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Kinetic theory of parametric excitation of acoustic waves in piezoelectric semiconductors

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Abstract. Using a kinetic description for electrons and the usual equation of motion for lattice displacement we have derived a general dispersion relation for acoustic waves in a piezoelectric semiconductor, in the presence of a strong high frequency electric field oscillating near the electron plasma frequency. Earlier hydrodynamic results valid for $k\lambda_e \ll 1$ (where k is the wave number of the acoustic wave and λ_e the electron mean free path) are rederived as a special case. For $k\lambda_e \gg 1$, two instability branches are discovered and magnitudes of the threshold electric field required to drive the acoustic wave unstable in each case, are obtained.

Keywords. Piezoelectric semiconductors; parametric effects; acoustic waves.

1. Introduction

One of us has recently investigated (Kaw 1973, hereafter referred to as I) the parametric excitation of ultrasonic waves in collision dominated piezoelectric semiconductors by the application of high frequency electromagnetic fields oscillating near the electron plasma frequency. This parametric excitation process is quite distinct from the ones considered by earlier workers (Chaban 1968, Levin and Chernozatonskii 1970, Epshtein 1969; the driving frequency, ω_0 is near acoustic frequency, ω in these works) in the sense that it involves the simultaneous excitation of electron plasma waves. The calculation was done using hydrodynamic equations for the electron 'fluid' and can therefore be applied only to the excitation of long wavelength acoustic waves satisfying the inequality $k\lambda_e \ll 1$. The calculation showed that in the range of validity of the hydrodynamic treatment, higher values of k have a lower threshold field of excitation. It is therefore of interest to extend the above calculation into the domain $k\lambda_e \gtrsim 1$. This is the aim of our present communication. The range of wavelengths $k\lambda_e \gtrsim 1$ obviously necessitates the use of a kinetic description for electrons.

Starting with a Boltzmann equation description for electrons in the presence of a uniform external oscillating electric field, with a number-conserving relaxation model for collisions (for simplicity we ignore the energy dependence of the relaxation time) and the usual equation of motion for lattice displacement, we have derived a general dispersion relation for small amplitude acoustic waves. For $k\lambda_e \ll 1$ and taking the hydrodynamic limit we recover the results of I. For

$k\lambda_e \gtrsim 1$ and acoustic wave frequency $\omega > \nu$ (the electron collision frequency) we find that the dispersion relation is strongly modified and leads to possible excitation of acoustic waves in two different ranges of wavelengths. For $\{\omega_0 - (\omega_p^2 + k^2 V_T^2)^{\frac{1}{2}}\} > 0$ we find the excitation of weekly damped acoustic waves; the threshold field for the excitation of these modes is rather small. For $\omega_0 < (\omega_p^2 + k^2 V_T^2)^{\frac{1}{2}}$ a purely growing mode (which is a strongly modified version of the usual acoustic mode) is obtained; the excitation of this mode requires the amplification of a rather intense high frequency field. The results of the above analysis are in close agreement with the generalised analysis of parametric instabilities discussed recently by Nishikawa (1968).

2. Derivation of the dispersion relation

Consider a piezoelectric semiconductor under the influence of an oscillating electric field $\boldsymbol{E}_0 \cos \omega_0 t$ with frequency ω_0 close to the electron plasma frequency ω_p. Under the influence of this field, electrons oscillate with a velocity, $(e\boldsymbol{E}_0/m\omega_0) \sin \omega_0 t$ whereas the heavier lattice ions remain relatively unperturbed. We wish to investigate the excitation of acoustic waves due to the oscillatory relative motion between the electrons and the lattice. For simplicity, we consider only the one-dimensional case where the acoustic waves propagate along $\boldsymbol{E}_0$.

In a piezoelectric semiconductor, a propagating acoustic wave has low frequency electron density fluctuations associated with it. The external field produces a nonlinear coupling between these low frequency fluctuations and high frequency electron plasma waves in such a manner that both are driven unstable (*see* I for a detailed physical discussion). The basic equations of motion for electrons and the lattice are

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{e}{m}\left(E_0 \cos \omega_0 t \frac{\partial f}{\partial v} + E\frac{\partial f_0}{\partial v}\right) = -\nu\left(f - (n/n_0)f_0\right) \qquad (1)$$

$$n = \int f \, \mathrm{d}\boldsymbol{v} \qquad (2)$$

$$\rho\frac{\partial^2 u}{\partial t^2} = C\frac{\partial^2 u}{\partial x^2} - \beta\frac{\partial E}{\partial x} \qquad (3)$$

$$\frac{\partial E}{\partial x} = \frac{4\pi e}{\epsilon} n - \frac{4\pi\beta}{\epsilon}\frac{\partial^2 u}{\partial x^2} \qquad (4)$$

where f is the perturbed electron velocity distribution and

$$f_0 \equiv f_0\left(v - \frac{eE_0}{m\omega_0}\sin \omega_0 t\right)$$

are the equilibrium density and distribution function of electrons, respectively, u is the lattice displacement and ϵ is the appropriate dielectric constant.* All other symbols have their usual meanings as reported in I. Equation (1) is the linearized microscopic equation for electrons with a BGK type collision term on the right hand side (Bhatnagar *et al* 1954). Equations (3) and (4) are the familiar lattice and Poisson equations with the piezoelectric coupling terms included in them.

* In I, we have taken ϵ due to the lattice to be different for low frequency (ϵ_0) and high frequency (ϵ_∞) perturbations. However, if all frequencies are smaller then the frequencies of characteristic oscillators describing the lattice, ϵ can be taken same for both type of perturbations, We thus assume $\epsilon_0 = \epsilon_\infty = \epsilon$ (Ehrenreich 1966).

To solve eq. (1), we shall seek the Fourier transform of $f(x, v, t)$ with respect to space and time defined by the relation

$$f(x, \boldsymbol{v}, t) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(k, v, \omega) \exp \mathrm{i}(kx - \omega t)\, \mathrm{d}k \mathrm{d}\omega \tag{5}$$

As it is convenient to calculate the perturbed electron distribution function in its own oscillating frame of reference, we shall change the independent variable $\boldsymbol{v}$ to V by the substitution

$$V = \boldsymbol{v} - (eE_0/m\omega_0) \sin \omega_0 t$$

and put

$$f(k, v, \omega) = F(k, V, \omega) \exp(\mathrm{i}\mu \cos \omega_0 t).$$

The quantity $\mu\, (= keE_0/m\omega_0{}^2)$ represents physically the ratio of electron excursion length in the high frequency field to the wavelength of a typical perturbation and the function $F(k, V, \omega)$ is the Fourier transform of $F(x, V, t)$. Using eqs (1) and (5) and the well known property of Bessel function $J_n(x)$, namely,

$$\exp(\mathrm{i}x \sin\theta) = \sum_{-\infty}^{+\infty} J_n(x) \exp(\mathrm{i}n\theta)$$

the Fourier-transformed equation in terms of $F(k, V, \omega)$ reduces to

$$F(k, V, \omega) = -\frac{e}{m}\frac{\partial f_0(V)}{\partial V} \sum_{l} \mathrm{i}^l J_l(-\mu)\, E(k, \omega + l\omega_0)/[\mathrm{i}\,(kV - \omega - \mathrm{i}\nu)]$$

$$+ \nu f_0(V)\, n_1(k, \omega)/\mathrm{i}n_0(kV - \omega - \mathrm{i}\nu) \tag{6}$$

where

$$n_1(k, \omega) = \int F(k, V, \omega)\, \mathrm{d}V$$

Similarly, eqs (3) and (4) in the transformed variables become

$$u(k, \omega) = \mathrm{i}k\beta E(k, \omega)/(\rho\omega^2 - Ck^2) \tag{3$'$}$$

$$\mathrm{i}kE(k, \omega) = \frac{4\pi\beta^2 k^2}{\epsilon} u(k, \omega) + \frac{4\pi e}{\epsilon} \sum_{p=-\infty}^{p=+\infty} \mathrm{i}^p J_p(\mu)\, n_1(k, \omega + p\omega_0) \tag{4$'$}$$

Choosing the equilibrium distribution for $f_0(V)$ in the form

$$f_0(V) = n_0(a/\pi)^{\frac{1}{2}} \exp(-aV^2), \quad a = 2T/\mathrm{m}$$

a form quite suitable for non-degenerate semiconductors, and integrating eq. (6) over velocity space, the resultant relation between $n_1(k, \omega)$ and $E(k, \omega)$ is given by

$$4\pi e n_1(k, \omega) = -\mathrm{i}k\epsilon\chi_e(k, \omega) \sum_{l} \mathrm{i}^l J_l(-\mu)\, E(k, \omega + l\omega_0) \tag{7}$$

where χ_e is the electron susceptibility function defined by the relation

$$\chi_e = \frac{1}{k^2 d^2}\{(1 + \zeta Z(\zeta)\}/\left\{1 + \frac{\mathrm{i}\nu\sqrt{a}}{k} Z(\zeta)\right\} \tag{8}$$

$d\,(= \sqrt{\epsilon T/4\pi n_0 e^2})$ being the electron Debye length, $Z(\zeta)$ the usual plasma dispersion function (Fried and Conte 1961) and finally $\zeta = (\omega + \mathrm{i}\nu)\sqrt{a}/k$. Eliminating $u(k, \omega)$ between eqs (3′) and (4′) and replacing the terms $4\pi e \mathrm{i}^l n_1(k, \omega + l\omega_0)$

and $ik\epsilon i^{l} E(k, \omega + l\omega_0)$, respectively, by $N(k, \omega + l\omega_0)$ and $E(k, \omega + l\omega_0)$, the two coupled set of recurrence relations between $N(k, \omega)$ and $E(k, \omega)$ can be written in the form

$$N(k, \omega) = -\chi_c(k, \omega) \sum_l J_l(-\mu) E(k, \omega + l\omega_0)$$

$$\{1 + \chi_l(k, \omega)\} E(k, \omega) = \sum_p J_p(\mu) N(k, \omega + p\omega_0) \tag{9}$$

the former being the revised form of eq. (7) while the latter is derived as a consequence of relations (3′) and (4′). $\chi_l(k, \omega)$ is the lattice susceptibility function given by the expression

$$\chi_l(k, \omega) = -4\pi\beta^2 k^2/\rho\epsilon(\omega^2 - Ck^2/\rho) \tag{10}$$

To derive the dispersion relation, we eliminate $E(k, \omega)$ in the set (9) and thus obtain an infinite determinantal relation between $N(k, \omega)$ and their harmonics (*i.e.* $N(k, \omega \pm \omega_0)$, etc.). We shall stipulate that the electron excursion lengths are much shorter than the wavelengths of the perturbation and therefore we shall retain only terms involving $N(k, \omega)$ and $N(k, \omega \pm \omega_0)$. This approximation enables us to truncate the infinite determinant to a 3×3 relation. Such a truncation procedure has been extensively used in the theory of parametric instabilities in plasmas; a detailed analysis of the validity of this approximation may be found in the work by Ott *et al* (1973). Assuming that $\chi_l(k, \omega \pm \omega_0) \approx 0$ (because the lattice responds weakly at high frequencies), after some simplifications, the dispersion relation can finally be put in the form

$$1 + \chi_l + \chi_e = -\frac{1}{4}\mu^2\chi_l(1 + \chi_e)\left\{\frac{1}{(1 + \chi_e^+)} + \frac{1}{(1 + \chi_e^-)}\right\} \tag{11}$$

where

$\chi_e^{\pm} = \chi_e(k, \omega \pm \omega_0)$ are the high frequency electron susceptibilities.

3. Solution of the dispersion relation

We shall now discuss the properties of the dispersion relation for certain special cases of interest. Equation (11) can be rewritten as

$$\omega^2 - \frac{Ck^2}{\rho} - \frac{4\pi\beta^2k^2}{\rho\epsilon} = \frac{4\pi\beta^2k^2}{\rho\epsilon}\left\{A - \frac{\chi_e}{1 + \chi_e}\right\} \tag{12}$$

where the expression for χ_l (eq. 10) is used in eq. (11). The quantity A defines the contribution arising due to the externally applied electric field and is given by

$$A = \frac{1}{4}\mu^2\left\{\frac{1}{(1 + \chi_e^+)} + \frac{1}{(1 + \chi_e^-)}\right\} \tag{13}$$

For $A = 0$ and $kV_T \ll \nu$ (V_T being the electron thermal velocity), one uses the large argument expansion of Z-function in χ_e and eq. (12) reduces to the known hydrodynamic result, *viz.*

$$\omega^2 = \frac{Ck^2}{\rho} + \frac{4\pi\beta^2k^2}{\epsilon\rho}\left\{1 + \frac{\omega_p^2}{(\omega_p^2 + k^2V_T^2 - i\nu\omega)}\right\} \tag{12 a}$$

This equation describes damped acoustic waves in piezoelectric semiconductors, the damping arising because of their interaction with electrons. For $A \neq 0$ and $kV_T \ll \nu$, the dispersion relation of I can be recovered as follows:

Equation (12) may be written as

$$\omega^2 - \frac{Ck^2}{\rho} - \frac{4\pi\beta^2k^2}{\epsilon\rho} + \frac{4\pi\beta^2k^2}{\rho\epsilon}\left[1 + \frac{i\omega\nu}{\omega_p^2} + \frac{2\omega_0\delta}{(\omega + \frac{1}{2}i\nu)^2 - \delta^2}\left(\frac{\mu^2}{8}\right)\right] = 0 \tag{12 b}$$

where

$$\delta = \omega_0 - \omega_R$$

$$\omega_R = (\omega_p^2 + k^2V_T^2)^{\frac{1}{2}}$$

and we have approximated $\chi_e \simeq -\omega_p^2/i\omega\nu$ and $\omega_0 = \omega_R$. Equation (12 *b*) is identical to eq. (9) of I if we binomially expand the square parenthesis in that equation, retaining first order terms only and assume $\epsilon_\infty = \epsilon_0 = \epsilon$ as discussed earlier*. It should be emphasised that eq. (12 *b*) is more exact than eq. (9) of I and the expansion of the latter becomes necessary because of certain approximations inherent in the hydrodynamic equations.

We now examine the excitation of acoustic waves for the range of wavelengths $kV_T \gg \nu$. We shall assume throughout our analysis that $\omega_0 \gg kV_T \gg \omega, \nu$. Using eq. (8) and the small argument expansion of the *Z*-function (Fried and Conte 1961) the electron susceptibility can be written as

$$\chi_e = \frac{1}{k^2d^2}\left\{1 + \frac{i\sqrt{\pi}\,(\omega + i\nu)}{kV_T}\right\} / (1 - \sqrt{\pi}\nu/kV_T) \tag{8 a}$$

We restrict our attention to two frequency ranges: (i) $|\omega| \ll \nu$ and (ii) $|\omega| \gtrsim \nu$. For these cases, the quantity A representing the effect of high frequency field term takes the following forms:

$$A \approx \mu^2\{\nu^2(\delta^2 + \tfrac{1}{4}\nu^2 + \tfrac{1}{2}\omega_0\delta) + 2\omega_0\delta^3 + 2i\nu\omega(\tfrac{1}{4}\nu^2 - \delta^2 + \omega_0\delta)\}/$$
$$8(\delta^2 + \tfrac{1}{4}\nu^2)^2; \qquad (|\omega| \ll \nu) \tag{13 a}$$

and

$$A \approx \mu^2(2i\nu\omega - 2\omega_0\delta - \nu^2)/8(\omega^2 - \tfrac{1}{4}\nu^2 - \delta^2 + i\nu\omega); \quad (|\omega| \gtrsim \nu) \tag{13 b}$$

Firstly, we shall examine the case when $|\omega| \ll \nu$. Substituting the expressions (8 *a*) and (13 *a*) in eq. (12), we obtain

$$\omega^2 - \frac{Ck^2}{\rho} - \frac{4\pi\beta^2k^2}{\epsilon\rho} = \frac{4\pi\beta^2k^2}{\epsilon\rho}\left\{\mu^2\left[\nu^2\left(\delta^2 + \frac{\nu^2}{4} + \frac{\delta\omega_0}{2}\right) + 2\omega_0\delta^3 + 2i\nu\omega\left(\frac{\nu^2}{4} - \delta^2 + \omega_0\delta\right)\right]\Big/ 8(\delta^2 + \tfrac{1}{4}\nu^2)^2 - \left[\frac{1}{(1 + k^2d^2)} + \frac{i\sqrt{\pi}\omega k^2d^2}{kV_T(1 + k^2d^2)^2}(1 - \sqrt{\pi}\nu/kV_T)^{-1}\right]\right\} \tag{14}$$

* In I, we have taken ϵ due to the lattice to be different for low frequency (ϵ_0) and high frequency (ϵ_∞) perturbations. However, if all frequencies are smaller than the frequencies of characteristic oscillators describing the lattice, ϵ can be taken same for both type of perturbations. We thus assume $\epsilon_0 = \epsilon_\infty = \epsilon$ (Ehrenreich 1966).

It is clear that the instability is excited when the imaginary part of the high frequency term overcomes the usual damping term on the right and side of eq. (14). Therefore the condition for the excitation of instability is

$$\tfrac{1}{4}\mu^2\nu\,(\tfrac{1}{4}\nu^2 - \delta^2 + \omega_0\delta)/(\tfrac{1}{4}\nu^2 + \delta^2)^2$$
$$> \sqrt{\pi}k^2d^2/kV_T(1 + k^2d^2)^2(1 - \sqrt{\pi}\nu/kV_T)$$

or

$$\frac{E_0^2}{4\pi n_0 T} > \frac{4\sqrt{\pi}}{(1 + k^2d^2)^2}\left(\frac{\omega_0}{\omega_P}\right)^4\left(\frac{\nu^2}{4} + \delta^2\right)^2 \Big/ kV_T\nu\left(1 - \frac{\nu\sqrt{\pi}}{kV_T}\right)\left(\frac{\nu^2}{4} - \delta^2 + \omega_0\delta\right) \quad (15)$$

The above criterion also defines a threshold needed for destabilization of acoustic waves. Further for instability it must be noted that $\delta > 0$ or $\omega_0 > \omega_R$ as it is evident from the dispersion relation (14). The minimum amplitude of the electric field required for parametric excitation can be simplified by optimizing the right hand side of the inequality (15). For ω_0 close to ω_P and $k^2d^2 \ll 1$, this threshold field turns out to be of order, $\nu\,(4\pi n_0 T/kV_T\omega_0)^{\frac{1}{2}}$. This result therefore leads us to the conclusion that the short wavelength acoustic instabilities can be triggered by the applied fields with lesser amplitudes.

In the aforesaid criterion for instability, we notice that the threshold condition is independent of the frequency of acoustic waves. This feature is a direct consequence to our earlier assumption that $|\,\omega\,| \ll \nu$. For $|\,\omega\,| \gtrsim \nu$, the quantity A is significantly altered (eq. 13 *b*) and hence the modified dispersion relation by virtue of eqs (12), (13 *b*) and (8 *a*) becomes

$$(\omega^2 + i\nu\omega - \tfrac{1}{4}\nu^2 - \delta^2)(\omega^2 + i\omega\omega_1 - \omega_2^2) + \lambda^2(\nu^2 + 2\omega_0\delta - 2i\nu\omega) = 0 \quad (16)$$

where

$$\omega_1 = \frac{4\pi\beta^2k^2}{\epsilon\rho}\,\frac{\sqrt{\pi}k^2d^2}{kV_T(1 + k^2d^2)^2}\left(1 - \frac{\nu\sqrt{\pi}}{kV_T}\right)^{-1},$$

$$\omega_2^2 = \frac{Ck^2}{\rho} + \frac{4\pi\beta^2k^2}{\epsilon\rho}\,\frac{k^2d^2}{1 + k^2d^2} \approx \frac{Ck^2}{\rho}$$

and

$$\lambda^2 = (\mu^2/8)(4\pi\beta^2k^2/\epsilon\rho) \quad (17)$$

Equation (16) is a fourth degree algebraic equation in ω with complex coefficients. We now investigate it following the methods developed by Nishikawa (1968). This equation admits of two kinds of growing roots depending on the sign of $\delta = \omega_0 - \omega_R$. For $\delta < 0$ we have a purely growing instability of the acoustic wave. The threshold electric field for this instability can be obtained by putting $\omega = 0$ in eq. (16). This gives

$$\left(\frac{keE_0}{m\omega_0^2}\right) = \left\{\frac{\epsilon C(\delta^2 + \nu^2/4)}{\pi\beta^2\omega_0\,|\,\delta\,|}\right\}^{\frac{1}{2}} \quad (18)$$

Optimizing the right side of eq. (18) with respect to $|\,\delta\,|$ we obtain

$$keE_0/m\omega_0^2 = (\epsilon C\nu/\pi\beta^2\omega_0)^{\frac{1}{2}}.$$

In contrast to the threshold electric field for $|\,\omega\,| \ll \nu$, we notice that in the present case, the amplitude critically depends on the piezoelectric coefficient in addition to its dependence on the wavelength of the perturbation. The dependence on k is the same as before, *i.e.* shorter wavelength modes are excited more readily.

For $\delta > 0$, we have an instability of the usual acoustic waves in the semiconductor. The threshold electric fields can be readily obtained assuming ω to be real, equating the real and imaginary parts of eq. (16) and eliminating the unknown quantity (*viz.*, ω) from the equations. Thus we obtain the two equations

$$(\omega^2 - \tfrac{1}{4}\nu^2 - \delta^2)(\omega^2 - \omega_2^2) - \nu\omega_1\omega^2 + 2\omega_0\delta\lambda^2 = 0 \tag{19 a}$$

$$\nu(\omega^2 - \omega_2^2) + \omega_1(\omega^2 - \tfrac{1}{4}\nu^2 - \delta^2) - 2\nu\lambda^2 = 0 \tag{19 b}$$

Eliminating ω^2 between eqs (19 *a*, *b*), it gives a complicated quadratic equation in λ^2 which can however, be readily solved. Making the additional assumptions $\omega_2 \gg \nu \gg \omega_1$ and $\delta \neq 0$ which are valid over a wide parameter range we get for the threshold λ^2

$$\lambda^2 = \frac{\omega_1}{2\omega_0}\left[\frac{(\omega_2^2 - \delta^2)^2 + \nu^2\omega_2^2}{\nu\delta}\right] \tag{20}$$

The right side has minimum value for $\delta = \omega_2$ and gives

$$(keE_0/m\omega_0^2) = \left(\frac{\epsilon C}{\pi\beta^2}\frac{\omega_1}{\omega_2}\frac{\nu}{2\omega_0}\right)^{\frac{1}{2}} \tag{21}$$

This threshold field is smaller than that of the purely growing instability, as is usually the case (Nishikawa 1968). Note that $\delta = \omega_2$ *i.e.* $\omega_0 = \omega_R + \omega_2$ corresponds to the case of perfect matching of the incident frequency to the sum of frequencies of the two decay waves; that is why the threshold field is minimum for $\delta = \omega_2$. Since (ω_1/ω_2) goes as k^2, in this case, the threshold field is independent of k.

4. Summary

We have extended the previous hydrodynamic calculation of parametric excitation of acoustic waves in piezoelectric semiconductors, into the short wavelength region $kV_T > \nu$ where a kinetic description of electrons is necessary. For $|\omega| > \nu$, we find that there are two branches of instability depending on the sign of $\delta = \omega_0 - (\omega_P^2 + k^2V_T^2)^{\frac{1}{2}}$; this conclusion is similar to that obtained by Nishikawa (1968) for general parametric instabilities. For $\delta < 0$, a purely growing instability is obtained. The threshold field for this instability does not depend on the damping of the low frequency mode; it depends inversely on wavenumber k and the piezoelectric coefficient β. For $\delta > 0$, we get the excitation of usual acoustic waves in the semiconductor. The threshold field E_0 is now independent of k and β. This is a consequence of the fact that the threshold λ^2 is now proportional to the low-frequency damping rate ω_1 and both go as $k^2\beta^2$, which therefore cancels out.

Finally, it should be pointed out that the use of the elastic equation for the description of acoustic waves limits our treatment to the excitation of non-dispersive acoustic phonons only. This puts a lower limit on the wavelength of the acoustic waves, namely $ka \ll 1$ where a is the interparticle spacing in the lattice (Kittel 1966). Thus the wavelength range for the validity of the above kinetic treatment is given by

$$a^{-1} \gg k \gg \nu/v_e$$

Since the typical interatomic spacing is a few angstroms and a typical electron mean free path is a few microns, the range of wavelengths for which the treatment is applicable is quite considerable.

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