Diffractive and nondiffractive components of the multiplicity distribution in pp collisions

D S NARAYAN
Tata Institute of Fundamental Research, Bombay 400005

MS received 13 February 1974

Abstract. Topological cross sections for diffractive and nondiffractive components in pp collisions are deduced on the basis of a dynamical model proposed earlier to explain the multiplicity distribution of charged particles. The model has an important prediction for the angular and momentum distributions of charged particles in diffractive events.

Keywords. pp collisions; topological cross sections; diffractive and nondiffractive components.

Several authors (Koba et al 1972, Harari and Rabinovici 1973, Van Hove 1973, Fiabkowski and Miettinen 1973, Lach and Malamud 1973) have attempted to explain the prong multiplicity distribution of charged particles in pp collisions by different approaches. Surveying these attempts, one finds that the prong multiplicity distribution is rather a gross feature of multiparticle production that can be fitted in many ways and not restrictive enough to lead to definite conclusions regarding the pattern of multiparticle production. However, Dao et al (1973) have recently reported measurements on the topological cross sections of a more restrictive type, which put strong constraints on the dynamical models which could be compatible with these results. In the new measurements, one makes a selection of events in which there is a slow moving proton. Dao et al have given explicitly the values of the topological cross sections for events corresponding to $|t| < 0.25 \text{ GeV}^2$, where $t$ is the four momentum transfer between the target and the detected proton. From the histograms presented in their paper, one can also deduce the topological cross sections for events corresponding to $|t| < 0.125 \text{ GeV}^2$ and $|t| < 0.5 \text{ GeV}^2$. Further the missing mass squared $M_x^2$, recoiling from the detected proton, has been obtained as a function of the number $n_c$ of charged particles for the different $|t|$ cuts, 0.125, 0.25 and 0.5 GeV$^2$.

The topological cross sections $\sigma_{n_c}^d$, corresponding to the events in the cut $|t| < 0.25 \text{ GeV}^2$, have been identified by Dao et al as the diffractive component and the difference $\sigma_{n_c} - \sigma_{n_c}^d = \sigma_{n_c}^{n,d}$ as the nondiffractive component, where $\sigma_{n_c}$ is the usual topological cross sections without any $|t|$ cut. The arbitrariness in the $|t|$ cut in specifying the diffractive component has been justified by citing that the cross sections do not change much when the $|t|$ cut is increased from 0.25 to 0.5 GeV$^2$.

The importance of the new results, in confronting the predictions of specific dynamical models to a test, has been noted. The nova models (Hwa 1970) which
Multiplicity distribution in pp collisions

predict $\sigma_{nc}^d \sim 1/n_c^2$ are seen to be in disagreement with the date. Further the
diffractive dissociation models with isotropic decay distribution and a cut off
in the momentum in the nova frame, would predict a peaking in the effective mass
plot for all charge topologies. Experimentally no such peaking is seen for $n_c \gtrsim 6$.
It has already been noted by Vander Velde (1972) at the Chicago Conference, on
the basis of data at 100 and 200 GeV, that only about 25\% of $\sigma_d^d$ and a smaller
percentage (if any) of $\sigma_{nc}^d (n_c \gtrsim 6)$ could be attributed to single diffraction dis-
sociation. A nova type of model with anisotropic angular distribution, motivated
by simple multiperipheral arguments, has been constructed recently by Kajantie
and Ruuskanen (1973). Though the disagreement with the effective mass plots
associated with usual nova models may be eliminated, the diffractive multiplicity
distribution at 303 GeV given in their paper seem to be in poor agreement with
the data, as $\sigma_{nc}^d$ falls too fast as $\exp\{- (n_c - 2)/2\}$.

Recently Chaudhary et al (1973) have proposed a model to explain the pattern
of multiplicity distribution in the entire energy range 5–300 GeV. In this model,
the multiplicity distribution is composed of a "specific" mixture of Poisson dis-
tributions. The individual components of this mixture would roughly corre-
spond to different $|t|$ cuts. The first component corresponds to the smallest $|t|$
cut but its value cannot be fixed on the basis of the model. If one makes the
identification that the first component roughly corresponds to the $|t|$ cut at 0.5
GeV$^2$, one can compare the predictions of the model for prong multiplicity dis-
tribution for diffractive and non-diffractive components with the experimental
data of Dao et al.

We find that, in spite of the arbitrariness in the $|t|$ cut, the theoretical results
for $\sigma_{nc}^d$ and $\sigma_{nc}^{n,d}$ are in reasonable agreement with experiment. The shape of the
distribution of $n_c^2 \times \sigma_{nc}^d$ as predicted by the model, compares well with the
experimental curve. Though the effective mass plots cannot be predicted straight
away from the model, rough bounds on the missing mass for different charge topo-
lologies can be inferred. These bounds are seen to be compatible with experiment.
Moreover, one does not expect, on the basis of the present model, sharp peakings
in the effective mass plots except for $n_c = 2$ and possibly $n_c = 4$.

We first briefly outline the model to the extent necessary for the present paper
and give relevant theoretical expressions. The results are then compared with
the experimental data. Finally we present the dynamical implications of our model
of diffraction and some theoretical ideas leading to such a dynamical picture.

According to the model of Chaudhary et al (1973) there are two groups of
particles produced in each collision, which are characterized by different number
distributions. One group of particles, called non-leading particles, may be identi-
fied with the central component or pionization and their distribution is a Poisson
in the number of charged pairs. The other group of particles, referred to as
leading particles, arise in the decay of two fast moving (in the CM system) baryon-
like excited objects. In the energy range which includes ISR energies, the masses
of these excited objects need not exceed a few GeV. The leading particles at any
given energy would have a variation between 2 and some maximum value $2R$,
where $R$ would be a function of energy. The number distribution of leading
charged particles is assumed to be a geometric distribution with $R$ terms. Further
the emission of leading and non-leading particles is assumed to be uncorrelated. The probability \( P_{ne} \) of having a total of \( n_e \) charged particles in an event can be written as:

\[
P_{ne} = \sum_{r=1}^{\frac{n_e}{2}} a_r \bar{P}_{\frac{n_e}{2}-r} \quad \text{if } \frac{n_e}{2} < R
\]

\[
= \sum_{r=1}^{R} a_r \bar{P}_{\frac{n_e}{2}-r} \quad \text{if } \frac{n_e}{2} > R
\]

(1)

where \( \bar{P}_m \) is the probability of having \( 2m \) charged non-leading particles and \( a_r \) is the probability of having \( 2r \) charged leading particles in an event. By our assumptions regarding the number distributions of leading and non-leading particles, one can write,

\[
\bar{P}_m = \exp \left( -\left( \langle n_{n,L} \rangle /2 \right) \right) \left[ \frac{\langle n_{n,L} \rangle /2}{m !} \right] (2)
\]

\[
a_r = \frac{1 - q^r}{1 - q} q^{r-1} (3)
\]

and

\[
\langle n_L \rangle = \frac{2}{1 - q} - \frac{2Rq^a}{1 - q^a} (4)
\]

where \( \langle n_L \rangle \) and \( \langle n_{n,L} \rangle \) are the average numbers of leading and non-leading charged particles. Further \( \langle n_L \rangle = \langle n_L \rangle + \langle n_{n,L} \rangle \) where \( \langle n_L \rangle \) is the average number of charged particles. According to our earlier discussion, the first term on the R.H.S. of (1) is identified with diffraction and the sum of the remaining terms with nondiffraction. The topological cross sections \( \sigma_{nc}^d \) and \( \sigma_{nc}^{n,d} \) can then be written as:

\[
\sigma_{nc}^d = \sigma_{ln} a_1 \bar{P}_{\frac{n_e}{2}} (5)
\]

\[
\sigma_{nc}^{n,d} = \sigma_{ln} \sum_{r=2}^{\frac{n_e}{2}} a_r \bar{P}_{\frac{n_e}{2}-r} \quad \text{if } \frac{n_e}{2} < R
\]

\[
= \sigma_{ln} \sum_{r=2}^{R} a_r \bar{P}_{\frac{n_e}{2}-r} \quad \text{if } \frac{n_e}{2} > R
\]

(6)

where \( \sigma_{ln} \) is the total inelastic cross section. The topological cross sections \( \sigma_{nc}^d \) and \( \sigma_{nc}^{n,d} \) are completely determined by eqs (2) to (6) if one knows the break up of the average number \( \langle n_L \rangle \) of the charged particles as a sum \( \langle n_L \rangle \) and \( \langle n_{n,L} \rangle \) and the value of \( R \). One can give a parametrization of \( \langle n_L \rangle \), \( \langle n_{n,L} \rangle \) and \( R \) as functions of energy or alternatively treat them as free parameters to be fixed at each energy so as to obtain a minimum \( \chi^2 \) fit to the experimental data of \( P_{nc} \). Table (1) in the paper of Chaudhary et al (1973) has been computed by the latter procedure. We make use of that table to compute the values of \( \sigma_{nc}^d \) and \( \sigma_{nc}^{n,d} \) according to (5) and (6). The results at 303 GeV for \( \sigma_{nc}^d \), \( \sigma_{nc}^{n,d} \) and \( n_e^2 \sigma_{nc}^d \) are shown in figures 1, 2 and 3 respectively and compared with the experimental cross sections corresponding to the cut \( |t| < 0.5 \text{ GeV}^2 \). Results for \( \sigma_{nc}^d \) at other energies are given.
Table 1. Calculated diffractive charged particle cross sections (in mb) and the charged multiplicity. The numbers in brackets are experimental values deduced from the histograms given by Vander Velde (1972).

<table>
<thead>
<tr>
<th>( P_{\text{lab}} ) (GeV/c)</th>
<th>19</th>
<th>69</th>
<th>102</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>d 2</td>
<td>9.08</td>
<td>5.07</td>
<td>4.76</td>
<td>2.81</td>
</tr>
<tr>
<td>d 4</td>
<td>4.66</td>
<td>5.87</td>
<td>5.35</td>
<td>4.00</td>
</tr>
<tr>
<td>d 6</td>
<td>1.20</td>
<td>3.39</td>
<td>3.00</td>
<td>2.91</td>
</tr>
<tr>
<td>d 8</td>
<td>0.20</td>
<td>1.31</td>
<td>1.12</td>
<td>1.39</td>
</tr>
<tr>
<td>d 10</td>
<td>0.026</td>
<td>0.38</td>
<td>0.32</td>
<td>0.50</td>
</tr>
<tr>
<td>d 12</td>
<td>0.0027</td>
<td>0.087</td>
<td>0.070</td>
<td>0.14</td>
</tr>
<tr>
<td>d 14</td>
<td>0.017</td>
<td>0.013</td>
<td>0.007</td>
<td></td>
</tr>
<tr>
<td>d 16</td>
<td>0.003</td>
<td>0.002</td>
<td>0.007</td>
<td></td>
</tr>
<tr>
<td>d 18</td>
<td>0.0003</td>
<td>0.0003</td>
<td>0.001</td>
<td>0.0003</td>
</tr>
</tbody>
</table>

Figure 1. Diffractive cross section \( \sigma_{\text{ne}}^d \) versus \( n_e \) = the number of charged particles in the collision. The experimental points are cross sections corresponding to the cut \( |t| < 0.5 \), deduced from the histograms of Dao et al. (1973).
Figure 2. Nondiffractive cross sections $\sigma_{nc}^{n-d}$ versus $n_c$. The experimental points correspond to the cut $|t| < 0.5$.

Figure 3. $n_c^2 \sigma_{nc}^{d}$ versus $n_c$.

in table 1. It may be noted that the distribution in (5) is not a Poisson distribution even though $P_m$ is a Poisson distribution.

According to our model of diffraction outlined above, diffractive events are those in which there are just two leading charged particles and some pionization. We assume that in these events only one of the incident protons is excited at a time and the events detected in the cut $|t| < 0.5$ GeV$^2$ correspond to the projectile excitation. Events corresponding to the target excitation would have much larger missing masses and one may assume that they go out of the $|t|$ cut. The situation is represented in figure 4. Using the notation of figure 4, the square of the missing mass is $M_x^2 = (P_x^* + \Sigma_i K_i)^2$ where the sum over $i$ is taken for both charged and neutral particles. Since the momenta $K_i$ refer to pionization, it is reasonable to assume that $\Sigma_i K_i = 0$. Moreover, as the values of $M^*$ are not expected to be much larger than the mass $M$ of the proton, one can approximate $E_x^* \approx E_x' \approx \frac{1}{S} (S - \Sigma_i \omega_i)$ where $E_x^*$, $E_x$ and $\omega_i$ are the energy components of $P_x^*$, $P_x'$ and $K_i$ in the cm system. With these approximations one gets

$$M_x^2 \approx M^*^2 + \sqrt{S} \left(\Sigma_i \omega_i\right)$$

(7)
In a two-prong event there can be no charged non-leading particles and the excited object $a^*$ (figure 4) would emit one or more neutral particles. According to experiment (Dao et al. 1973), the number of neutral particles in a two-prong event is $\sim 1$. This would imply that the emission of neutral non-leading particles should be rare in two prong events. One then finds that $M^2_x \sim M^{*2}$ and as the values of $M^*$ are limited, one gets a peaked distribution for $M^2_x$ in two-prong inelastic events. The situation would change somewhat for 4-prong events but the peaking may be expected to be still seen in four-prong events. For $n_e \geq 6$, there would be considerable fluctuation both in the number of neutral pionization particles and the total energy carried by all the pionization particles. This would give a broad spread to the values of $M^2_x$. A rough lower bound on the value of $M^2_x$ can be obtained if one assumes that no neutral particles are emitted in the central component and that the charged particles in it are all pions with energies not much larger than the pion mass $m_\pi$. This gives according to (7), the inequality

$$M^2_x \geq [\langle M^{*2} \rangle + \sqrt{S (n_e - 2)} m_\pi]$$  \hspace{1cm} (8)

Similarly a rough upper bound can be obtained if one assumes that in the pionization, the number of neutral particles is half the number of charged particles and that there are very few pionization particles with energies greater than, say, $2 \langle P_\perp \rangle$ where $\langle P_\perp \rangle$ is the average transverse momentum of such particles. This gives

$$M^2_x \leq [\langle M^{*2} \rangle + 3 \sqrt{S (n_e - 2)} \langle P_\perp \rangle]$$

Taking $\langle M^{*2} \rangle \sim 4 \text{ GeV}^2$, $\langle P_\perp \rangle \sim 0.3 \text{ GeV}/c$, one finds at 303 GeV that, in 4-prong events $11 < M^2_x < 47$, in 6-prong events $17 < M^2_x < 90 \text{ GeV}^2$ and in 8-prong events $24 < M^2_x < 133$. These rough lower and upper bounds on the values of $M^2_x$ are consistent with the observed distribution of $M^2_x$.

If the model described in this paper for diffraction is valid, it has an important experimental consequence, namely, that all the charged particles excepting the two leading ones are due to pionization. They have small momenta and an angular distribution nearly isotropic in the cm system. This unexpected feature of the model is amenable to experimental test using the ISR machine. This can be achieved simply by triggering for events in which there is a leading proton within a given $|t|$ cut, say $|t| > 0.5 \text{ GeV}^2$. One then looks at the angular and momentum distribution of the other charged particles. Contrary to what one expects from
the usual models of diffraction, one would find, on the basis of the present model, a considerable fraction of particles, particularly in high multiplicity events, to be emitted at large angles and with small momenta.

While specifying the pattern of diffraction according to the present model, the physical ideas associated with the origin of such a pattern has not been discussed. A possible mechanism for the realization of the present model is the one represented in figure 4 in which two Pomerons are exchanged leading to, besides an excited baryon, a Pomeron-Pomeron interaction resulting in a low mass cluster which decays into a number of pion pairs. A different mechanism is provided, at a classical level, by the model of random fragmentation (Narayan 1971) proposed by the author to explain the features of multiparticle production. One has to refer to the original paper for details. We would like to point out, here, some crude resemblance between the physical ideas underlying the above model with those of the parton model (Feynman 1969). In the spirit of the parton model, the proton may be regarded as composed of a heavy parton having the quantum numbers of a proton and a sea of light partons. What has been referred to as core-cloud collision in the fragmentation model would correspond to the interaction of a heavy parton with a light parton, giving rise to a baryon-like excited object. The cloud-cloud interaction leads always to pionization. This unexpected and surprising result is attributed to a new aspect of physics which is postulated to emerge at high energies. Due to the Lorentz contraction of the hadrons and the extremely short duration of the time of interaction in the CM system, the interaction gets localized to small cells which keep contracting with energy in such a way that the energy per cell remains roughly constant. This localization may be compared to point-like interactions in the parton picture. In the latter model, it is, however, difficult to see how the collision of two light partons always leads to pionization, except for the collision of so-called “wee” partons. As a matter of speculation, one may imagine that when two light-partons in a cell interact to emit a pair of pions, the energy involved in the interaction is just the energy contained in the cell which is small, while the cell itself is nearly at rest in the CM system.

Acknowledgement

The author would like to thank B S Chaudhary and V Gupta for useful discussions.

References

Dao F T and Whitmore J 1973 NAL–Pub–73/74–Exp 2000·000 and 2600·200
Fiabkowski K and Miettinen H I 1973 Phys. Lett. 43B 61
Harari H and Rabinovici E 1973 Phys. Lett. 43B 49
Hwa R C 1970 Phys. Rev. D 1 1790
Kajantie K and Ruuskanen P V 1973 Phys. Lett. 45B 149
Van Hove L 1973 Phys. Lett. 43B 65