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Analysis of proton-carbon inelastic cross sections measured in satellite experiment and upper limit on primary cosmic ray deuteron flux in the energy range 20 to 600 GeV

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Abstract. An analysis has been made of the experimental results of Akimov *et al* on the inelastic cross sections of proton on proton and carbon targets in the energy range 20 to 600 GeV obtained from artificial earth satellites. It is found that an upper limit of 4% at 95% confidence level can be set on the fraction of deuterons relative to the flux of protons in the primary cosmic radiation at energies in the range 20 to 60 GeV. There is an indication for a rise of (29 ± 7) mb in the inelastic cross section of proton against carbon in the energy range of 200 to 600 GeV over and above what is expected from Glauber's theory. If this rise has to be interpreted as due to contamination from cosmic ray deuterons, the fraction of deuterons relative to protons needed is $(15 \pm 4)\%$ in this energy region.

Keywords. Inelastic cross section (20 to 600 GeV); Glauber multiple scattering theory; flux of deuterons in primary cosmic rays.

1. Introduction

In the last few years interest in the hadron-nucleus collisions have increased due to the success of Glauber multiple scattering theory (Glauber 1959, 1970) in understanding the experimental data. At present we know absorption cross sections of various charged hadrons on different nuclear targets up to 60 GeV from accelerators (Denisov *et al* 1973) and they all seem to be accounted for in terms of the Glauber theory. In the present paper an attempt has been made to understand the cosmic ray data of Akimov *et al* (1970) where inelastic cross sections of protons on protons and on carbon nuclei were measured in the energy range 20 to 600 GeV.

Akimov *et al* (1970) have made measurements on total inelastic cross sections of singly charged primary cosmic ray particles (protons and deuterons) on carbon and hydrogen, in the energy region 20 to 600 GeV, using satellites named as Proton I to Proton III. They have used carbon and polyethylene targets. The incident energy was measured by a total absorption calorimeter of 3 interaction mean free paths. The experiment detected inelastic events of three types: (i) no outgoing charged particles, (ii) ≥ 2 charged particles, and (iii) total γ-ray energy > 500 MeV. Basically it means that the experiment did not detect elastic scatterings both coherent and incoherent. They have deduced inelastic cross sections (σ_{inel}) of proton-proton and proton-carbon in the energy range 20 to 600 GeV. Since deuterons cannot be discriminated from protons in the experiment of Akimov *et al* we shall first estimate in section 2 the deuteron flux by comparing their p-p data with the precision data on σ_{inel} known up to about 2000 GeV from accelerators. A better estimate of deuteron flux in the energy region 20 to 60 GeV will be made in section 3.1. In section 3 we shall discuss in detail the proton-carbon data in terms of the Glauber scattering theory.

2. Estimation of deuteron flux

In figure 1 *a* is shown the inelastic p-p (σ^{s}_{inel}) data of Akimov *et al.* The solid curve in figure 1 *a* in the energy region 20 to 60 GeV, is a line drawn through the precision accelerator data, $\sigma^{\text{accl}}_{\text{inel}}$. The $\sigma^{\text{accl}}_{\text{inel}}$ in the energy region 20 to 60 GeV has been obtained as follows:

$$\sigma^{\text{accl}}_{\text{inel}} = \sigma_{\text{tot}}\,(\text{pp}) - [(1 + \alpha^2)\,\sigma^2_{\text{tot}}]/(16\pi B) \tag{1}$$

where σ_{tot} is the total p-p cross section, α is the ratio of the real to the imaginary part of the forward scattering amplitude and B is the slope of the p-p scattering (Denisov *et al* 1971, Beznogikh *et al* 1973). Error on the points in this energy range is 0·1 mb. For energies more than 100 GeV we have used $\sigma^{\text{accl}}_{\text{inel}}$ as obtained at NAL (Chapman *et al* and Charlton *et al* 1972) and at ISR (Amaldi *et al* 1973). The solid curve beyond 60 GeV is a guide-line through these data points.

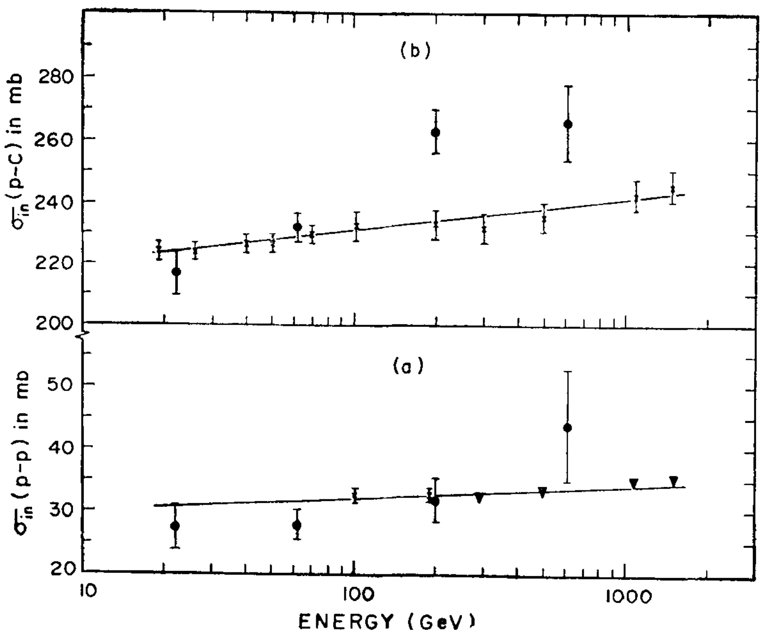

Figure 1. (*a*) Inelastic cross section of proton-proton is plotted *vs* energy of the incident protons. The four points of Akimov *et al* are shown as ●. The accelerator points are: NAL data as ×, ISR data as ▼. The solid line is drawn through accelerator points. (*b*) Inelastic cross section of proton-carbon is plotted *vs* energy of the incident protons. The four points of Akimov *et al* are shown as ●. The calculated points or the basis of Glauber theory are shown as × . The solid line through the calculated points is drawn for guidance.

Table 1. Details of σ_{inel} (pp) and estimation of fraction of deuterons

Incident energy (GeV)	σ_{inel} (pp) Akimov *et al* (mb)	σ_{inel} (pp) Accelerator (mb)	Fraction of deuterons
22	$27 \cdot 7 \pm 3 \cdot 6$	$30 \cdot 8 \pm 0 \cdot 1$	$-0 \cdot 11 \pm 0 \cdot 12$
62	$28 \cdot 2 \pm 2 \cdot 3$	$31 \cdot 8 \pm 0 \cdot 1$	$-0 \cdot 12 \pm 0 \cdot 08$
200	$32 \cdot 1 \pm 3 \cdot 8$	$32 \cdot 7 \pm 1 \cdot 2$	$-0 \cdot 02 \pm 0 \cdot 12$
610	$44 \cdot 3 \pm 9 \cdot 3$	$33 \cdot 5 \pm 0 \cdot 4$	$+0 \cdot 35 \pm 0 \cdot 30$

Let us now assume that the singly charged particles of primary cosmic rays are composed of a fraction ϵ of deuterons and $(1 - \epsilon)$ protons at a given primary energy. The fraction ϵ can be obtained from the following relation:

$$\epsilon = [\sigma^{s}_{inel}/\sigma^{accl}_{inel} - 1]/(K - 1) \tag{2}$$

where $K = \lambda_p/\lambda_d$, *i.e.*, the ratio of the interaction mean free paths of protons and deuterons in hydrogen. The value of K is $1 \cdot 91$ (Denisov *et al* 1971). In this way we have estimated the fraction of deuterons among the singly charged particles. The result is summarised in table 1.

We see from table 1 that the fraction of deuterons among singly charged particles is consistent with zero at all the four energy regions. Assuming that there is no energy dependence of deuterons with respect to protons, the best value of ϵ will be given by the weighted mean of the four points and its value is:

$$\langle \epsilon \rangle = (-0 \cdot 1 \pm 0 \cdot 06)$$

From this we estimate the upper limit on the fraction of deuterons among the singly charged particles as $0 \cdot 12$ with 95% confidence (two standard deviation value). This upper limit corresponds to $3 \cdot 7\%$ for the same energy/nucleon of deuterons as that of protons. A better estimate on the upper limit of deuterons is made in section 3.1.

3. Proton-carbon inelastic cross section

In figure 1 *b* we have shown the inelastic cross section of proton-carbon as obtained by Akimov *et al.* (1970). We have calculated the expected inelastic cross section of p-carbon using the Glauber theory in the following way:

(i) Density distribution of nucleons is assumed to be of harmonic oscillator type:

$$\rho(r) = C[1 + \delta r^2/a_0^2] \exp[-r^2/a_0^2] \tag{3}$$

where

$$C = 4/[(\sqrt{\pi} a_0)^3 \cdot A], \quad \delta = (A - 4)/6 \quad \text{and} \quad A = 12.$$

The rms radius is given by

$$\langle r^2 \rangle^{\frac{1}{2}} = a_0 (5/2 - 4/A)^{\frac{1}{2}}.$$

The choice of the harmonic oscillator type of function for density of nucleons in carbon follows from its earlier use in fitting electron scattering data (Hofstadter 1957) and hadron scattering data up to 60 GeV/c by Denisov *et al* (1973).

(ii) The value of $\langle r^2\rangle^{\frac{1}{2}}$ is taken as 2·30 fermi as obtained by Denisov *et al* (1973) by fitting their absorption cross sections of p-carbon in the energy range 20 to 60 GeV.

(iii) The nucleon-nucleon scattering amplitude is taken to be:

$$f(q) = [k\sigma_{\text{tot}}\,(\text{pp})/4\pi]\,(\text{i} + \alpha)\exp[-Bq^2/2] \tag{4}$$

(iv) The nuclear profile function with the density distribution as in (i) is calculated to be:

$$\Gamma(\boldsymbol{b}) = 1 - \left\{1 - (T/A)\exp(-b^2/R^2)\left[A - \frac{4a_0^{\,2}\,\delta}{R^2}(1 - b^2/R^2)\right]\right\}^A \tag{5}$$

where

$$T = [(1 - \text{i}\alpha)\,\sigma_{\text{tot}}\,(\text{pp})/2\pi R^2], \qquad R^2 = a_0^{\,2} + 2B,$$

and $\boldsymbol{b}$ is the impact parameter vector.

(v) The absorption cross section is calculated by:

$$\sigma_{\text{abs}} = \int\{1 - |\,1 - \Gamma(b)\,|^2\}\,\text{d}^2 b \tag{6}$$

This leads to a value of σ_{abs} as (251 $\pm$ 3) mb in the energy range 20–60 GeV which is in good agreement with the experimental value of Denisov *et al* which is (248 $\pm$ 2) mb in the same energy range.

(vi) It has already been mentioned in the introduction that the apparatus of Akimov *et al.* did not detect quasi-elastic scattering; therefore the inelastic cross section thus detected by them is given by:

$$\sigma_{\text{inel}} = \sigma_{\text{abs}} - \sigma_{\text{quasi-el}}$$

(vii) The expression for $\sigma_{\text{quasi-el}}$ is given by:

$$\sigma_{\text{quasi-el}} = N_{\text{eff}}\sigma_{\text{el}}^{\text{pp}}$$

where N_{eff} is the effective number of nucleons of the target taking part in the scattering and $\sigma_{\text{el}}^{\text{pp}}$ is the elementary p-p elastic cross section. The value of N_{eff} is taken to be 3·4 as obtained by Belletini *et al* (1966) for collisions of 21·5 GeV protons with carbon nuclei.

The inelastic cross section of p-carbon, thus obtained, is shown in figure 1 *b* and the solid curve is the guide-line through the calculated points. The errors on the calculated points are due to uncertainties in the slope parameter, the real to imaginary part of the forward scattering amplitude, the value of N_{eff} and the $\sigma_{\text{el}}^{\text{pp}}$. The magnitude of these uncertainties amount to (i) 3 mb in the energy region 20 to 60 GeV and (ii) 5 mb for energies $>$ 60 GeV. We have listed in table 2 the values of cross sections as obtained by Akimov *et al* and our calculated values from Glauber theory.

From figure 1 *b* and table 2 we see that the points of Akimov *et al* at 22 GeV and 62 GeV are in good agreement with the calculated ones, whereas the points at 200 GeV and 610 GeV are larger than the calculated ones by (30·0 $\pm$ 8·6) mb

Table 2. Details of σ_{incl} (p–C)

Incident energy (GeV)	σ_{inel} (p–C) Akimov *et al* (mb)	σ_{inel} (p–C) calculated from Glauber theory (mb)
22	217±7	224±3
62	232±5	229±3
200	263±7	233±5
610	266±12	239±5

and (27 ± 13) mb respectively. It is to be noted that the rise of the cross section σ_{inel} (p-carbon) is unlikely to be due to any systematic error since (*a*) σ_{inel} (pp) agrees well with the accelerator measurements in the range 20–600 GeV and (*b*) σ_{inel} (p-carbon) agrees with accelerator measurements at 20 and 60 GeV. Now the question arises whether this latter discrepancy can be accommodated by an admixture of deuterons among the incident protons. This is discussed below.

3.1. *Is the rise in* σ_{inel} (p-C) *due to contamination of deuterons* ?

We noted above that there is a discrepancy in the measured σ_{inel} (p-C) with respect to Glauber calculation in the energy range 200 to 600 GeV. It is better, therefore, to estimate first the fraction of deuterons among the singly charged cosmic ray particles in the energy region 20 to 60 GeV—as there are accelerator data in σ_{inel} (p-p) as well as in σ_{inel} (p-C) in this energy range—and then extrapolate this to higher energy range. For this we estimate ϵ from σ_{inel} (p-C) at 20 and 60 GeV in the way described by eq. (2) of section 2. We have used $K = \lambda_{p-c}/\lambda_{d-c}$ to be 1·9 in agreement with the experimental observation of Binon *et al* (1970) on the absorption of antideuterons in carbon. The values of ϵ thus obtained are (− 0·03 ± 0·04) and (+ 0·01 ± 0·03) at 22 and 62 GeV respectively. Taking a weighted mean of these two values of ϵ and the two values listed in table 1 from σ_{inel} (p-p) at 22 and 62 GeV, we obtain:

$$\langle \epsilon \rangle = (-0{\cdot}02 \pm 0{\cdot}02)$$

From this we estimate an upper-limit on the fraction of deuterons among the singly charged particles in the energy range 20 to 60 GeV, as 0·04 with 95% confidence level. This upper limit corresponds to 1% for the same energy/nucleon of deuterons as that of protons. This estimate is better than the estimates made by Ganguli *et al* (1967) and Apparao (1973).

Now, let us see whether we can explain the rise of σ_{inel} (p-C) in the energy region 200 to 600 GeV with the assumption of 4% as deuterons, an upper limit estimated above, among the singly charged particles. This would give rise to an increase of 0·04 (K—1), *i.e.*, 3·6%, which corresponds to a rise of only 8·6 mb. But the experimental rise to be accounted for are (*see* table 2): (30·0 ± 8·6) and (27 ± 13) mb at 200 and 610 GeV respectively. In order to account for this rise the fractions of deuterons that are needed are: (14 ± 4)% at 200 GeV and (12 ± 6)% at 610 GeV—the weighted mean of these two values is (13·4 ± 3·3)%. The latter value corresponds to $(15 \pm 4)\%$ of deuterons with respect to primary protons. Thus we see that we need a larger fraction of deuterons in the energy range 200 to 600 GeV if we have to explain the observed rise in $\sigma_{\rm inel}$ (p-C). Below we examine other theoretical considerations to see whether they can naturally accommodate the rising $\sigma_{\rm inel}$ (p-C) *i.e.*, without the hypothesis of deuteron contamination.

3.2. *Other theoretical considerations*

Recently Maor and Nussinov (1973) using two components for the nucleon-nucleon amplitude in the Glauber formalism have predicted a faster rise for nucleon-nucleus cross section than expected from the normal single exponential representation of the amplitude as in eq. (4). But their predicted rise is effective for energies > 1000 GeV and hence cannot explain the experimental rise discussed above. We would also like to mention that some theoretical calculations (Trefil 1971) predicted a decrease of $\sigma_{\rm inel}$ (p-C) at energies > 100 GeV due to multistep regeneration through inelastic intermediate steps; such a decrease is definitely ruled out from our analysis.

One might wonder whether the observed increase in the cross sections at energies above 60 GeV could have something to do with our neglect of dynamical position correlations of nucleons inside the target nuclei of carbon. It looks to us that the effects of these correlations would not produce an energy dependent rise at such high energies. At kinetic energies of a few GeV already the incident nucleons move with velocities close to the speed of light ($\beta = 1$) and the motion of nucleons in the nuclus ($\beta \lesssim 0{\cdot}2$) appear ' frozen ' to the incident nucleon. Position correlation should then have just the effect of normalising the free parameter a_0 in eq. (3). A discussion of this problem by Yennie (1971) reveals that the effect of these correlations is more important for quasi-elastic (incoherent) scattering than for total cross sections. The analysis of Glauber and Matthiae (1970) of proton-nucleus quasi-elastic scattering at 19 GeV/*c* does not show that correlation effects are significant. Thus we can exclude the possibility of a remote connection between the observed rise in the carbon cross section and the effects of position correlation between nucleons in the target nucleus.

4. Summary

We made a detailed analysis of the experimental data of Akimov *et al.* for inelastic cross sections of proton-carbon and proton-proton collisions in the energy range 20 to 600 GeV. Our present analysis has been made by using Glauber's theory and the information from accelerator energies regarding $\sigma_{\rm inel}$ (p-p) in the energy range 20 to 1500 GeV and $\sigma_{\rm inel}$ (p-C) in the energy range 20 to 60 GeV. We draw the following conclusions:

(i) The deuteron flux in the primary cosmic rays is consistent with zero in the energy range 20 to 60 GeV. A good upper-limit with 95% confidence level on the fraction of deuterons with respect to primary protons in the energy range 20 to 60 GeV is 4%. For the same energy per nucleon, the above upper limit is 1%.

(ii) There is an indication for a rise of (29 ± 7) mb in the inelasic cross sections of proton-carbon collisions in the energy range 200 to 600 GeV over and above what is expected from the Glauber's multiple scattering theory*. This leads to the prediction of absorption cross section at NAL in the energy range 200 to 600 GeV as (288 $\pm$ 8) mb, whereas the expected value from Glauber's theory is (258 $\pm$ 3) mb.

(iii) If however the rise in σ_{inel} (p-C) is interpreted as due to possible contamination from deuterons, its contribution with respect to primary protons has to be (15 $\pm$ 4)% in the energy range 200–600 GeV as compared to the upper limit of 4% in the energy range 20 to 60 GeV.

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* An earlier analysis made by Balashov and Korenman (1970) compared the absorption cross section as obtained from Glauber's theory with the experimental results of Akimov *et al*. Their procedure is not correct, because one must subtract quasi-elastic cross section from σ_{abs} for comparison with the results of Akimov *et al.*