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Phonon drag resistivity of potassium*

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Abstract. The phonon drag resistivity for potassium is calculated by solving the Boltzman equations for both the electrons and phonons as opposed to the conventional method of Ziman where the phonon equation is not considered. By an application of the Schwartz inequality we can show that the drag resistivity in the present formalism is larger than that obtained by the conventional method. We substantiate this result by numerical calculation for potassium at very low temperatures, using a realistic phonon spectrum obtained from inelastic neutron scattering data.

Keywords. Phonon drag; alkali metal; electrical resistivity; transport coefficient.

1. Introduction

The effect of non-equilibrium phonons, the so-called phonon drag effect, on the transport coefficients of metals is known to be quite significant at very low temperatures where the electron-phonon interaction is the principal mechanism by which the phonon system could reach equilibrium. Measurement of thermal conductivity of potassium by Archibald *et al* (1967) shows that the least important mechanism for phonon equilibrium is the phonon-phonon scattering. Phonon impurity scattering can be ruled out if the sample is sufficiently clean.

In view of the fact that the ideal electrical resistivity of potassium has recently been a subject of extensive investigation, it is necessary to study the phonon drag resistivity more carefully. For this one may refer to the works of Rice and Sham (1970), Ekin (1971), Trofimenkoff and Ekin (1971) and Gugan (1971).

The earlier work of Sondheimer (1956) vastly overestimates the drag resistivity ρ_g because of the incorrect assumption that the phonon impurity scattering, rather than the phonon-electron scattering, is the chief mechanism for the phonon to reach to equilibrium. More recently Huebener (1966) obtained some numbers for ρ_g from experimental data on thermopower and thermal resistivity above 20 K where the phonon drag effect is not important as rightly concluded by him. Recently Kaveh and Wiser (1972) have calculated the drag resistivity of potassium using Ziman's (1963) formula where a trial function is used for the phonon out of balance distribution function.

It was first observed by Gurevich (1945, 1946) that the Boltzman equations for electrons and phonons are interrelated. Later, Bailyn (1958) pointed out that

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the phonon out of balance function can be explicitly written in terms of electron out of balance function so that there is no need to use a trial function for the phonons.

In this paper we shall show explicitly that the resistivity derived from the simultaneous solutions of the Boltzman equations for both electrons and phonons should give a better limit than the conventional method of Ziman (1963) which has been used by authors such as Kaveh and Wiser (1972) and Huebener (1966). In fact, one can show that there exists a definite relation of inequality between the results obtained by these two methods. We shall substantiate this by explicit numerical calculation for potassium at very low temperatures.

In section 2 we shall outline the theory and in section 3 we shall present the numerical results and the relevant discussions.

2. Theory

Let us take the electron distribution function f_k with wavevector $\boldsymbol{k}$ and the phonon distribution function N_{qj} with wavevector $\boldsymbol{q}$ and polarisation j to be of the form

$$\begin{aligned} f_k &= f_k^0 - g_k f_k^{0\prime}, \\ N_{qj} &= N_{qj}^0 - G_{qj} N_{qj}^{0\prime}, \\ g_k &\equiv \phi_k E_x, \\ f_k^0 &\equiv [\exp\{(\epsilon_k - \mu)/k_B T\} + 1]^{-1} \\ N_{qj}^0 &\equiv [\exp(\hbar\omega_{qj}/k_B T) - 1]^{-1} \end{aligned} \tag{1}$$

Here E_x is the applied electric field, in the x-direction, ϵ_k and $\hbar\omega_{qj}$ are the electron and phonon energies respectively, μ is the chemical potential. $f_k^{0\prime}$ and $N_{qj}^{0\prime}$ are the derivatives of the thermal equilibrium functions f_k^0 and N_{qj}^0 with respect to the corresponding energy variable. From now on we shall suppress the polarisation index j of the phonons for notational simplicity, although its presence will always be kept in mind.

The electrical conductivity σ is,

$$\begin{aligned} \sigma &= \Sigma_k \, e v_{kx} f_k^{0\prime} \phi_k \\ &= \langle (e\hbar/m) k_x f_k^{0\prime}, \phi_k \rangle \end{aligned} \tag{2}$$

Here v_{kx} is the x-component of the electron velocity, e is the charge and m the effective mass of the conduction electrons. Here we have assumed a free-electron-like constant energy surface for the conduction electrons which is justified for the alkali metals like Na or K. To find the unknown function ϕ_k we appeal to the Boltzmann equation for the electron, which can be written as:

$$-\left(\frac{\partial f_k}{\partial t}\right)_{\text{drift}} = \left(\frac{\partial f_k}{\partial t}\right)_{\text{coll}} = \sum_{k'} [p(k', k) f_{k'} (1 - f_k) - p(k, k') f_k (1 - f_{k'})] \tag{3}$$

Here $p(k, k')$ is the probability of transition from a state k to k'. In the context of electron-phonon interaction this is given by:

$$p(k, k') = \sum_q \lambda_{kk'q} [N_q \bar{\delta} \bar{\triangle} + (N_q + 1) \delta^+ \triangle^+] \tag{4}$$

$$\left.\begin{aligned} \lambda_{kk'q} &\equiv \frac{\pi}{Mn\omega_q} |(\boldsymbol{k}' - \boldsymbol{k}) \cdot \hat{e}_{qj}|^2 V^2 (|\boldsymbol{k}' - \boldsymbol{k}|) \\ \delta^{\pm} &\equiv \delta(\epsilon_{k'} - \epsilon_k \pm \hbar\omega_q) \\ \triangle^{\pm} &\equiv \sum_G \delta(\boldsymbol{k}' - \boldsymbol{k} \pm \boldsymbol{q} \pm \boldsymbol{G}) \end{aligned}\right\} \quad (5)$$

Here V is the electron ion pseudopotential, M is the ion-mass, n is the density of the conduction electrons, $\hat{e}_{qj}$ is the unit vector along the polarisation j of the phonon $\boldsymbol{q}$, and $\boldsymbol{G}$ is a reciprocal lattice vector (which should not be confused with the phonon out of balance function).

Similarly the Boltzman equation for the phonons can be written as,

$$-\left(\frac{\partial N_q}{\partial t}\right)_{\text{drift}} = \left(\frac{\partial N_q}{\partial t}\right)_{\text{coll}} = \sum_{\boldsymbol{k},\boldsymbol{k}'} \lambda_{kk'q} [(N_q + 1) f_k (1 - f_{k'}) - N_q f_{k'} (1 - f_k)] \delta^+ \triangle^+ \quad (6)$$

Both the drift terms are known functions. Therefore, the simultaneous solutions of eqs (3) and (6) will give the distribution functions f_k and N_q. To arrange the equations in a more convenient form, let us define two quantities P and L as,

$$P(\boldsymbol{k}, \boldsymbol{k}', \boldsymbol{q}) = A(\boldsymbol{k}, \boldsymbol{k}', \boldsymbol{q}) [\bar{\delta}\bar{\triangle} + \delta^+ \triangle^+]$$

$$L(\boldsymbol{k}, \boldsymbol{k}', \boldsymbol{q}) = A(\boldsymbol{k}, \boldsymbol{k}', \boldsymbol{q}) (\bar{\delta}\bar{\triangle} - \delta^+\delta^+) \quad (7)$$

$$A(\boldsymbol{k}, \boldsymbol{k}', \boldsymbol{q}) \equiv \frac{\lambda_{kk'q} f_k^0 f_{k'}^0}{k_B T \left| \exp\left(\frac{\mu - \epsilon_k}{k_B T}\right) - \exp\left(\frac{\mu - \epsilon_{k'}}{k_B T}\right) \right|} \quad (8)$$

With the help of eqs (7) and (1) the collision integrals (3) and (6) can be simplified to linear order in g_k and G_q as:

$$\left(\frac{\partial f_k}{\partial t}\right)_{\text{coll}} = \sum_{\boldsymbol{k}',\boldsymbol{q}} [P(\boldsymbol{k}, \boldsymbol{k}', \boldsymbol{q}) (g_{k'} - g_k) - L(\boldsymbol{k}, \boldsymbol{k}, \boldsymbol{q}) G_q]$$

$$\left(\frac{\partial N_q}{\partial t}\right)_{\text{coll}} = -\frac{1}{2} \sum_{\boldsymbol{k},\boldsymbol{k}'} [P(\boldsymbol{k}, \boldsymbol{k}', \boldsymbol{q}) G_q - L(\boldsymbol{k}, \boldsymbol{k}', \boldsymbol{q}) (g_{k'} - g_k)] \quad (9)$$

If the phonon system was assumed to be in thermal equilibrium, *i.e.*, G_q to be zero, then the phonon equation was unnecessary and one would get the ordinary transport coefficients by solving the electron equation alone. The G_q term in eq. (9) thus introduces the phonon drag effect. The second equation of (9) readily gives us G_q in terms of the electron variables as,

$$G_q = \sum_{\boldsymbol{k},\boldsymbol{k}'} \frac{L(\boldsymbol{k}, \boldsymbol{k}', \boldsymbol{q})}{S(q)} (g_{k'} - g_k) \quad (10)$$

where

$$S(q) \equiv \sum_{\boldsymbol{k}_1, \boldsymbol{k}_2} P(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{q})$$

Substituting this value of G_q in the electron equation (9) and remembering that the $(\partial f_k / \partial t)_{\text{drift}}$ is $(e\hbar/m) k_z f_k^{0\prime} E_z$, one gets the equation for the electron out of balance function ϕ_k to be

$$X(\boldsymbol{k}) = (H_0 - H_1)\,\phi_k$$

with $$X(\boldsymbol{k}) \equiv (e\hbar/m)\,k_x f_k'. \tag{11}$$

The operators H_0 and H_1 are defined in the following way:

$$\left.\begin{aligned} H_0\phi_k &= \sum_{k'q} P(\boldsymbol{k}, \boldsymbol{k}', \boldsymbol{q})\,(\phi_k - \phi_{k'}) \\ H_1\phi_k &= \sum_{k', q} \frac{L(\boldsymbol{k}, \boldsymbol{k}', \boldsymbol{q})}{S(\boldsymbol{q})} \sum_{k_1, k_2} L(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{q})\,(\phi_{k_1} - \phi_{k_2}) \end{aligned}\right\} \tag{12}$$

It can be shown that the operators H_0, H_1 and $(H_0 - H_1)$ are all positive definite. The positive definite character of H_0 and $(H_0 - H_1)$ were shown by Bailyn (1958). We shall show in Appendix 1 that H_1 is also positive definite which has some interesting consequence, as we shall see, in phonon drag resistivity.

Although it is not possible to solve an equation like (11) exactly, one can put some variational limit on the quantity like $\langle \phi_k, X(\boldsymbol{k}) \rangle$ from the fact that $(H_0 - H_1)$ is positive definite. Comparing with eq. (2) one can see that this quantity is nothing but the electrical conductivity σ. Thus applying the principle of Schwartz inequality (Appendix 2) on equation (11) we find that the resistivity ρ (σ^{-1}) has the limit

$$\rho \leqslant \frac{\langle \phi_t, (H_0 - H_1)\,\phi_t \rangle}{|\langle \phi_t, X(\boldsymbol{k}) \rangle|^2} \tag{13}$$

where ϕ_t is some trial function for ϕ_k.

The term of H_1 comes from the fact that the phonons were out of equilibrium. Therefore, this term is explicitly the phonon drag effect. Also $\langle \phi_t, H_1\phi_t \rangle$ is positive definite (Appendix 1). Therefore, the phonon drag explicitly reduces the total resistivity which is consistent with the physical picture that the non-equilibrium phonons produces an additional electric current along the general drift direction of the electrons.

3. Comparison with Ziman's theory

Ziman (1963) developed the phonon drag theory in a completely different manner. Instead of setting up the Boltzman equation for the phonons he uses a trial functior q_x for the phonon out of balance function. This does not affect the ordinary resistivity ρ_{ord} which is proportional to $\langle \phi_t, H_0\phi_t \rangle$. However, the drag resistivity ρ_g is drastically changed.

We can write Ziman's expression for the drag resistivity $\rho_g(Z)$ in terms of our notation as,

$$\rho_g(Z) = \frac{P_{1L}^{\,2}}{J_0^{\,2} P_{LL}} \tag{14}$$

where J_0 is $\langle X(k), \phi_t \rangle$ with k_x as the trial function ϕ_t. Also,

$$\left.\begin{aligned} P_{1L} &= \sum_{k, k', q} A(\boldsymbol{k}, \boldsymbol{k}', \boldsymbol{q})\, Q_x q_x \bar{\delta}\, \bar{\Delta} \\ P_{LL} &= \sum_{q} \tfrac{1}{2} q_x^{\,2} S(q) \end{aligned}\right\} \tag{15}$$

with

$$\boldsymbol{Q} = \boldsymbol{q} + \boldsymbol{G}$$

In the present formalism the drag resistivity ρ_g which is $\langle \phi_t, H_1 \phi_t \rangle / J_0^2$ can be written as,

$$\rho_g = \frac{1}{J_0^2} \sum_{k, k', q} \frac{k_x L(k, k', q)}{S(q)} \sum_{k_1, k_2} L(k_1, k_2, q)(k_{1x} - k_{2x}) \tag{16}$$

Writing L from eq. (7) and remembering that interchange of k' and k is equivalent to interchange of $\delta^+ \triangle^+$ and $\bar{\delta}\bar{\triangle}$, one gets,

$$\rho_g = \frac{1}{J_0^2} \sum_{q} \frac{2}{S(q)} \left| \sum_{k, k'} A(k, k', q)\, Q_x \bar{\delta}\bar{\triangle} \right|^2 \tag{17}$$

We can rewrite $\rho_g(Z)$ and ρ_g as:

$$\left.\begin{aligned} \rho_g(Z) &= |\sum_q f_1 f_2|^2 / \sum_q f_3 \\ \rho_g &= \sum_q \left[\frac{f_1^2 f_2^2}{f_3}\right] \end{aligned}\right\} \tag{18}$$

where

$$\left.\begin{aligned} f_1 &\equiv \sum_{k, k'} A(k, k', q)\, Q_x \delta \bar{\triangle}, \\ f_2 &\equiv q_x, \\ f_3 &\equiv \tfrac{1}{2} J_0^2 \cdot q_x^2 S(q). \end{aligned}\right\} \tag{19}$$

It is easy to show that $\rho_g(Z)$ is smaller than ρ_g in the following way. From Schwartz inequality,

Table 1. The non-drag resistivity ρ_{ord} (divided by T^5, and in units of 10^{-14}), and the ratios (ρ_g/ρ_{ord}) and $(\rho_g(Z)/\rho_{ord})$. For the electron ion potential we have used the Bardeen (1937) pseudopotential model, the parameters being those of Hasegawa (1964).

T K	$\frac{\rho_{ord}}{T^5}\left(\frac{10^{-14}\ \Omega\text{-cm}}{^\circ K^5}\right)$	$\frac{\rho_g}{\rho_{ord}}$	$\frac{\rho_g(Z)}{\rho_{ord}}$
1·0	3·55	0·996	0·935
1·5	3·98	0·903	0·890
2·0	6·50	0·744	0·431
3·0	18·8	0·526	0·041
4·0	29·3	0·475	0·004
5·0	32·7	0·461	0·0003
6·0	31·4	0·459	10^{-5}
8·0	24·2	0·457	0·00015
10·0	17·6	0·455	0·00031

$$\left|\sum_q f_1 f_2\right|^2 = \left|\sum_q \frac{f_1 f_2}{\sqrt{f_3}} \cdot \sqrt{f_3}\right|^2$$

$$< \sum_q \left(\frac{f_1^2 f_2^2}{f_3}\right) \times \sum_q f_3$$

Therefore

$$\frac{|\sum_q f_1 f_2|^2}{\sum_q f_3} < \sum_q \left(\frac{f_1^2 f_2^2}{f_3}\right)$$

i.e.,

$$\rho_g(Z) < \rho_g \tag{20}$$

Since both ρ_g and $\rho_g(Z)$ are subtracted from the ordinary resistivity to give the total resistivity which is an upper bound, we claim that our method will produce a better upper bound than Ziman's method when phonon drag is considered. We shall present the result of the numerical calculation of $\rho_g(Z)$ and ρ_g for potassium at very low temperatures and will see that $\rho_g(Z)$ is indeed much smaller than ρ_g even at a very low temperature.

4. Results and discussion

To carry on the numerical calculations for the drag resistivity, according to the formulae in Appendix 3, we have generated a fairly accurate phonon spectrum by using six nearest neighbour force constants obtained by Woods *et al* (1962) from inelastic neutron scattering data.

To do the integrations over $\boldsymbol{q}$, we have exploited the fact that the irreducible 1/48 (which is a tetrahedron) of the first Brillouin zone holding a solid angle $4\pi/48$ to the centre of the zone is equivalent to the rest of the zone provided all the reciprocal lattice vectors G are considered simultaneously. However, because of the restriction that $|\boldsymbol{q} + \boldsymbol{G}| \leq 2k_F$ (*see* Appendix 3), it turns out that only a limited number of $\boldsymbol{G}$ need to be considered.

The results (ρ_g/ρ_{ord}), $(\rho_g(Z)/\rho_{ord})$ and ρ_{ord} calculated from eqs (21), (23) and (24) are presented in table 1. We can see that $\rho_g(Z)$ is much smaller than ρ_g above 1·5 K. The reason is that in Ziman's theory the phonon function G_q is simply q_x contrary to our result which is (*see* eq 10) $\sim (q_x + G_x)$. Therefore, Ziman's theory will give a different result when Umklapp scattering ($\boldsymbol{G} \neq 0$) starts becoming important. It should be remembered that the lowest G is of the order of $2k_F$ for potassium. Therefore, in order to satisfy the condition that $|\boldsymbol{q} + \boldsymbol{G}| \leq 2k_F$, $|\boldsymbol{Q}|$ should be $|\boldsymbol{q} - \boldsymbol{G}|$. Thus for normal processes ($\boldsymbol{G} = 0$), $(\boldsymbol{Q} \cdot \boldsymbol{q})$ is positive in eq (23) for $\rho_g(Z)$, whereas it is negative for $\boldsymbol{G} \neq 0$. Therefore, $\rho_g(Z)$ is actually the square of the difference of two terms, one for normal scattering and the other for Umklapp scattering. By inspection of eqs (14) and (15) we can say that $\rho_g(Z) \propto (P_{1L}^N - P_{1L}^U)^2$, where P_{1L}^N and P_{1L}^U are the magnitudes of P_{1L} for normal and Umklapp scattering respectively. At very low temperature P_{1L}^N dominates and P_{1L}^U is negligible. As temperature increases P_{1L}^N and P_{1L}^U become comparable to each other and eventually P_{1L}^U dominates over P_{1L}^N. In fact, in the temperature region from 3 K to 10 K, P_{1L}^N and P_{1L}^U are not much different from each other. This accounts for the drastically low value of $\rho_g(Z)$ in this temperature region.

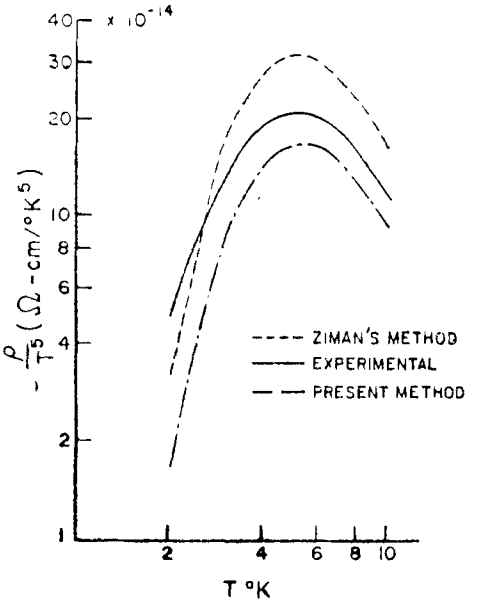

Figure 1. The ideal electrical resistivity of potassium as a function of temperature. The theoretical results, both in Ziman's method and in the present method, have been corrected for phonon drag effect. The experimental results are those of Ekin (1971).

However, the effect of Umklapp scattering is completely different in the case of ρ_g (eq 21). Even though Q_s might be negative for Umklapp processes, ρ_g is the sum of the squares of these terms. Therefore, there is no cancellation of various contributions. Although the effect of Umklapp scattering is manifested in a general reduction of (ρ_g/ρ_{ord}), it is not so drastic as in the case of $(\rho_g(Z)/\rho_{ord})$.

To compare the theoretical calculations with experimental results of Ekin (1971) we plot the total ρ which is $(\rho_{ord} - \rho_g)$ as a function of temperature as presented in figure 1. In this respect it is worth noting that anything better than order of magnitude agreement between measured and calculated resistivities of pure metals at very low temperatures is very unusual. Also there is a considerable amount of disagreement among various experimental results (for example, *see* the data of Garland *et al* (1968), of Natale *et al* (1968) and of Ekin (1971). In any event, the experimental results seem to be closer to ρ_g calculations than to $\rho_g(Z)$ calculations. Also, it must be remembered that electron-electron scattering also gives rise to temperature-dependent ideal resistivity. Calculation of Wilkins *et al* suggests that the calculated value of this contribution is about 60% of the observed ideal electrical resistivity at 2 K. If this contribution is considered Ziman's calculation will yield results farther away from the experimental results, whereas our results will approach the experimental data closer.

It should also be noted that at very low temperature when there is virtually no Umklapping, the drag resistivity ρ_g (also $\rho_g(Z)$) and the ordinary resistivity ρ_{ord} are identical to each other so that the total resistivity would become zero. This vanishing of resistivity was first suggested by Peierls (1930) who pointed out that there is apparently no means of dissipating the crystal momentum produced in the electron phonon system if only normal scattering ($G = 0$) is present.

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Appendix 1

Here we want to show that the operator H_1 defined ir eq (12) is positive definite. For any function g_k,

$$H_1 g_k = \sum_{k', k_1, k_2, q} \frac{L(k, k', q)}{S(q)} \cdot L(k_1, k_2, q)(g_{k_1} - g_{k_2})$$

Therefore

$$(g_k, H_1 g_k) = \sum_{k, k', q, k_1, k_2} gk \frac{L(k, k', q)}{S(q)} L(k_1, k_2, q)(g_{k_1} - g_{k_2})$$

From the definition of L in (7), it is clear that $L(k, k', q) = -L(k', k, q)$. Therefore interchanging the dummy variables k, k', we can rewrite,

$$\langle q_k, H_1 g_k \rangle = \frac{1}{2} \sum_q \frac{1}{S(q)} \left| \sum_{k, k'} (g_k - g_{k'}) L(k, k', q) \right|^2 \geqslant 0$$

Therefore H_1 is positive definite.

Appendix 2

Given an equation of the form $X(k) = H\phi_k$ where H is positive definite, we want to show that $\langle X(k), \phi_k \rangle$ has some lower bound. Let ϕ_t be some arbitrary function. Since H is positive definite, we have, for any a,

$$\langle (\phi_k - a\phi_t), H(\phi_k - a\phi_t) \rangle \geqslant 0$$

or,

$$\langle \phi_k, H\phi_k \rangle + a^2 \langle \phi_t, H\phi_t \rangle - 2a \langle \phi_k, H\phi_t \rangle \geqslant 0.$$

This quantity has minimum value when

$$a = \langle \phi_t, X(k) \rangle / \langle \phi_t, H\phi_t \rangle$$

and when ϕ_t becomes identical to the true solution ϕ_k, this quantity becomes zero. For this value of a, we have,

$$\langle \phi_k, X(k) \rangle \geqslant \frac{\langle \phi_t, X(k) \rangle^2}{\langle \phi_t, H\phi_t \rangle}$$

i.e.,

$$\rho \leqslant \langle \phi_t, H\phi_t \rangle / \langle \phi_t, X(k) \rangle^2$$

The equality sign holds good when ϕ_t is identical with ϕ_k.

Appendix 3

The working formulae for $\rho_g(Z)$ and ρ_g can be obtained from eqs (14), (15) and (17), by performing the integrations over k and k' for a given value of q, consistent with the delta functions $\bar{\Delta}$ and $\bar{\delta}$. The results are as follows:

$$\rho_g = \frac{1}{J_0^2} \sum_q \frac{f(q)}{\Gamma(q)} \left\{ \sum_G \frac{|Q \cdot \hat{\epsilon}_q|^2}{Q} V^2(Q) Q_z \right\}^2 \tag{21}$$

where

$$f(q) = \left(\frac{1}{4\pi^3}\right)^2 \frac{\pi^2}{\hbar M n k_B T} \cdot \frac{K_F^2}{v_F^2} N_q^0 (N_q^0 + 1)$$

$$\Gamma(q) = \sum_G \frac{|Q \cdot \hat{\epsilon}_q|^2}{Q} V^2(Q)$$

$$Q = q + G. \tag{22}$$

J_0 is the current $\langle (e\hbar/m)\, k_x f_k^{0\prime}, k_x \rangle$ and is equal to $(ek_F S_F/12\pi^3\hbar)$. Here S_F is the area of the Fermi surface; k_F and v_F are the Fermi momentum and Fermi velocity respectively. Similarly the expression for $\rho_\theta(Z)$ as given in eq (14) can be written as

$$\rho_\theta(Z) = \frac{1}{J_0^2} \frac{\left\{\sum_q f(q) \sum_G \frac{|Q \cdot \hat{e}_q|^2}{Q} V^2(Q)\, Q \cdot q\right\}^2}{\sum_q q^2 f(q) \Gamma(q)} \tag{23}$$

The ordinary or the non-drag part of the resistivity ρ_{ord} is simply $1/J_0^2 \langle k_x, H_0 k_x \rangle$ H_0 is defined by eq (12). After doing the necessary algebra, it is given by

$$\rho_{ord} = \frac{1}{J_0^2} \sum_q f(q) \sum_G \frac{|Q \cdot \hat{e}_q|^2}{Q} V^2(Q)\, Q^2. \tag{24}$$

It should be remembered that the absolute values of Q $(q + G)$ in the above equations are restricted to be less than or equal to $2k_F$. This restriction arises from the momentum delta functions $\bar{\triangle}$.

References

Archibald M A, Dunick J E and Jericho M H 1967 *Phys. Rev.* **153** 78
Ashcroft N W 1966 *Phys. Lett.* **23** 48
Bardeen J 1937 *Phys. Rev.* **52** 688
Bailyn M 1958 *Phys. Rev.* **112** 1587
Ekin J W 1971 *Phys. Rev. Lett.* **26** 1550
Garland J C and Bowers R 1968 *Phys. Rev. Lett.* **21** 1007
Gugan D 1971 *Proc. Roy. Soc. Ser.* A **325** 223
Gurevich L 1945 *J. Phys. (U.S.S.R.)* **9** 477; 1946 **10** 67
Huebener R P 1966 *Phys. Rev.* **146** 502
Hasegawa A 1964 *J. Phys. Soc. Japan* **19** 504
Kaveh, Moshe and Wiser, Nathan 1972 *Phys. Rev. Lett.* **29** 1374
Natale G G and Rudnick I 1968 *Phys. Rev.* **167** 687
Peierls R 1930 *Ann. Phys. LPZ* (5), **4** 121
Rice T M and Sham L 1970 *Phys. Rev.* **B 1** 4546
Sondheimer E H 1956 *Can. J. Phys.* **34** 1246
Trofimenkoff P N and Ekin J W 1971 *Phys. Rev.* **B 4** 2392
Woods A D B, Brockhouse B N, March R H, Stewart A T and Bowers R 1962 *Phys. Rev.* **128** 1112
Ziman J M 1963 *Electrons and phonons* (Clarendon Press, Oxford)
Wilkins J W and Lowrence W E 1970 O.N.R. contract N00014-67-A-6077-0010, Technical report No. 28