tag{6.7}$$

That is, the *metrics* for our universes of constant density are identical with the metrics that in conventional general relativity correspond to empty universes. These solutions are

$$\left.\begin{aligned} &(a)\; k = +1, \quad S = \cosh\left(t\sqrt{\Lambda/3}\right) \\ &(b)\; k = 0, \quad\;\; S = \exp\left(t\sqrt{\Lambda/3}\right) \\ &(c)\; k = -1, \quad S = \sinh\left(t\sqrt{\Lambda/3}\right) \end{aligned}\right\} \tag{6.8}$$

For $k = 0$ we have simply the steady state cosmology. Denoting the Hubble constant by T, the constant density ρ satisfies $\kappa\rho T^2 = 6$, as in the steady-state solution of the usual C-field cosmology. In our view, the only acceptable cosmological models in any theory are those with $k = +1$, since only these correspond to universes containing a finite amount of matter. In our case this leads to a universe with a contracting phase followed by an expanding phase. This will lead to Olber's paradox unless the matter is non-radiating in the contracting phase (Lord 1974).

With $\rho = 2\rho_0$, $C = \text{constant} + t\,(f\rho)^{\frac{1}{2}}$. It is somewhat disturbing that the constant must be $-\infty$ to comply with (6.2). However, only the derivatives of C enter into (5.7), so perhaps this peculiarity should not be taken too seriously. The undifferentiated field C is unobservable.

To summarise: the creation-field theory we have presented differs from the usual one in that only universes with constant density are allowed. The usual C-field theory admits also universes which asymptotically approach a state of uniform density for large t. Whether this is an advantage or a disadvantage is arguable. One of the difficulties with general relativity (and modifications to it including Hoyle-Narlikar theory) is that it provides a very large number of alternative cosmological models with no clear criterion for deciding which one should correspond to the real universe. Our theory, in conjunction with the requirement $k = +1$ leads to only *one* model if pressure terms are neglected.

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