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# The filamentary structure of the sunspot magnetic fields

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**Abstract.** The filamentary structure of the magnetic fields as well as the coherent radiations that emanate from a sunspot are explained considering solar burst as a non-equilibrium process. Methods of irreversible statistical mechanics have been applied to the problem of an electron gas in a constant magnetic field to explain the above features. We have obtained the non-equilibrium distribution function in the self-consistent field approximation. The dielectric function, we obtained, is a function of time, besides being a function of frequency and wavevector. We have thus taken the non-linearity of the system as well. This theory explains many features of stria bursts, chain bursts as well as the type III bursts. This also accounts for the bunching of the magnetic field lines as a consequence of quantisation of flux in the Landau sense.

**Keywords.** Filamentation; bunching of magnetic fields; coherent radiation; sunspots.

## 1. Introduction

Various theories have been put forward to explain the phenomena of solar flare and the associated mechanisms (De Jager 1968). However in the recent past, more and more experimental results have brought out fine structures in the flare both in the visible and radio range (de la Nöe and Boischot 1972) and the theories have either to be modified drastically or recast to explain the new results.

The observational results have shown that most of the radiations are nonthermal in origin and theoreticians have started realising (Friedman and Hamberger 1969) that one should invoke plasma turbulence to explain these phenomena rather than the usual hydrodynamics or magnetohydrodynamics. In spite of this, Coppi and Friedland (1971) tried to develop a magnetohydrodynamic theory invoking the microscopic instabilities such as tearing mode instabilities to explain the solar flare. Their theory however cannot account for the phenomenon of bunching of magnetic lines (Moreton and Severny 1968) or the high degree of coherence observed in the radiation. This clearly shows that one cannot realistically assume an MHD scheme and that the phenomena may be far removed from a thermodynamic equilibrium. It is also questionable that particles in a system which is radiation dominant have a collision integral which is Boltzmannian in nature, so that the MHD equations (which are nothing but moments of the Boltzmann equations) could be applicable to this system at all.

We propose in this paper to investigate this problem *de novo* from a microscopic standpoint. We shall consider the problem of an electron gas in a constant magnetic field (initially) and after switching on the interactions, see how the system evolves in space and time. We assume the system to be inhomogeneous and obtain the one particle distribution function $f(\boldsymbol{q}, \boldsymbol{p}, t)$ starting from a Liouville representation. We shall sum up all the diagrams of the order $(e^2c/m)^n$ and obtain a dielectric function and show that many of the observed features could be explained on the basis of this distribution function. The methods adopted here have been developed earlier (Pratap *et al* 1972 *a*) and the peculiar feature is that the expansion does not involve an assumption on the magnetic field intensity or temperature and therefore one can take the various temperature limits and magnetic field limits. These have been discussed in two recent papers (Pratap 1974 *a*, *b*).

In this paper we propose to show that in applying the techniques of non-equilibrium statistical mechanics to this problem, one can explain the bunching of electrons, thereby obtain filamentation, in density as well as magnetic field and radiation. Physically the electrons describe small eddies, the size of which is being determined by the temperature as well as the original magnetic field intensity and this rearranges the field, etc., in a filamentary structure. We do not assume a Maxwellian at any stage, nor does $f(\boldsymbol{q}, \boldsymbol{p}, t)$ go over to a Maxwellian asymptotically in time. Therefore, the system is not a hydromagnetic one, and is nearer to a turbulent one, even though the energy spectrum is very much different from a Kolmegrov's power law. Under some approximation, one can obtain the power law and this will be discussed in a latter communication.

## 2. Formulations of the problem

The system consists of charged particles in a magnetic field $\boldsymbol{H}$ which we take for convenience in the Z directions of a Cartesian coordinate system. We label one particle of the system by $P$ denoting the test particle and the remaining as the field particles denoted by $l$. The interaction is through the electromagnetic field (the transverse component) designated by $\lambda$. We then have the total Hamiltonian of the system as

$$\mathcal{H} = \mathcal{H}_P + \sum_l \mathcal{H}_l + \sum_\lambda \mathcal{H}_\lambda \tag{2.1}$$

where

$$\mathcal{H}_P = \frac{1}{2m_p}\left[\boldsymbol{p}_P - \frac{e_P}{2c}\boldsymbol{q}_P \times \boldsymbol{H} - \frac{e_P}{c} A_P(\boldsymbol{q}_P, J_\lambda, \omega_\lambda)\right]^2$$

$$\mathcal{H}_l = \frac{1}{2m_l}\left[\boldsymbol{p}_l - \frac{e_l}{2c}\boldsymbol{q}_l \times \boldsymbol{H} - \frac{e_l}{c} A_l(\boldsymbol{q}_l, J_\lambda, \omega_\lambda)\right]^2$$

and

$$\mathcal{H}_\lambda = \nu_\lambda (J_\lambda + J_{-\lambda}) \tag{2.2}$$

We shall now make the following transformations:

$$m_P\, \boldsymbol{u} = \boldsymbol{p}_P - \frac{e_P}{2c}\boldsymbol{q}_P \times \boldsymbol{H} - \frac{e_P}{c} A_P \tag{2.3}$$

and

$$\boldsymbol{P}_l = \boldsymbol{p}_l - \frac{e_l}{2c}\boldsymbol{q}_l \times \boldsymbol{H} = \boldsymbol{a}\,(2m_l\tilde{\omega}_l J_l)^{\frac{1}{2}} \cos \omega_l$$

$$+ \boldsymbol{b}\,(2m_l\tilde{\omega}_l J_l)^{\frac{1}{2}} \sin \omega_l + \boldsymbol{c} p_{3l} \tag{2.4}$$

where $\varpi$ is the cyclotron frequency ($= eH/m_i c$), $J_i$ and $\omega_i$ are the action angle variables. The transformation (2.4) has been possible because of the fact that the motion of the particle can be resolved into one along the magnetic field and the other perpendicular to the magnetic field. The motion perpendicular to the magnetic field is like that of a harmonic oscillator in a coordinate system moving in the direction of $\boldsymbol{H}$ with a velocity of $p_{3i}/m_i$. With these transformations, the Hamiltonian can be written as

$$\mathcal{H}_P = m_P u^2/2$$

$$\mathcal{H}_i = \frac{1}{2m_i} P_i^2 - \frac{e_i}{m_i c} (\boldsymbol{P}_i \,.\, \boldsymbol{A}_i) \tag{2.5}$$

where we have retained only the first order in the electric charge, since the square term in the Hamiltonian does not contribute towards the self-consistent field approximation. With these, we can write the Liouville density

$$\rho = \rho\,(\boldsymbol{u}, \boldsymbol{q}_P \,;\; J_i, \omega_i, p_{3i}, \boldsymbol{q}_i \,;\; J_\lambda, \omega_\lambda, t) \tag{2.6}$$

and the Liouville's equation as

$$\frac{\partial \rho}{\partial t} + \dot{\boldsymbol{u}}_p \,.\, \frac{\partial \rho}{\partial \boldsymbol{u}} + \dot{\boldsymbol{q}}_P \,.\, \frac{\partial \rho}{\partial \boldsymbol{q}_P} + \dot{J}_i \frac{\partial \rho}{\partial J_i} + \dot{\omega}_i \frac{\partial \rho}{\partial \omega_i} + \dot{p}_{3i} \frac{\partial \rho}{\partial p_{3i}}$$

$$+ \dot{\boldsymbol{q}}_i \,.\, \frac{\partial \rho}{\partial \boldsymbol{q}_i} + \dot{J}_\lambda \frac{\partial \rho}{\partial J_\lambda} + \dot{\omega}_\lambda \frac{\partial \rho}{\partial \omega_\lambda} = 0 \tag{2.7}$$

Using the equation (2.3) and Hamilton's equations of motion, we can obtain the time derivatives occurring in (2.7) and on substituting these, we can rewrite equation (2.7) as

$$\frac{\partial \rho}{\partial t} + \mathrm{i}\mathcal{L}\rho = \frac{e}{c} (\mathrm{i}\delta L)\, \rho \tag{2.8}$$

where

$$\mathrm{i}\mathcal{L} = L_P + L_i + L_\lambda \tag{2.9}$$

with

$$L_P = \dot{\boldsymbol{u}}_P \,.\, \frac{\partial}{\partial \boldsymbol{q}_P} + \vec{\Omega}_P \times \boldsymbol{u}_P \,.\, \frac{\partial}{\partial \boldsymbol{u}_P} \qquad (\Omega_P = e_P H/m_P c)$$

$$L_i = \varpi_i \frac{\partial}{\partial \omega_i} + \frac{\boldsymbol{P}_i}{m_i} \,.\, \frac{\partial}{\partial \boldsymbol{q}_i}$$

$$L_\lambda = \nu_\lambda \frac{\partial}{\partial \omega_\lambda} \tag{2.10}$$

and

$$\mathrm{i}\delta L = \mathcal{A}_P + \mathcal{B}_P + \mathcal{A}_i + \mathcal{B}_i \tag{2.11}$$

wherein

$$\mathcal{A}_P = a_P \left(\frac{8c^2}{V\nu_\lambda}\right)^{\frac{1}{2}} \nu_\lambda \frac{\partial}{\partial \mathrm{u}} \,.\, \boldsymbol{e}_\lambda \sqrt{J_\lambda} \sin \omega_\lambda \cos (\boldsymbol{K}_\lambda \,.\, \boldsymbol{q}_P)$$

$$\mathcal{B}_P = a_P \left(\frac{8c^2}{V\nu_\lambda}\right)^{\frac{1}{2}} (\boldsymbol{u} \,.\, \boldsymbol{e}_\lambda) \cos (\boldsymbol{K}_\lambda \,.\, q_P) \left\{ \frac{\partial}{\partial \omega_\lambda} \sqrt{J_\lambda} \cos \omega_\lambda \frac{\partial}{\partial J_\lambda} \right.$$

$$- \frac{\partial}{\partial J_\lambda} \sqrt{J_\lambda} \cos \omega_\lambda \frac{\partial}{\partial \omega_\lambda} \Big\}$$

$$\mathscr{A}_l = \alpha_l \left(\frac{8c^2}{V\nu_\lambda}\right)^{\frac{1}{2}} \sqrt{J_\lambda} \cos \omega_\lambda \, O_l^\lambda$$

$$\mathscr{B}_l = \alpha_l \left(\frac{8c^2}{V\nu_\lambda}\right)^{\frac{1}{2}} (\boldsymbol{P}_l \, . \, e_\lambda) \cos (\boldsymbol{K}_\lambda \, . \, \boldsymbol{q}_l) \left\{ \frac{\partial}{\partial \omega_\lambda} \sqrt{J_\lambda} \cos \omega_\lambda \frac{\partial}{\partial J_\lambda} \right.$$

$$\left. - \frac{\partial}{\partial J_\lambda} \sqrt{J_\lambda} \cos \omega_\lambda \frac{\partial}{\partial \omega_\lambda} \right\} \tag{2.12}$$

In writing the above operators we have defined the interaction vector potential and other quantities as in (Pratap 1967), and the operator $O_l^\lambda$ is also defined in (Pratap *et al* 1972 *a*). The essential difference between the present formulations and the previous ones is in the definition of $A_p$ and also $L_p$. It may be noted that the two terms in $L_p$ also do not commute amongst themselves and one has to do a Baker-Hausdorff expansion (Pratap *et al* 1972 *b*) and rewrite $\exp(-\tau L_p)$ as

$$\exp(-\tau L_p) = \exp\left(\tau \boldsymbol{u} \times \vec{\Omega} \, . \, \frac{\partial}{\partial \boldsymbol{u}}\right) \exp\left[-\sin \Omega\tau \frac{\boldsymbol{u}}{\Omega} + \frac{\vec{\Omega} \, . \, \boldsymbol{u}}{\Omega^3} \right.$$

$$\left. \times (\sin \Omega\tau - \Omega\tau) \vec{\Omega} + \frac{1 - \cos \Omega\tau}{\Omega^2} \boldsymbol{u} \times \vec{\Omega}\right] . \frac{\partial}{\partial \boldsymbol{q}} \tag{2.13}$$

With the above definitions of the operators, we can write the formal solution of equation (2.8) as

$$\rho(t) = \exp(-i\mathscr{L}t\left[\sum_{j=0}^{\infty} \left(\frac{e}{c}\right)^j \int_0^t \mathrm{d}t_1 \int_0^{t_1} \mathrm{d}t_2 \ldots \int_0^{t_{j-1}} \mathrm{d}t_j O_j \rho(0) \right. \tag{2.14}$$

where

$$O_j = e^{iLt_1} (i\delta L) e^{-iL(t_1 - t_2)} (i\delta L) e^{-iL(t_2 - t_3)} (i\delta L) \ldots e^{-iL(t_{j-1} - t_j)} (i\delta L) e^{-iLt_j} \tag{2.15}$$

Solution given by (2.14) contains all the correlations of all time scales. In obtaining the one particle distribution function, we have to extract a subset of infinite terms and sum them and this is what we shall do in the next section.

## 3. One particle distribution function

We shall develop the one-particle distribution function with the interaction time as the characteristic time scale, *viz*. the inverse of the plasma frequency. Thus we shall take all the terms of the order of $(e^2c/m)$ where $c$ is the concentration $(N \to \infty, V \to \alpha, N/V = c)$. We shall essentially be working in the first Born approximation and the details are given in Pratap (1967). Thus in the self-consistent field approximation, we can write the distribution function as

$$f(\boldsymbol{u}, \boldsymbol{q}, t) = \frac{z_p^2}{mV} 8\pi e^2 \int_0^t \mathrm{d}t_1 \int_0^{t_1} \mathrm{d}t_2 e^{-L_p(t-t_1)} \frac{\partial}{\partial \boldsymbol{u}} \, . \, e_\lambda \, (\boldsymbol{u}_p \, . \, \overrightarrow{e_\lambda + \alpha})$$

$$\times \cos(ky + \overline{\boldsymbol{K}_\lambda + \boldsymbol{b}k} \, . \, \boldsymbol{Q}) \, \rho(t_1 t_2) \oint \mathrm{d}z e^{-iz(t_1 - t_2)} \frac{iz}{(z^2 - \nu_\lambda^2)(1 - \epsilon(z))} \tag{3.1}$$

where now $\vec{\alpha}$ is a vector defined in (C.5) of Pratap *et al* (1972 *a*) and

$$Q = -(u + u \times \vec{\Omega}\tau)\frac{\sin \Omega\tau}{\Omega} + \frac{(\vec{\Omega} \cdot u)(\sin \Omega\tau - \Omega\tau)}{\Omega^3}\vec{\Omega} + $$

$$+ \left(\frac{1 - \cos \Omega\tau}{\Omega^2}\right)(u + u \times \vec{\Omega}\tau) \times \vec{\Omega} \qquad (\tau = t_1 - t_2) \qquad (3.2)$$

and

$$\epsilon(z) = 1 - \frac{\chi(z)}{z^2 - v_{\lambda'}^2} \qquad (3.3)$$

Explicit expression for the function $\chi(z)$ are given in Pratap (1974) and various limits such as high and low temperature as well as strong and weak magnetic field strengths are discussed at length in the above reference. We propose to consider the filamentation process in the plasmas in this paper.

## 4. Filamentation in plasmas

The one particle distribution function given in (3.1) can now be integrated with respect to the momenta variables (not with respect to $\boldsymbol{u}$) and we then obtain the density distribution. One could actually see that in the direction normal to the magnetic field, we have a cosine distribution and since the distribution function is positive definite, we get a spatial distribution in the density as well. In figure 2 we have made a polar plot of the distribution function, and in this we have drawn the isodensity curves. It may be seen that we get closed curves inside the circle. The circle may be treated as the sunspot, then these closed curves are feet of the isodensity tubes as well as isogauss tubes. Thus these tubes arise out of the sunspot. The tubes need not always be normal to the surface, but this can be derived from the expressions given above. It may be seen that the argument of the cosine function is a complicated function of position, velocity and time, and time appears as both linear and harmonic terms.

Thus at points $y$ defined by

$$ky + (\boldsymbol{K}_\lambda + \boldsymbol{b}k) \cdot \boldsymbol{Q} = (2n + 1)\pi/2 \qquad (4.1)$$

we have the distribution function becoming zero. Thus in spite of the system being inhomogeneous, we do get allowed and forbidden regions in the phase space as well as in the configuration space, over which the particles, fields as well as radiation get bunched up and obtain the phenomenon of filamentation which are functions of space and velocity besides being a function of time.

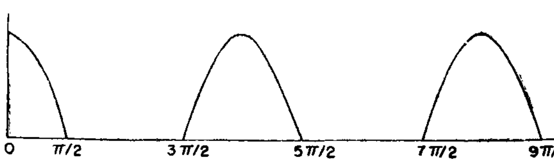


**Figure 1.** A plot of the distribution function as a function of space in a direction perpendicular to the magnetic field. Since the distribution function is positive definite, we get allowed and forbidden zones in space and thereby obtain filamentation. The scale length is $k^{-1}$.

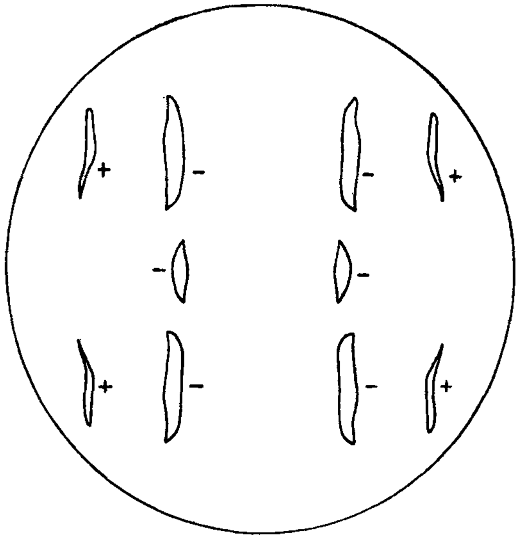

**Figure 2.** A polar plot of the distribution function wherein we have drawn isodensity lines. The plot has been made on an arbitrary scale. The closed contours are the feet of the tubes on the photosphere and the tubes are normal to the Sun's surface. The slope of the tubes can be obtained by differentiating $f(\boldsymbol{p}, \boldsymbol{q}, t)$ with respect to the variables.

In the weak field approximation, $\chi(z)$ can be written as

$$\chi = -2\omega_{pi}^2\left(\frac{4\hbar K_{3\lambda}^2}{m_i\varpi_i}\right)^{\frac{1}{2}}\left(\frac{K_{3\lambda}p_{30_i}}{m_i}\right)^2(e_\lambda \cdot c)\ (e_{\lambda'} \cdot C)\ J_0(\sqrt{2x})$$
$$\times\left[z^2-\left(\frac{K_{3\lambda}\,p_{30i}}{m_i}\right)^2\right]^{-1}\frac{d}{dp_{30i}}\left[1+\left\{\exp\beta\left(\frac{p_{30i}^2}{2m_i}-\frac{\hbar\varpi}{2}-\mu\right)\right\}\right]^{-1} \quad (4.2)$$

In the classical limit (4.2) becomes

$$\chi = -2\omega_{pi}^2\left(4\,\frac{\omega_{pi}}{\varpi}\,\frac{K_{3\lambda}^2}{K_D^2}\right)^{\frac{1}{2}}\left(\frac{k_{3\lambda}\,p_{30i}}{m_i}\right)^2\left[z^2-\left(\frac{K_{3\lambda}\,p_{30_i}}{m_i}\right)^2\right]^{-1}$$
$$\times(e_\lambda \cdot c)\ (e_{\lambda'} \cdot c)\ J_0(\sqrt{2x})\left(\frac{\beta p_{30i}^2}{m_i}\right)\exp\left(-\beta p_i^2/2m_i\right) \quad (4.3)$$

where $x = 4\omega_{pi}k^2/\varpi K_\lambda^2$. Taking the inverse Laplace transform appearing in (3.1), by substituting for $\epsilon(z)$ from (3.3), we get after some manipulations,

$$\oint dze^{-i_s(t_1-t_2)}\frac{iz}{(z^2-\nu_\lambda^2)\left(1-\epsilon(z)\right)} = \oint dze^{-i_s(t_1-t_2)}\,iz\left[\left(\frac{1}{2}+\frac{\gamma^2-\nu_{\lambda'}^2}{4\psi}\right)\right.$$
$$\times\left(z^2-\left(\frac{\nu_{\lambda'}^2+\gamma^2}{2}-\psi\right)\right)^{-1}+\left(\frac{1}{2}-\frac{\gamma^2-\nu_{\lambda'}^2}{4\psi}\right)$$
$$\left.\times\left(z^2-\left(\frac{\nu_{\lambda'}^2+\gamma^2}{2}+\psi\right)\right)^{-1}\right] \quad (4.4)$$

where

$$\gamma = \frac{K_{3\lambda}p_{30i}}{m}\ ;\ \nu_{\lambda'} = \nu_\lambda + \nu\ ;\ \psi^2 = \frac{(\nu_{\lambda'}^2-\gamma^2)^2}{4} - x;\ x = 4\hbar k^2/m_i\varpi_i \quad (4.5)$$

One can now take the inverse Laplace transform of (4.4) and write it as

$$\left(\frac{1}{2}+\frac{\gamma^2-\nu_{\lambda'}{}^2}{4\psi}\right)\cos\left[\left(\frac{\nu_{\lambda'}{}^2+\gamma^2}{2}-\psi\right)^{\frac{1}{2}}(t_1-t_2)\right]$$
$$+\left(\frac{1}{2}-\frac{\gamma^2-\nu_{\lambda'}{}^2}{4\psi}\right)\cos\left[\left(\frac{\nu_{\lambda'}{}^2+\gamma^2}{2}+\psi\right)^{\frac{1}{2}}(t_1-t_2)\right] \tag{4.6}$$

If we take the positive root of (4.5), then (4.6) will remain a harmonic function if and only if

$$\frac{\nu_{\lambda'}{}^2+\gamma^2}{2}-\psi\geqslant 0 \tag{4.7}$$

We call this a reality condition and this would make the argument of the cosine function real. If we take the lower condition, we get

$$\nu_{\lambda'}{}^2=(\nu_\lambda+\nu)^2=2\omega_{pi}{}^2\left(\frac{4\hbar K_{3\lambda}{}^2}{m_i\tilde{\omega}_i}\right)^{\frac{1}{2}}\mathrm{e}^{-x/4}\,(e_\lambda\cdot c)\,(e_{\lambda'}\cdot c)\,J_0\,(\sqrt{2}x)$$
$$\times\,p_{30i}\,\frac{\mathrm{d}}{\mathrm{d}p_{30i}}\left[1+\exp\left\{\beta\left(\frac{p_{30}{}^2}{2m_i}-\frac{\hbar\tilde{\omega}}{2}-\mu\right)\right\}\right]^{-1} \tag{4.8}$$

Equation (4.8) which comes out of a nonabsorption condition from the inverse Laplace transform, is an equation defining the modifying frequency $\nu$ $(=ck)$—a frequency which modifies the photon frequency.

## 5. Discussion

The above results can be interpreted in two ways:

(*a*) Considering the electron gas, one can obtain the natural frequencies with which the system would oscillate. These are given by the solutions of the equation (3.3).

(*b*) If we consider the plasma as a dielectric, then any radiation, passing through the medium, would get modified as according to (4.8). Thus if the r.h.s. is less than $\nu\lambda$, then the frequency is shifted towards the red and we thereby observe a red shift. It may be remembered that $\nu_\lambda$ $(=ck_\lambda)$ is in an arbitrary direction while $\nu\,(=bck)$ is in the $y$ direction. Hence (4.8) is a highly transcendental equation for a given $\nu_\lambda$ and therefore it may have more than one solution, and also because $J_0\,(\sqrt{2x})$ have infinite number of zeros. It may so happen that for a given frequency in the visible region, it may have only a single red shifted frequency in the visible or infrared region and the rest of them may be in the radio or higher wave length region. Furthermore, the right hand side is dependent on $\nu_{\lambda 3}$, *i.e.*, the $Z$ component of the frequency, and hence the red shift is dependent on the original frequency as well. As $x=4\hbar k^2/m\tilde{\omega}$ the equation (4.8) is highly transcendental.

Secondly for those values of $x$ such that

$$(8\hbar k^2/m\tilde{\omega})^{\frac{1}{2}}=j_i \tag{5.1}$$

where $j_i$ is the $i$th zero of the Bessel function of the first kind and order zero, the second term in (3.3) would vanish. In the most general case we have the condition (5.1) as

$$A\,(2\hbar/m\tilde{\omega})^{\frac{1}{2}}=j_i \tag{5.2}$$

where $A$ is a function of time defined by (A.6) (Pratap *et al* 1972 *a*). Condition (5.2) is a time-dependent one, and that this frequency would make the dielectric function unity, *i.e.*, the refractive index becomes unity. Radiation with this time-dependent frequency emerges out of the system uninhibited and thereby a tunnelling of radiation takes place. These radiation frequencies therefore serve as windows in the frequency spectrum. The time dependence of the dielectric function through $A$ in the argument of the Bessel function is due to the fact that we have taken non-linear effects also in the formulation.

The third feature is the bunching of the magnetic field, density, radiation, etc. As has already been pointed out, the scale length of these filamentations ($k^{-1}$) is dependent on the parameters of the plasma and especially on the Fermi radius $(\hbar/m\tilde{\omega})^{\frac{1}{2}}$ which is a direct consequence of the Landau quanta of energy ($\hbar\tilde{\omega}$). It also depends on the temperature, plasma frequency, etc., as can be seen from equation (4.8). These features will not be seen in an MHD approach. It can be observed in the records published by de la Nöe (1972) that the chain bursts have a slope, with the lower frequency emission occurring earlier than the higher frequencies. This shows that the frequencies do depend on the time as has been obtained in this paper.

Finally the radiation that is coming out of the system has a frequency width small compared to the original frequency. This is an indication of the high degree of coherence. This problem will be presented in a separate paper. This method developed here can also be used to obtain quantitative estimates of the transfer of energy from the longitudinal mode to the transverse mode and *vice versa* in the presence of the magnetic field and when inhomogeneities are present. This is also reserved for a future occasion.

## Appendix

In this appendix, we propose to show the transition from the quantum state to classical state adopted in deriving (4.3) from (4.2). We constructed the initial state of the system from the density matrix using Landau wave functions which are solutions of the Schrödinger equation using the Hamiltonian operator defined in (2.2) without the interaction potential. One can easily see that the limit $\hbar \to 0$ will make $p^2 (= \hbar/i \nabla)^2$ vanish and thereby reduce the Schrödinger equation to a singular differential equation. To avoid this difficulty, we use the definition of Fermi momentum (Landau and Lifshitz 1959)

$$p_F = (3\pi^2 c)^{\frac{1}{3}} \hbar \tag{A.1}$$

where $c\,(= N/V,\ N,\ V \to \infty)$ is the concentration. Remembering that the Ferm momentum is a statistical concept, and that the concentration appearing in (A.1) is in the thermodynamic limit, one can take this to the classical limit of thermal momentum. Therefore, in the classical limit one can replace $\hbar$ by $mV_T/(3\pi^2 c)^{\frac{1}{3}}$. In this limit (A.1) can be rewritten as

$$\frac{T^3}{c^2} = 9\pi^4 \left[\left(\frac{m}{k}\right)^{\frac{1}{2}} \left(\frac{\hbar}{m}\right)\right]^6 \tag{A.2}$$

and setting values of the quantities, we get a plot of $\ln T = \frac{2}{3} \ln c - 10$ giving a line of slope 2/3 and making an intercept of $-10$ on the $\ln T$ axis and 15 on $\ln c$ axis. Thus for $T \sim 10^{50}$K, $C \sim 10^{22}$. For a system with its state defined by any point on the upper part of the line we have thermal momentum $\gg (3\pi^2 c)^{\frac{1}{3}} \hbar$ and hence (A.1) fails. This gives the usual classical limit of high temperature. This is the case one gets by the erroneous procedure of taking, $\hbar \to 0$. But for a system represented by a state point below the line, we get a low temperature, high density limit in which quantum effects become more important. This does not imply that the quantum effects are important only at temperatures close to the $T \to 0$ K limit. On the other hand even at ordinary temperatures, at high density limits, " quantum like " effects would dominate. This is precisely what we observe at the solar photosphere and chromosphere.

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