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# Gravitational charges, $f$-gravity and hadron masses

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**Abstract.** Two types of fundamental gravitational charges are suggested by quantization of the angular momentum (*i.e.* $J = n\hbar$, where $n$ is an integer or half integer) occurring in the uncharged and charged Kerr metrics. These charges turn out to be $e/\sqrt{\alpha}$ and $e/\alpha$, where $e$ is the unit electric charge and $\alpha$ the fine structure constant. The use of strong ($f$) gravity leads to corresponding fundamental masses $M_1(f) \sim 2 \cdot 2 \times 10^{-24}$ g and $M_2(f) \sim 2 \cdot 3 \times 10^{-23}$ g. It is postulated that the hadrons are composed of these fundamental entities (christened oms here). Thus mesons are *diomic* particles and baryons are *triomic* particles. This has a close resemblance to the quark model but here we deal with gravitational charges. The charges constituting hadrons are bound together by strong ($f$) gravity which is super strong compared to nuclear forces. Various hadron masses are obtained as the vibrational excitations of these composite units. The above model is capable of accounting for quantum numbers such as spin, baryon number, strangeness and isospin.

**Keywords.** Gravitational charges, quantum black holes, $f$-gravity, hadron mass spectrum.

## 1. Introduction

The concept of the gravitational charge defined as $\sqrt{G_N}M$, where $G_N$ is the Newtonian gravitational constant and $M$ the mass, in analogy with the electric charge has been reemphasized recently by Motz (1972). Keeping in view the fundamental unit of electric charge, there has been a search for a fundamental unit of mass. The quantization of electric charge is connected with the dual property of matter arising from the existence of both electric and magnetic charges. This has been elegantly demonstrated by Dirac (1931, 1948) and Schwinger (1969) (*see* also Sinha and Sivaram 1973). Thus the quantization of the gravitational charge can be achieved by a procedure similar to the Dirac-Schwinger quantization condition. In fact Motz has shown that by quantizing one of the angular momentum components in the general relativistic equation of motion of two bodies as well as from the Machian definition of inertial mass in a rotating universe, the following quantization condition for the gravitational charge is obtained:

$$G_N M^2/c = \hbar \tag{1}$$

($\hbar$ = Planck constant, $c$ = speed of light).
This is analogous to the Dirac-Schwinger relation connecting the electric ($e$) and magnetic ($f$) charges, namely,

$$ef/c = \hbar \tag{2}$$

Earlier Motz (1962) had shown that the application of the Weyl principle of gauge invariance to the Einstein-Ricci tensor gives rise to the same condition, $G_N M^2 = \hbar c$, for a particle of inertial mass $M$ obeying the Dirac equation. The above procedure leads to a quantized unit of mass given by

$$M = (\hbar c/G_N)^{\frac{1}{2}} \tag{3}$$

This mass turns out to be of the order of $2 \cdot 2 \times 10^{-5}$ g. This mass constructed solely from $\hbar$, $c$ and $G_N$ thus unites quantum mechanics, relativity and Newtonian gravity and is referred to as the Planck mass (*see* Wheeler 1962). Motz attaches considerable significance to this entity and has named it " uniton ". He further claims that this may play a universal role in determining the masses of elementary particles, in cosmological problems such as energy generation of quasars and the solar neutrino puzzle.

The purpose of the present paper is three-fold. First, we shall obtain the same quantum condition for the gravitational charge by making use of the Kerr solution of the field equations of general relativity. Secondly, by making use of the charged Kerr metric we shall derive a new condition which gives another quantized mass which, in addition to $\hbar$, $c$ and $G_N$, involves, the fundamental electric charge $e$. We believe that this entity containing the four most important physical constants has greater claim for universality. Thirdly, we shall show that on using the $f$ gravity coupling constant $G_f \sim 10^{38}\, G_N$, these two fundamental masses are scaled down to roughly the nucleon mass. Finally, a model for hadrons will be suggested wherein they are composed of these gravitational charges held together by strong gravitational interactions. The ensuing mass spectrum is consistent with that suggested earlier (Sivaram and Sinha 1973 *a*, 1974 *a*).

## 2. Kerr metric and mass quantization condition

Kerr (1963) gave an exact particular solution of the Einstein vacuum field equations $E_{\mu\nu} = 0$, where $E_{\mu\nu}$ is the Einstein-Ricci tensor. This is believed to represent the gravitational field of a rotating body. This solution is best described by the line element due to Boyer and Lindquist (1967) given by

$$\begin{aligned} ds^2 &= g_{11} dr^2 + g_{22} d\theta^2 + g_{33} d\phi^2 \\ &\quad + g_{30} d\phi dt - g_{00} dt^2 \end{aligned} \tag{4}$$

where

$$\left.\begin{aligned} g_{11} &= (r^2 + a^2\cos^2\theta)/(r^2 - 2mr + a^2) \\ g_{22} &= (r^2 + a^2\cos^2\theta) \\ g_{33} &= \left(r^2 + a^2 + \frac{2mra^2\sin^2\theta}{r^2 + a^2\cos^2\theta}\right)\sin^2\theta \\ g_{30} &= \frac{4mra\sin^2\theta}{r^2 + a^2\cos^2\theta} \end{aligned}\right\} \tag{5}$$

and

$$g_{00} = \left(1 - \frac{2mr}{r^2 + a^2\cos^2\theta}\right) \tag{6}$$

Here $m$ is the geometrized mass $m = G_N M/c^2$, $M$ being the mass of the body, $a$ is the geometrized angular momentum given by $a = J/Mc$, $J$ being the total angular momentum of the body. The 'event horizon' or the one way membrane, which is located inside the surface $g_{00}=0$, is described by the equation (*see* for example Zeldovich and Novikov 1971)

$$r^2 + a^2 - 2mr \equiv \triangle = 0 \tag{7}$$

This has two roots given by

$$r_{\pm} = m \pm \sqrt{m^2 - a^2} \tag{8}$$

where $r_+$ defines the 'event horizon'. Since we have the ultimate unit of spinning body in mind, we assume that the angular momentum $J$ of the body can be quantized in integral or half-integral multiples of $\hbar$. Thus we write

$$J = n\hbar \tag{9}$$

where $n$ is positive integer (or half-integer). Accordingly, we get

$$a = n\hbar/Mc \tag{10}$$

We shall call a body having this quantized angular momentum a kind of "quantum black hole", with a real value of $r_+$.

Now for $r_+$ to be real (cf equation (8)) we must have $m^2 - a^2 \geqslant 0$, *i.e.* $m \geqslant a$. Using eq. (10) and $n = 1$, we get the condition

$$G_N M^2 = \hbar c \tag{11}$$

This shows that for $r_+$ to be real the quantum black hole must have a mass not less than

$$M_1 = (\hbar c/G_N)^{\frac{1}{2}} \sim 2{\cdot}2 \times 10^{-5}\,\mathrm{g} \tag{12}$$

As a result of the quantization of the angular momentum $J$, the mass $M$ will be quantized as $M = (n\hbar c/G_N)^{\frac{1}{2}}$.

The foregoing derivation of the quantization condition is much simpler and has the added advantage of directly bringing in the quantized angular momentum of the entity through the Kerr metric. As stated earlier the relation (11) involves the three fundamental constants $\hbar$, $c$ and $G_N$. This mass therefore carries the quantum, relativistic and gravitational characteristics. However, the important attribute of electric charge is still missing. As the unit of electric charge is most universal in elementary particle physics, the above approach must be generalized to include this attribute also.

We shall obtain a fundamental mass having all the four attributes from the charged Kerr metric.

## 3. Charged Kerr metric and a new universal mass

Carter (1968) has given the appropriate form of the Kerr metric describing the gravitational field of a rotating body having an electric charge. Again using Boyer-Lindquist coordinates the $g_{00}$ component for the charged Kerr metric turns out to be

$$g_{00} = \left(1 - \frac{2mr}{r^2 + Q^2 + a^2\cos^2\theta}\right) \tag{13}$$

where $Q$ is the geometrized charge and is given by $Q^2 = Ge^2/c^4$; here we have taken $e$ to be the fundamental electric charge as that of the electron. As in the previous section, $a = J/Mc$. It may be noted that for $a = 0$, the expression (13) reduces to the well-known solution given by Reissner (1916) and by Nordström (1918) of the Einstein field equation for a charged massive body.

The event horizon corresponding to equation (13) is now given by

$$r^2 - 2mr + a^2 + Q^2 \equiv \triangle_Q = 0 \tag{14}$$

Hence, the roots are

$$r_\pm = m \pm (m^2 - a^2 - Q^2)^{\frac{1}{2}} \tag{15}$$

where as before $r_+$ defines the 'event horizon'. Let us now consider the equation of the event horizon in the Reissner-Nordström solution, namely,

$$\left(1 - \frac{2G_N M}{rc^2} + \frac{G_N e^2}{c^4 r^2}\right) \equiv \triangle_{RN} = 0 \tag{16}$$

From this we get

$$r_\pm = G_N M/c^2 \pm (G_N^2 M^2/c^4 - G_N e^2/c^4)^{\frac{1}{2}} \tag{17}$$

This shows that for $r_+$ to be real we must have $M^2 \geqslant e^2/G$. It may be noted that $M^2 = e^2/G$ does not yield a real value for $r_+$ when we make use of equation (15). Choosing the next integral value for $M^2$, *i.e.* $2e^2/G$, gives us (with $m = G_N M/c^2$)

$$m^2 = 2G_N e^2/c^4 \tag{18}$$

Using this value of $m^2$ and with '$a$' given by equation (10) together with the inequality for $r_+$ to be real, *i.e.*

$$m^2 - a^2 - Q^2 \geqslant 0$$

gives us the quantization condition for the mass, *i.e.*

$$G_N M^2 = (n\hbar c/e)^2 \tag{19}$$

Thus with $n = 1$, we get a new fundamental inertial mass

$$M_2 = \hbar c/\sqrt{G_N} e \tag{20}$$

which is of the order of $2 \cdot 3 \times 10^{-4}$ g.

The above expression for mass contains the four most important fundamental constants of modern physics, namely, $G_N$ for Newtonian gravitation, $e$ for electrodynamics, $c$ for relativity and $\hbar$ for quantum mechanics. We believe that this mass has greater claim for universality than the Motz uniton. By analogy with $M_1$, we shall refer to entities such as $M_2$ as charged quantum black holes. *It may be noted that these 'bare' quantum black holes having masses $M_1$ and $M_2$ are unobservable.*

## 4. $f$-gravity and the fundamental masses

In the foregoing sections, we have used the Newtonian gravity (characterized by the coupling constant $G_N$) in all the calculations. In the field theory of gravitation, namely Einstein's linearized field equations, the interactions are mediated by massless gravitons and hence infinite range. However, the discovery of $f$-mesons having quantum numbers identical with the massless gravitons (*e.g.* $J^P = 2^+$) strongly suggests the possibility of a strong gravity field mediated by these spin $2^+$ mesons. Isham *et al.* (1971) have shown that the $f$-gravity field couples strongly with hadronic matter. Also, recently the present authors (Sivaram and Sinha 1973 *b*, 1974 *a*) have shown that we must necessarily invoke the strong gravity ($f$-gravity) mediated by massive $f$-mesons if one admits the possibility of general relativity playing some role in determining the masses of elementary particles. It was also shown that the range of this $f$-gravity is extremely short ($\sim 10^{-14}$ cm) as determined by the mass of the $f$-meson. In contradistinction to Newtonian gravity, the coupling constant for $f$-gravity, namely, $G_f$ is very large. This is easily estimated by equating the Schwarzchild radius (with $G_f$) of a nucleon (where $g_{00}$ vanishes) to the Compton length (*see* Sivaram and Sinha 1973 *b*, 1974 *a*). This gives

$$G_f = \hbar c / 2M_N^2 \qquad (21)$$

$$\sim 6 \cdot 7 \times 10^{30} \text{ dynes cm}^2 \text{ gm}^{-2} \text{ (CGS units)}$$

This is consistent with the empirical value suggested by Salam (1973), namely, $G_f \sim 6 \cdot 6 \times 10^{30}$ (CGS unit). Thus

$$G_f = 10^{38} \, G_N.$$

With this value of the $f$-gravity coupling constant, equation (12) gives

$$M_1(f) = (\hbar c / G_f)^{\frac{1}{2}} \approx 2 \cdot 2 \times 10^{-24} \text{ g} \qquad (22)$$

If we were to allow for $J = n\hbar$, the value $n = \frac{1}{2}$, the above mass would turn out to be $1 \cdot 6 \times 10^{-24}$ g which is the proton mass. Similarly, we find that

$$M_2(f) = \hbar c / \sqrt{G_f} e \approx 2 \cdot 3 \times 10^{-23} \text{ g} \qquad (23)$$

*i.e.* one order of magnitude larger than the nucleon mass. Thus, we see that the fundamental masses $M_1(f)$ and $M_2(f)$ are not preposterously large. On the other hand, they are close to the baryon masses. *As remarked earlier, these bare quantum black holes are unobservable.*

## 5. Gravitational charges and hadron masses

As remarked earlier the gravitational charge is defined as $M\sqrt{G}$. If we multiply equations (12) and (20) by $\sqrt{G_N}$ or equations (22) and (23) by $\sqrt{G_f}$ we get the quantities $(\hbar c)^{\frac{1}{2}}$ and $(\hbar c / e)$. These can be rewritten as

$$g_1 = (\hbar c / e^2)^{\frac{1}{2}} \, e = e / \sqrt{\alpha} \qquad (24)$$

and

$$g_2 = (\hbar c / e^2) \, e = e / \alpha \qquad (25)$$

where $\alpha$ is the fine-structure constant. Thus we get two kinds of gravitational charges $g_1$ and $g_2$. It is remarkable that we get the same values of the fundamental gravitational charges, whether we use $G_N$ or $G_f$. This, of course, indicates that the charge is truly fundamental quantity. The above analysis shows that the use of Newtonian gravitational constant leads to very large values for the fundamental inertial masses $M_1$ and $M_2$, whereas the use of $G_f$ gives much smaller values for these masses, *i.e.* $M_1$ $(f)$ and $M_2$ $(f)$. Nevertheless, the interaction strength between these entities remains the same. This is because for the large coupling constant $G_f$, the masses are correspondingly scaled down, the reverse being true for a smaller value of $G$, *i.e.* $G_N$.

In further calculations, we shall make use of the fundamental gravitational charges $g_1$ and $g_2$. At this stage we postulate that the hadrons are composed of these fundamental gravitational charges. Before we proceed further, it is important as to what names are giver to these fundamental entities. The names of particles ending with -on has gone on without settling the question of elementarity. Accordingly we make a slight deviation. We split the word atom and choose the second half, *i.e.* OM. Thus these fundamental gravitational charges will be referred to as oms. Perhaps it may be noted in passing that this word in Sanskrit means the ultimate reality.

The postulate is that the mesons are composed of two gravitational charges each with $e/\sqrt{\alpha}$ and the baryons are composed of three charges, namely, two with $e/\sqrt{\alpha}$ and the third having $e/\alpha$. Thus the mesons are *diomic* particles and baryons are *triomic* particles. This model has close similarity with the quark model and $SU_3$ symmetry. However, the difference comes from the fact that quarks are electrically charged particles and oms are gravitationally charged particles and are responsible for the masses. They may also have electrical charges in addition. Then they will have the property of Schwinger's dyons (Schwinger 1969).

Let us consider the case of mesons first. Before considering the excitations of this linear diomic unit, we discuss the equilibrium separation between these two gravitational charges $(e/\sqrt{\alpha})$. For this, a simple calculation yields the desired result. The energy of the system (kinetic + potential energies) is given by*

$$E = (\pi^2 \hbar^2/2\mu r^2) - e^2/\alpha r \tag{26}$$

where the first term is the kinetic energy $p^2/2\mu$, with $p = \hbar k$, $k = 2\pi/d$, where $\mu$ is the reduced mass $= (\frac{1}{2})\, M_1\, (f)$ and $d = 2r$; the second term is the attractive potential energy of interaction between the two gravitational charges. The minimization of energy with respect to $r$ gives the equilibrium separation $r_{0_1}$

$$r_{0_1} = \hbar^2 \pi^2\, \alpha/\mu e^2 \tag{27}$$

$$\approx 2\cdot 8 \times 10^{-13} \text{ cm.}$$

If we were to displace the gravitational charges from the equilibrium configuration

* The relativistic expression for the energy

$$E = \sqrt{\frac{\hbar^2 \pi^2 c^2}{r^2} + \mu^2 c^4} - \frac{e^2}{\alpha r}$$

also gives

$$r_{01} = \frac{\alpha \hbar^2 \pi^2}{\mu e^2} \left(1 - \frac{e^4}{\alpha^2 \hbar^2 \pi^2 c^2}\right)^{\frac{1}{2}} \approx \alpha \hbar^2 \pi^2/\mu e^2$$

the same as equation (27) obove.

by an infinitesimal displacement, it is easy to show that a restoring force sets in (Sivaram and Sinha 1973 *a* and *b*). The corresponding force constant is given by

$$K_f = 2e^2/\alpha r_{0_1}{}^3 \tag{28}$$

The frequency of oscillation ($\omega_1$) corresponds to the energy

$$\hbar\omega_1 = \hbar\sqrt{\frac{K_f}{\mu_1(f)}} = 35\ \text{MeV} \tag{29}$$

where $\mu_1(f)$ is the reduced mass. For calculating the excited states, we make use of the energy eigenvalues of the relativistic harmonic oscillator (Harvey 1972). This is given by

$$E_n = \hbar\omega_1\left[(n+\tfrac{1}{2}) + \tfrac{3}{4}b\,(n^2+n+\tfrac{1}{2})\right] \tag{30}$$

As indicated in earlier papers (Sivaram and Sinha 1973 *a*, 1974 *a*) $\frac{3}{4}b \sim 1$. Thus

$$E_n = \hbar\omega_1\left[(n+1)^2\right] \tag{31}$$

Using this expression we get the following spectrum of meson masses (table 1). Let us consider the case of baryons. As remarked earlier, the baryons are assumed to comprise of three gravitational charges, *i.e.* two having magnitude $e/\sqrt{\alpha}$ and the third with $e/\alpha$. We assume that the three gravitational charges have a triangular disposition. Proceeding in the same manner as done for the case of mesons with gravitational charges, it can be easily shown that the equilibrium radius $r_{0_2}$ turns out to be

$$\begin{aligned} r_{0_2} &= \hbar^2\pi^2\alpha^{3/2}/e^2 M_1(f) \\ &\approx 4\cdot 8 \times 10^{-14}\ \text{cm.} \end{aligned} \tag{32}$$

The corresponding force constant for the oscillation of the mass $M_2(f)$ relative to the other two is given by

$$K_2 = 2e^2/\alpha^{3/2} r_{0_2}{}^3 \tag{33}$$

The corresponding frequency is

$$\omega_2 = \sqrt{K_2/\mu_2(f)} \tag{34}$$

**Table 1.** Meson mass spectrum

| $n$ | $E_n$ (in MeV) | Possible Identification |
|---|---|---|
| 0 | 35 | ground state |
| 1 | 140 | (139, pions) |
| 2 | 315 | |
| 3 | 560 | (549, $\eta$ 500, K) |
| 4 | 875 | (890, K*) |
| 5 | 1260 | {(1264, *f*) (1247, $KK^0$)} |

**Table 2.** Baryon mass spectrum (with $n_p = 0$)

| $n_m$ | $E_n$ (in MeV) | Possible Identification |
|---|---|---|
| 0 | 835 | (Unexcited " parton ") |
| 1 | 940 | (938, p) |
| 2 | 1115 | (1115, $\Lambda$, $\Sigma$) |
| 3 | 1360 | (1320, $\Xi$) |
| 4 | 1675 | (1672, $\Omega^-$) |

where the reduced mass

$$\frac{1}{\mu_2(f)} = \frac{1}{M_2(f)} + \frac{1}{2M_1(f)}.$$

Thus $\mu_2(f) = 1 \cdot 7\ M_1(f)$.

This gives

$$\omega_2 = 1 \cdot 29 \times 10^{24}\ \mathrm{c/s}.$$

The corresponding energy

$$\hbar\omega_2 = 800\ \mathrm{MeV}.$$

The total vibrational energy of the system will comprise the excitation of the oscillator with frequency $\omega_2$ and another with $\omega_1$. In point of fact, the latter corresponds to the excitation of the meson clouds and the former to the excitation of $M_2(f)$ relative to the meson cloud. The total vibrational energy is given by

$$E_n = \hbar\omega_2 (n_p + 1)^2 + \hbar\omega_1 (n_m + 1)^2 \tag{35}$$

For baryons, we choose $n_p = 0$, *i.e.* the mode $\omega_2$ is not excited. Thus the various states for the baryons are given in table 2.

It is interesting to note that the above sequence is in the order of increasing strangeness with $\triangle S = 1$, for each consecutive mass. In fact, as shown earlier (Sivaram and Sinha 1973 *a*, 1974 *a*) in a different context $(n_m - 1) = |S|$. The mass splitting within individual multiplets ( $\Sigma$ $\Xi$ etc.) which involves the dependence on the other degrees of freedom such as isospin has not been considered so far. Such internal degrees of freedom can be described within a general relativistic framework in which the two de Sitter spaces, namely $SO_{4,1}$ and $SO_{3,2}$ appear. In this, one of them can be treated as an external event space and the other as internal structure space. These internal degrees of freedom can be identified as internal rotational and vibrational modes. It is worth mentioning that the " isorotational " excitation of the units considered above gives the right order of energy. For example, $E_{\mathrm{rot}} = (\hbar^2/2I)\ K(K+1)$, with $I = \mu_1(f)\ r_{0_1}^2$, gives the various splittings of the energy levels of the order of 2 to 4 MeV. This is of the same order as the observed splitting within the multiplets. A detailed account will be given in a later communication including a group theoretical analysis of the model.

## 6. Concluding remarks

In the foregoing sections, we have shown that for obtaining the mass spectrum of hadrons as the vibrational excited states of some fundamental composite units one has to invoke the role of strong ($f$) gravity. This indicates a possible role for Einstein's general relativity (but with the coupling constant $G_f$) in determining the structure and the masses of elementary particles (*see* also Sivaram and Sinha 1974 *a*).

In summary, the following important points considered in the present paper may be noted. The fundamental gravitational charges were obtained by quantizing the angular momentum occurring in uncharged and charged Kerr metrics (Sivaram and Sinha 1974 *b*). This procedure, in effect, also amounts to the quantization of the fundamental electric charge by postulating gravitational charges ($M\sqrt{G}$). This has close resemblance with Dirac's work of quantizing electric charge by postulating a fundamental magnetic charge (in this context Barut's (1971) work on dyon may have some relevance).

It should be noted that one could choose half-integral values in the quantization condition for the masses. This will not affect the masses calculated to any appreciable extent. We believe that each of the gravitational charges mentioned above may have half-integral spin $(\frac{1}{2})\hbar$. Mesons, being made up of two such charges with opposite spins, will have zero net spin. On the other hand, the baryons which consist of three gravitational charges will have the residual half-integral spin of the third object. Other combinations are also possible and one cannot rule out the possibility of $(3/2)\hbar$ spin for baryons and $\hbar$ for mesons in some excited states. The above model is thus capable of accounting for all the four quantum numbers, *i.e.* the baryon number $n_B = (n_p + 1)$, strangeness $|S| = (n_m - 1)$, the spin as noted above and the isospin being related to the " rotation " (spin) in isospace of the composite system.

In sections 2 and 3, we have discussed the charged and uncharged Kerr metrics for Einstein's conventional field equations involving weak Newtonian constant $G_N$. In section 4, we have invoked the role of $f$-gravity with the coupling constant $G_f$. This amounts to modifying the Einstein equations to $E_{\mu\nu} = k_f T_{\mu\nu}$, where $E_{\mu\nu}$ and $T_{\mu\nu}$ are the Einstein's tensor and energy-momentum tensors respectively and $k_f = 8\pi G_f/c^4$. A detailed discussion of the modified field equations with coupling constants of different interactions and modifications to incorporate the short-range forces involving $f$-gravity *via* the redefined " cosmological " constant has been given elsewhere (*see* Sivaram and Sinha 1973 *b*, 1974 *a* and *c*; Lord, Sinha and Sivaram 1974, Sivaram, Sinha and Lord 1974).

The quantization conditions imposed in sections 2 and 3 are reminiscent of the conditions used by Bohr for hydrogen atom and Dirac for magnetic charges. It would thus appear that the quantization conditions have to be developed further for a more complete theory. This will be discussed in subsequent papers.

Finally, it is worth mentioning that the ground states of both mesons and baryons are unobservable. We believe they are the singlet states in each case, having zero values of charge, isospin and hypercharge. This point will be further elaborated in a group theoretical analysis of this model to be published later.

*Note added in proof:—*

From the form of interaction energy between the gravitational charges $g_1$ and $g_2$, it is easy to estimate the strong interaction coupling constant which turns out to be $e/\alpha^{3/4} \sim 30e$.

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