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Analysis of cosmic ray data on nucleon-nucleus collisions and its implication on high energy behaviour of nucleon-nucleon total cross section

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Abstract. From existing cosmic ray measurements of the *inelastic* collision cross sections of nucleons on nuclei of carbon, iron and lead in the range of energies 10^2 to 10^4 GeV as well as the measurements of cross sections on air nuclei in the extensive air shower (EAS) regions (10^5 to 10^8 GeV), we conclude that the Glauber multiple scattering theory is adequate to account for the data. Recent suggestion of Maor and Nussinov to parametrize the nucleon-nucleon total cross section with a component growing proportional to $\ln^2 E$ (E is the incident energy) is at variance with the EAS data. However the data are consistent with a nucleon-nucleon total cross section rising no faster than $\ln E$ in these energy regions.

Keywords. Inelastic collision cross section of nucleons (10^2 to 10^8 GeV); Glauber multiple scattering theory; nucleon-nucleon total cross sections.

1. Introduction

Measurements made recently with intersecting storage rings (ISR) at CERN by Amaldi *et al* (1973) and Amendolia *et al* (1973) in the energy range 500 to 2000 GeV have conclusively brought out the rise in the total cross section in p-p collisions. The rise in total cross section was also concluded by Yodh *et al* (1972) from their analysis of cosmic ray data. The behaviour of nucleon-nucleon total cross section much beyond ISR energies, which is of great theoretical importance, has to come from cosmic ray experiments. One such analysis was made by Ganguli and Subramanian (1973) of cosmic ray air-shower data in the energy range 10^6 to 10^8 GeV. This analysis, hereafter referred to as G–S, showed that the growth of the p-p total cross section beyond ISR energies is unlikely to be faster than $\ln s$ where s is the square of the total CM energy. In this paper we analyse the existing cosmic ray data on nucleon-nucleus collisions using Glauber multiple scattering theory (Glauber 1959, 1970) in order to deduce the possible behaviour of nucleon-nucleon total cross section at very high energies. The new consideration brought in this paper is the possible rise of nucleon-nucleus collision cross sections due to an expanding halo around the nucleon (Udgaonkar and Gell-Mann 1962, Maor and Nussinov 1973) at energies $> 10^{12}$ eV.

We would like to stress that the inelastic cross sections as measured in cosmic ray experiments refer to meson production. This means that

$$\sigma_{\text{inel}} = \sigma_{\text{abs}} - \sigma_{\text{quasi-el}} \tag{1}$$

Table 1. Nucleon-carbon inelastic cross sections

Primary energy (GeV)	σ_{inel} (mb)	Reference	Remarks
22	217± 7	Akimov *et al* (1970)	Satellite experiment
62	232± 5		
200	263± 7		
610	266±12		
100	217±13	Bozoki *et al* (1970)	Neutron primary
130	213±24		
150	214±12	Alakoz *et al* (1971)	Neutron primary
300	186±17		
400	245±55	Anoshin *et al* (1971)	Proton primary
600	218±23	Rubtsov *et al* (1973)	

where σ_{abs} $(= \sigma_{\text{tot}} - \sigma_{\text{el}})$ is the absorption cross section in the nucleus and $\sigma_{\text{quasi-el}}$ is the quasi-elastic scattering cross section where one (or more) nucleons are knocked out from the nucleus and the incident particle does not undergo any appreciable change*.

In section 2 we bring out all the existing cosmic ray data. Calculations using Glauber multiple scattering theory are described in section 3. Discussion of the results follows in section 4 and summary in section 5.

2. Details of the existing cosmic ray data

There are basically three types of cosmic ray experiments that give rise to σ_{inel}: (A) Observation of direct interaction of cosmic ray particles with specific targets along with a total absorption calorimeter to measure the primary energy, (B) zenith angle distribution of extensive air shower (EAS) frequencies which give interaction mean free path of primaries with air nuclei, and (C) comparison of the unaccompanied cosmic ray proton spectra at mountain altitudes with the primary proton spectrum; this third aspect we shall not discuss in this paper as it has been dealt with in detail by Yodh *et al* (1972). The experimental results from (A) and (B) are summarised below.

(A) In this category data exist for carbon, iron and lead nuclei. The data for carbon and iron which are presented in tables 1 and 2 are corrected, wherever necessary, for pion contamination in the atmosphere. The data of Akimov *et al* (1970) refer to satellite experiments. The data for lead shown in table 3 are not corrected for pion contamination as the correction is negligible compared to the errors. The data of carbon, iron and lead are also plotted in figures 1 *a*, *b* and 2 *a*.

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* Analyses made by Balashov and Korenman (1970) and Auger and Lombard (1973) assumed $\sigma_{\text{inel}} = \sigma_{\text{abs}}$ which is not correct. The latter authors did not also correct the data for iron for contamination from pions in the atmosphere.

Table 2. Proton-iron inelastic cross sections

Primary energy (GeV)	σ_{inel} (mb)	Reference
88	797±11	Jones *et al* (1970)
121	780±14	
177	836±22	
262	810±11	
370	804±17	
565	784±32	
750	880±53	
400	700±35	Bashindjhagyan *et al* (1971)
≈250	760±37	Andronikashvili *et al* (1967)

Table 3. Proton-lead inelastic cross sections

Primary energy (GeV)	σ_{inel} (mb)	Reference
100	1650±170	Alakoz *et al* (1968)
200	1583±125	Denisov (1968)
300	1583±167	
400	1765±350	Anoshin *et al* (1971)
3000	1625±250	Chubenko *et al* (1973)
5000	1810±110	
9000	1835±160	
3200	1729±146	Akashy *et al* (1963)

he method of obtaining inelastic collision cross section of protons against air nuclei by studying the zenith angle dependence of the frequency distribution of EAS of a given size (number of electrons in the shower) and same age of development can be found in the works referred to in table 4 (*see* also Hayakawa 1969) and is contained in the relation

$$I(\theta) = I(0) \exp\left(-\frac{X \sec\theta}{\lambda_{\text{tr}}}\right) \tag{2a}$$

where $I(\theta)$ is the intensity of showers occurring at zenith angle θ at an atmospheric depth X, and λ_{tr} is the proton-air inelastic collision mean free path. As discussed in detail by G-S, it is not λ_{tr} that one directly observes in the experiment because very low fractional energy transfer collisions do not significantly contribute to air shower growth. Denoting by λ_{obs} the mean free path deduced from the experiment, G-S have shown that these are related by

$$\lambda_{\text{tr}} = \lambda_{\text{obs}}\left[1 - \int_{\eta_m}^{1} \eta^{(\gamma-1)} P(\eta)\, \mathrm{d}\eta\right] \tag{2b}$$

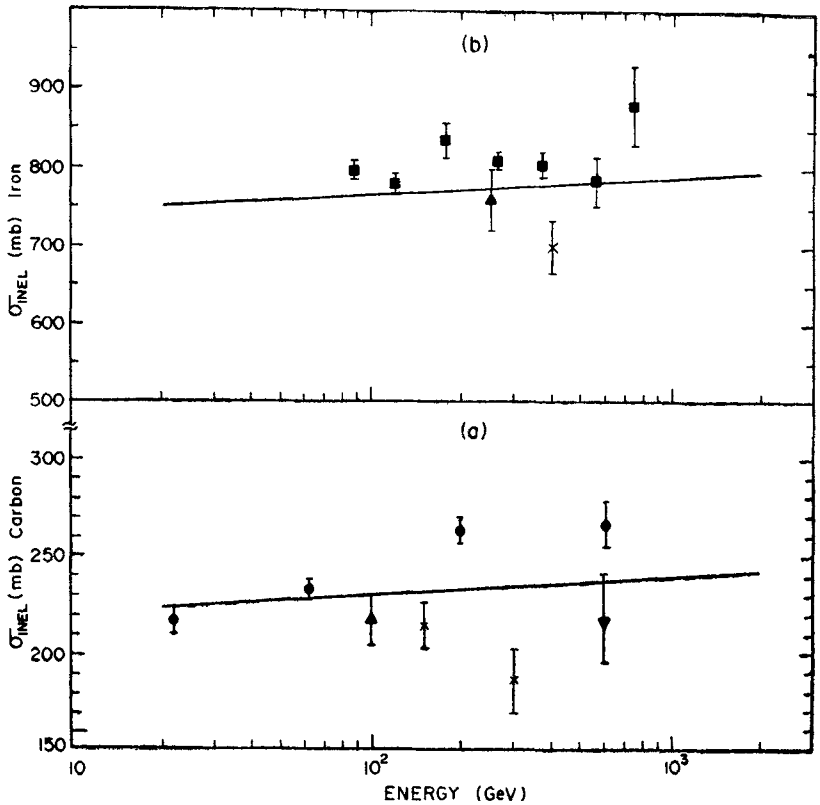

Figure 1. (*a*) Inelastic cross section of nucleon-carbon is plotted *vs* energy of the incident nucleons. Data points are: ●—Akimov *et al* (1970); ▲—Bozoki *et al* (1970); ×—Alakoz *et al* (1971) and ▼-Rubtsov *et al* (1963). The curve represents the values calculated on the basis of the Glauber theory. (*b*) Inelastic cross section of proton-iron is plotted *vs* energy of the incident protons. Data points are: ■—Jones *et al* (1970); ×—Bashindjhagyan *et al* (1971) and ▲—Andronikashvili *et al* (1967). The curve represents the values calculated on the basis of Glauber theory.

where γ is the exponent of the differential energy spectrum of primary particles and $P(\eta)$ is the probability of having elasticity η (≤ 1) in the p-air collisions. For $P(\eta)$ we use the same distribution as that obtained at 19 GeV using various nuclear targets (Liland and Pilkuhn 1969).

We assume that the $P(\eta)$ distribution in the EAS energy region to be the same as deduced at much lower energies as above. To a great extent the above assumption seems justified from ISR data when one looks at the x distribution of outgoing protons in reactions $p + p \rightarrow p +$ anything, where $x = p^*_{11}/p^*_{max}$ and $\eta = x$ at these energies. The x distribution does not significantly change from 20 GeV to 2000 GeV. The trend of $P\langle\eta\rangle$ above ISR energies which is generally required to explain air shower data (Sreekantan 1972) is if at all such that the collisions become more catastrophic, *i.e.*, mean η represented by $\langle\eta\rangle$ decreasing. So long as $\langle\eta\rangle$ does not increase, our estimates of p-air nucleus cross sections become upper bounds.

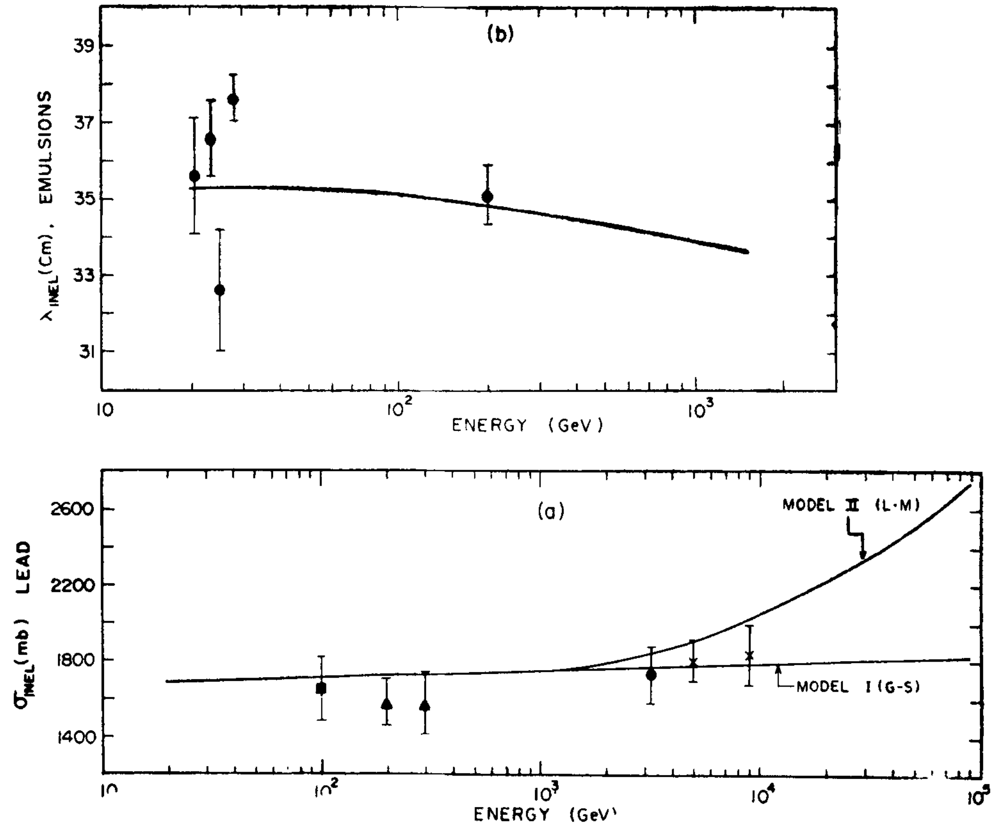

Figure 2. (*a*) Inelastic cross section of proton-lead is plotted *vs* energy of the incident protons. Data points are: ■—Alakoz *et al* (1968); ▲—Denisov (1968), ×—Chubenko *et al* (1973) and ●—Akashy *et al* (1963). The curve up to 1000 GeV represents the values calculated on the basis of the Glauber theory. Beyond 1000 GeV the curves are based on model I and model II (*see* the text). (*b*) Accelerator data of inelastic mean free path of protons in nuclear emulsion is plotted *vs* energy of the incident protons (refer text for sources of data). The curve represents the values of inelastic mean free path calculated on the basis of the Glauber theory.

A conservative value of η_m taken is $0 \cdot 7$. If η_m is chosen less than $0 \cdot 7$, the air shower is expected to show an increase in the number of muons by more than 40% which will be excluded by the experimental resolution which is $\sim 25\%$. The best experimental resolution is obtained at the largest of the air shower energies because of the large numbers of particles involved.

Thus with the assumed $P(\eta)$ distribution and $\eta_m = 0 \cdot 7$, one gets $\lambda_{tr} = 0 \cdot 83\ \lambda_{obs}$. It might be mentioned that due to the steeply decreasing nature of primary cosmic ray spectrum ($\gamma \approx 3$) the correction factor relating λ_{tr} and λ_{obs} in eq. (2 *b*) is not very sensitive to the choice of η_m so as to alter the conclusions arrived at in this paper. The values of λ_{tr} are shown in column 3 and the calculated λ_{abs} (p-air), using λ_{tr}, are shown in column 4 of table 4. The values of σ_{abs} (p-air) are plotted in figure 3.

Finally it may be mentioned that it is not necessary to make any further assump-

Table 4. Details of EAS data and estimation of σ_{abs} (p-air)

Primary energy (GeV)	λ_{obs} (gm/cm^2)	λ_{tr} (gm/cm^2)	σ_{abs} (mb)	Predicted σ_{abs} (mb)		Reference for λ_{obs}
				Model I	Model II	
10^5	91 ± 3	$75{\cdot}5 \pm 2{\cdot}5$	330 ± 12	333	528	Catz *et al*(1972)
$>10^5$	$85{\cdot}7 \pm 4{\cdot}5$	71 ± 4	350 ± 20	..	..	Capdevielle *et al* (1973)
$5{\cdot}10^5$	87 ± 7	72 ± 6	345^{+31}_{-27}	345	649	Sitte *et al* (1958)
$2{\cdot}10^6$	95 ± 10	79 ± 8	315^{+38}_{-29}	355	749	Matano *et al* (1963)
$6{\cdot}10^6$	92 ± 10	76 ± 8	326^{+40}_{-33}	360	816	
$2{\cdot}10^7$	80 ± 5	66 ± 4	376^{+24}_{-33}	370	884	
$6{\cdot}10^7$	82^{+8}_{-5}	68^{+7}_{-4}	366^{+24}_{-33}	380	937	
$2{\cdot}10^8$	84^{+11}_{-8}	70^{+9}_{-7}	356^{+40}_{-41}	385	987	

tions about air shower development to deduce the collision cross section using eq. (2 *a*) since energy resolution factors of the detector system employed, etc., are common to both $I(\theta)$ and $I(0)$. Of course one need not take the values of energy quoted in column 1 of table 4 literally; they represent central values in a range perhaps of a factor of 2 of the central value. This aspect hardly matters in the display of data in figure 3.

3. Nucleon-nucleus cross section in Glauber theory

3.1. *Calculation of absorption cross section*

The cross sections of nucleon-nucleus scattering have been obtained using Glauber theory of multiple scattering. For the light nuclei (carbon and air) we have taken the density distribution of nucleons to be of harmonic oscillator type and the details are described in our earlier paper (Ganguli *et al* 1973). For the heavier nuclei the inelastic cross sections have been obtained in the following way:

The elastic amplitude for scattering of a nucleon by a nucleus is given by

$$F(\boldsymbol{q}) = \frac{ik}{2\pi}\int \exp(i\boldsymbol{q}\cdot\boldsymbol{b})\,\{1 - [1 - \textstyle\int \Gamma(\boldsymbol{b}-\boldsymbol{s})\,\rho(\boldsymbol{r})\,\mathrm{d}^3 r]^A\}\,\mathrm{d}^2 b \tag{3}$$

where k is the incident momentum, $\boldsymbol{q}$ is the 3-momentum transfer, A is the mass number of the nucleus and $\boldsymbol{s}$ is the projection of $\boldsymbol{r}$ on the impact parameter plane $\boldsymbol{b}$. The density distribution of nucleons, $\rho(r)$, is taken to be of Woods-Saxon type:

$$\rho(r) = \rho_0\left[1 + \exp\frac{r-c}{a}\right]^{-1} \tag{4}$$

with $\int \rho(r)\,\mathrm{d}^3 r = 1$,

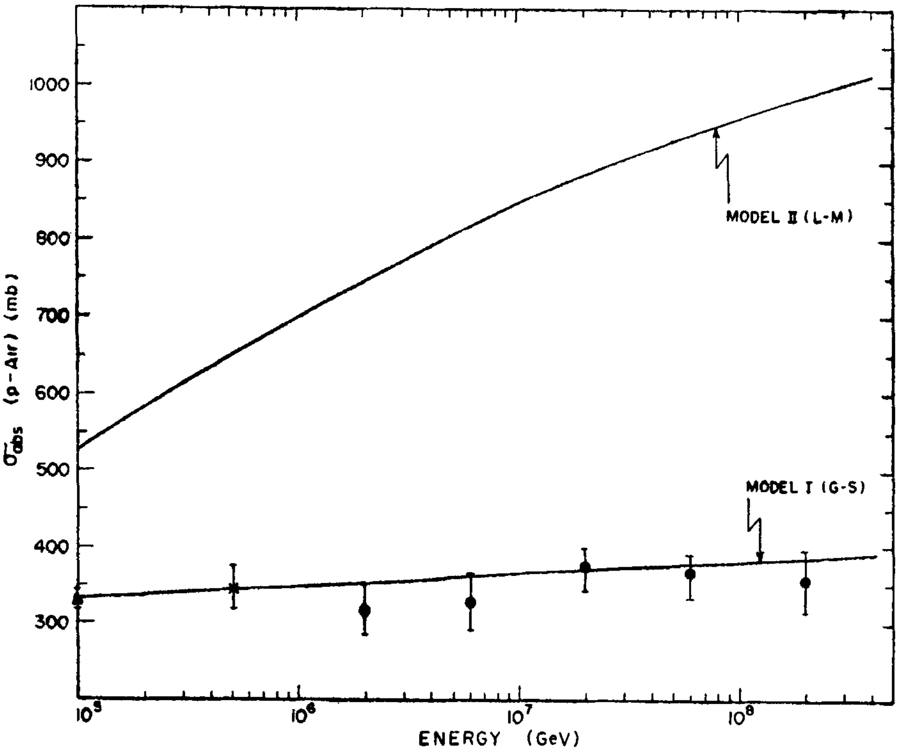

Figure 3. Absorption cross section of proton-air, as obtained from extensive air showers of cosmic rays, is plotted *vs* energy of the incident protons. Data points are: ▲—Catz *et al* (1972); ×—Sitte *et al* (1958) and ●—Matano *et al* (1963). The two curves represent the values calculated on the basis of model I and model II (*see* the text).

where a is taken to be 0·545 fm and the half-density radius c is taken to be

$$c = 1\cdot07\, A^{1/3} \tag{5}$$

The function Γ is the Fourier transform of the nucleon-nucleon scattering amplitude $f(q)$:

$$\Gamma(b) = \frac{1}{2\pi i k}\int \exp(-\,i\boldsymbol{q}\cdot\boldsymbol{b})\, f(q)\, d^2 q \tag{6}$$

If the nucleon-nucleon scattering amplitude $f(q)$ is taken as

$$f(q) = \frac{k\sigma_{tot}}{4\pi}(i + \alpha)\exp(-\,Bq^2/2) \tag{7}$$

where σ_{tot} is the total cross section, α is the ratio of the real to imaginary part of the forward scattering amplitude and B is the slope parameter, the function Γ reduces to

$$\Gamma(b) = \sigma_{tot}(1 - i\alpha)\exp(-\,b^2/2B)/(4\pi B) \tag{6a}$$

From this it follows that

$$\int \Gamma(\boldsymbol{b} - \boldsymbol{s})\,\rho(\boldsymbol{r})\, d^3 r = \tfrac{1}{2}\sigma_{tot}(1 - i\alpha)\,\tilde{T}(b) \tag{8}$$

where

$$\tilde{T}(b) = \frac{1}{B}\int_{0}^{\infty} T(s)\, I_0\,(bs/B) \exp\left(-\,(b^2 + s^2)/2B\right) s\,\mathrm{d}s \tag{9}$$

$$T(s) = \int_{-\infty}^{\infty} \rho\,(s, z)\,\mathrm{d}z \tag{10}$$

and I_0 is the modified Bessel function of zeroth order. The profile function of the nucleus, Γ^A, is thus given by:

$$\Gamma^A = 1 - [1 - \sigma_{\text{tot}}\,(1 - \mathrm{i}\alpha)\,\tilde{T}(b)/2]^A \tag{11}$$

The absorption cross section is then given by:

$$\sigma_{\text{abs}} = \int \{1 - |\,1 - \Gamma^A\,|^2\}\,\mathrm{d}^2 b \tag{12}$$

We have made numerical integration to obtain σ_{abs}.

For data points of air and lead nuclei where the primary energy is much more than 2 TeV (1 TeV = 10^3 GeV), we assume $\alpha = 0$ and the scattering amplitude $f(q)$ is calculated using the following two models:

Model I (ln s rise in σ_{tot})

In this we have used the parametrization of σ_{tot} (mb) as given by G-S:

$$\sigma_{\text{tot}} = 24\cdot 9 + 39\cdot 9\, s^{-\frac{1}{2}} + 2\cdot 08 \ln s \tag{13}$$

where s is in unit of GeV^2. This parametrization gives a good fit to the accelerator data up to ISR energies with $\chi^2 = 23$ for 32 data points. For the slope parameter, B, in GeV^{-2}, we have used the parametrization of Bartenev *et al* (1973) in the energy range 8 to 400 GeV:

$$B = 8\cdot 23 + 0\cdot 556 \ln s \tag{14}$$

Model II (ln^2 s rise in σ_{tot})

Here we use the parametrization of Maor and Nussinov (1973) where they use two components for the scattering amplitude as suggested by Leader and Maor (1973):

$$f(q) = f_1\,(q) + f_2\,(q) \tag{15}$$

with

$$f_j\,(q) = (\mathrm{i}k\sigma_j/4\pi) \exp\,(-\,B_j q^2/2),\ j = 1, 2 \tag{7a}$$

where

$$\sigma_1 = 38\cdot 4 \text{ mb};\quad \sigma_2 = 0\cdot 49\,[\ln\,(s/122)]^2 \text{ mb}$$

$$B_1 = 10\cdot 8 \text{ GeV}^{-2};\quad B_2 = 5\cdot 0\,[\ln\,(s/122)]^2 \text{ GeV}^{-2}$$

3.2. *Calculation of inelastic cross section*

For the data of category (A), section 2, we need to calculate inelastic nucleon-nucleus cross section. This is done by estimating $\sigma_{\text{quasi-el}}$ and subtracting it from σ_{abs}. The expression for $\sigma_{\text{quasi-el}}$ is given by (Belletini *et al* 1966):

$$\sigma_{\text{quasi-el}} = N_{\text{eff}}\sigma_{\text{el}}^{\text{pp}}$$

where N_{eff} is the effective number of nucleons of the target nucleus taking part in the scattering and $\sigma_{\text{el}}^{\text{pp}}$ is the elementary p-p elastic cross section. The values of N_{eff} are taken to be 3·4, 6·1 and 9·5 for carbon, iron and lead respectively (Belletini *et al* 1966). For energies more than 2 TeV we have used $\sigma_{\text{el}}^{\text{pp}} = 7\cdot 6$ mb and for lower energies we have used the accelerator measurements.

The cross sections thus calculated are shown as curves in figures 1*a*, *b* and 2*a*.

3.3. *Error on the calculated curves*

Error on the calculated curves basically arises from uncertainties in (i) nuclear radius parameters, and (ii) values of N_{eff}. Total error on the calculated curves comes out to be 2–3% of the cross sections.

3.4. *Check on the calculated curves from accelerator measurements*

Among the elements considered in this paper accelerator measurements exist for carbon and lead. Belletini *et al* (1966) made measurements with protons of 20 GeV, and Denisov *et al* (1973) with protons in the range 20 to 60 GeV. Their results on absorption cross sections agree very well with present calculation:

	Belletini *et al* (1966), 20 GeV	Denisov *et al* (1973), 20–60 GeV	Present calculation
σ_{abs} (p-C)	254±6	248±2	251±4 mb
σ_{abs} (p-Pb)	1750±125	1812±35	1771±40 mb

Recently a measurement on the inelastic mean free path of 200 GeV protons with emulsion nuclei has been made by Cüer *et al* (1973). Their value obtained from the track scanning method is:

$$\lambda_{\text{inel}}\,(\text{p-Eml}) = (35\cdot 1 \pm 0\cdot 8)\ \text{cm}$$

We have calculated σ_{inel} for different nuclei of emulsion and weighted them according to their contents in emulsion. This leads to inelastic mean free path at 200 GeV as

$$\lambda_{\text{inel}}\,(\text{p-Eml}) = (34\cdot 8 \pm 0\cdot 6)\ \text{cm}$$

which again is in good agreement with the measurement. We have also shown in figure 2*b* the expected λ_{inel} (p-Eml) up to 1500 GeV incident protons, along with the existing measurements in nuclear emulsions (Meyer *et al* 1963, Cvijanovich *et al* 1961, Bizzeti *et al* 1963, Baudinet-Robinet *et al.* 1962, Jain *et al* 1961, Cüer *et al* 1973; we have taken weighted mean of the three data points at 27, 27 and 28 GeV).

4. Discussion

4.1. *Nucleon-carbon inelastic cross section*

We see from figure 1 that except for the two points of Akimov *et al* (1970) at 200 and 610 GeV, the experimental data are compatible with the Glauber calculation. In an earlier paper (Ganguli *et al* 1973) we have pointed out that the data of Akimov *et al* mentioned above could be interpreted as due to contamination of deuterons in the primary cosmic ray protons to the extent of (15±4)% in the energy range 200 to 600 GeV.

4.2. *Proton-iron inelastic cross section*

Within statistical errors, data for iron agree with the expected values from Glauber theory.

4.3. *Proton-lead inelastic cross section*

Here the data exist up to 9 TeV and hence we have used the two models (section 3) for comparison. Beyond 1000 GeV the predictions from the two models differ.

Because of the large experimental error on the measured points it is not possible to distinguish between the two models.

4.4. *Proton-air absorption cross section*

Proton-air data exist from 10^5 to $2 \cdot 10^8$ GeV, the energy region which is not accessible to the present generation of accelerators. It has been pointed out by G-S that this energy region is not sufficient to distinguish between the models demanding asymptotically constant total cross sections for p-p and ln s rise in p-p total cross section (here s is the square of the total CM energy). Nevertheless, this energy region is sufficient to distinguish between the models demanding $\ln^2 s$ rise and ln s rise in the total cross section of p-p. This is demonstrated in figure 3 where we have plotted σ_{abs} (p-air) as a function of energy. Predicted values from the two models are also listed in columns 5 and 6 of table 4. We see from figure 3 that ln s rise in total cross section of p-p can explain the data extremely well whereas the $\ln^2 s$ rise as per model of Leader and Maor (1973) is in complete disagreement with the measurements.

5. Summary

We made a detailed analysis of cross sections from the existing cosmic ray measurements in carbon, iron, lead and air in terms of Glauber's theory. Up to 2000 GeV our input information on p-p scattering is based on accelerator results. Beyond 2000 GeV our input information on p-p scattering is based on two models:

Model I: ln s rise in total cross section,
Model II: $\ln^2 s$ rise in total cross section.

Accelerator measurements of proton-nucleus cross sections exist up to 200 GeV and they are in good agreement with our calculation.

For carbon and iron where the cosmic ray data exist up to about 1000 GeV, inelastic cross sections are compatible with calculations using the Glauber theory. For lead the data exist up to 9 TeV, but because of the large error in the experimental data we cannot distinguish between the two models.

Absorption cross section data on p-air are the most useful ones as they cover the energy region from 10^5 to $2 \cdot 10^8$ GeV. Calculation made using model I fits the data extremely well, whereas the prediction of model II is in complete disagreement with the experimental data.

We conclude that the growth of the p-p total cross section from ISR energies to extensive air-shower energies is unlikely to be faster than ln s.

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