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High frequency Brillouin scattering in metals and gaseous plasmas

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Abstract. Spontaneous and stimulated Brillouin scattering are studied in metals and gaseous plasmas, for an incident laser frequency ω_1 greater than the corresponding plasma frequency ω_{pl} in the medium. The calculation of threshold powers for the stimulated scattering in aluminium metal and non-degenerate Al-plasmas shows that their values become quite small as ω_1 approaches ω_{pl} For the case of backward wave scattering we also estimate the critical power above which a temporal instability sets in such media. It is argued that this instability may be one of the factors for anomalously large absorption of high power laser beams in laser-induced plasmas.

Keywords. Brillouin scattering; laser-induced plasma; aluminium plasma.

1. Introduction

Many new experimental results (Fleury 1970) have been obtained using high resolution laser Brillouin spectroscopy. The Brillouin scattering provides an excellent method for determining the elastic constants and low frequency phonon dispersion relations in solids. It is also useful in the study of many types of phase transitions. In these experiments, if the magnitude of the electric field amplitude of the incident laser beam exceeds a certain threshold value $|\mathscr{E}_{1\text{th}}|$, one observes the well-known stimulated Brillouin scattering (SBS), instead of only the spontaneous process (Chiao *et al.* 1964). When the incident field is increased further, beyond another critical value $|\mathscr{E}_{1c}|$, one is faced, in certain cases, with a temporal instability in the medium. There is a runaway creation of the scattered light wave inside the medium. Since SBS can be thought of as a parametric down conversion process involving the incident light wave, the scattered Stokes wave and a sound wave, this instability corresponds to an anomalously large absorption of the incident flux. In this paper we investigate the mechanism of the Brillouin scattering process in metals and gaseous plasmas to understand the nature of stimulation thresholds and temporal instabilities in such systems.

For an incident laser frequency greater than the plasma frequency of a solid state or gaseous plasma, the problem of Brillouin scattering has been discussed earlier by Ron (1963). With the fabrication of very high frequency lasers, and the need for understanding the heating process of a plasma by a high power optical laser, this problem is of renewed interest now. In section 2 of this paper, we derive an expression for the spontaneous Brillouin scattering cross section in a transparent plasma. We use the Nordheim-Bardeen model for the electron-phonon interaction to determine these cross sections in an aluminium plasma. Both the degenerate and non-degenerate plasmas are considered. In section 3, we consider the problem of SBS to obtain expressions for the threshold field $\mathscr{E}_{1th}$ and the gain coefficient G (per unit length) for the backward wave scattering in which the Stokes wave and the sound wave propagate in opposite directions. The value of the critical field at which the instability sets in the medium is also estimated in this section. We find that both the threshold and the critical field decrease as the incident laser frequency approaches the plasma frequency when the attenuation of the light wave is small compared to that of the sound wave. This has been investigated in detail, as a function of temperature T and electron density n in the case of the aluminium plasma. It is shown that the anomalous absorption due to temporal instability in a plasma may be playing a dominant role in recent experiments on heating of solids by high powered laser beams. The results are discussed in section 4.

2. Spontaneous scattering cross section

To discuss the motion of conduction electrons in a metal we use the well-known parabolic one band effective-mass approximation. Thus, in this model the electronic motion is identical to the case of a gaseous plasma, except for the new effective-mass m^*. From our considerations, we neglect other fully occupied bands, which contribute to the usual photoelastic coupling in insulators. For a gaseous plasma these do not come into picture in any case. Since in the continuum model for the ions, the electron-phonon interaction of the longitudinal acoustic phonon only is dominant, we will ignore the coupling of transverse acoustic phonons.

Because of the Coulomb interaction of the electrons, the bare acoustic phonon frequency and the bare electron-phonon interaction are renormalized due to polarization effects. The effective Hamiltonian for the inelastic scattering of the photon from "real" acoustic longitudinal phonons is

$$H_{eff} = H_0 + H_{eL} + H_{eR} \tag{1}$$

where H_0 describes the unperturbed system of electrons, photons and phonons H_{eL} describes the electron-phonon interaction and H_{eR} represents the electron-photon interaction. These are given in our model by

$$H_0 = \sum_{k\sigma} E_k C^\dagger_{k\sigma} C_{k\sigma} + \sum_{q\lambda} \hbar\omega_q a^\dagger_{q\lambda} a_{q\lambda} + \sum_{Q} \hbar\omega_Q b^\dagger_Q b_Q \tag{2}$$

$$H_{eL} = \sum_{k\sigma} \sum_{Q} \frac{\langle k \mid g_{-Q}\cdot\hat{\xi}_{-Q} \mid k+Q \rangle}{\epsilon_l(Q,\, \omega_Q)} C^\dagger_{k\sigma} C_{k+Q\sigma} (b^\dagger_Q + b_{-Q}) \tag{3}$$

$$H_{eR} = \left(\frac{e^2}{2m^* c^2}\right) \sum_{k\sigma} \sum_{q\lambda} \sum_{q'\lambda'} \left(\frac{2\pi\hbar c^2}{V\omega_q \epsilon(\omega_q)}\right)^{\frac{1}{2}} \left(\frac{2\pi\hbar c^2}{V\omega_{q'}\epsilon(\omega_{q'})}\right)^{\frac{1}{2}} \times$$

$$\times [\hat{e}_{q\lambda}\cdot\hat{e}^*_{q'\lambda'} a_{q\lambda} a^\dagger_{q'\lambda'} C^\dagger_{k+q-q'\sigma} C_{k\sigma} + H.C] \tag{4}$$

where, $C^\dagger_{k\sigma}$ and $C_{k\sigma}$ are creation and destruction operators for the electrons of wavevector k, spin index σ and energy E_k; $a^\dagger_{q\lambda}$ and $a_{q\lambda}$ are creation and destruction operators for photons of wavevector q, polarization λ and frequency ω_q ; $b^\dagger_Q$ and b_Q are creation and destruction operators for the longitudinal acoustic phonons of wavevector Q and frequency ω_Q. The polarization vectors of the photons and phonons are denoted by $\hat{e}_{q\lambda}$ and $\hat{\xi}_Q$, respectively. The dielectric function $\epsilon_l(Q, \omega_Q)$ is the longitudinal electronic response function (Lindhard 1954); and $\epsilon(\omega_q)$ is the optical transverse dielectric function of the medium. The bare electron-phonon matrix element can be expressed in terms of the bare electron-ion potential $v(\boldsymbol{r}-\boldsymbol{R})$ (Jha and Woo 1971) as

$$\langle k \,|\, g_{-Q}\cdot\hat{\xi}_{-Q} \,|\, k+Q\rangle = -\left(\frac{\hbar}{2MN\omega_Q}\right)^{\frac{1}{2}} \langle k \,|\, \sum_{R} \nabla v(\boldsymbol{r}-\boldsymbol{R})\cdot\hat{\xi}_{-Q}\exp(-\mathrm{i}\boldsymbol{Q}\cdot\boldsymbol{R}) \,|\, k+Q\rangle \tag{5}$$

The spontaneous differential scattering cross section of a photon in mode 1 $(1 \equiv \omega_1,\ \boldsymbol{q}_1,\ \hat{e}_1)$ to another photon in mode 2 $(2 \equiv \omega_2, \boldsymbol{q}_2, \hat{e}_2)$ with the creation of a longitudinal acoustic phonon in mode λ $(\lambda \equiv \omega, Q, \hat{\xi})$ (Jha and Woo 1971), in our model, therefore, becomes

$$\left(\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Theta \mathrm{d}\Omega}\right)_{spont.} = \left(\frac{e^2}{m^*c^2}\right)^2 \frac{\omega_2}{\omega_1}(N_\lambda+1)\left(\frac{\epsilon(\omega_2)}{\epsilon(\omega_1)}\right)^{\frac{1}{2}} |A_{12\lambda}|^2\,\delta(\Omega-\omega) \tag{6}$$

with the dimensionless amplitude

$$A_{12\lambda}(Q,\omega) = (\hat{e}_1\cdot\hat{e}_2^*) \sum_{k\sigma} \frac{\langle k \,|\, g_{-Q}\cdot\hat{\xi}_{-Q} \,|\, k+Q\rangle}{\epsilon_l(Q,\omega)} \frac{f(E_{k+Q})-f(E_k)}{E_{k+Q}-E_k-\hbar\omega-i\delta} \tag{7}$$

In the above equations $\Omega=\omega_1-\omega_2$ is the change in the laser frequency, $\hbar\boldsymbol{Q} = \hbar\boldsymbol{q}_1 - \hbar\boldsymbol{q}_2$ is the change in the momentum, $f(E_k)$ is the Fermi-Dirac electronic distribution function and N_λ is the initial number of thermal phonons in the mode λ.

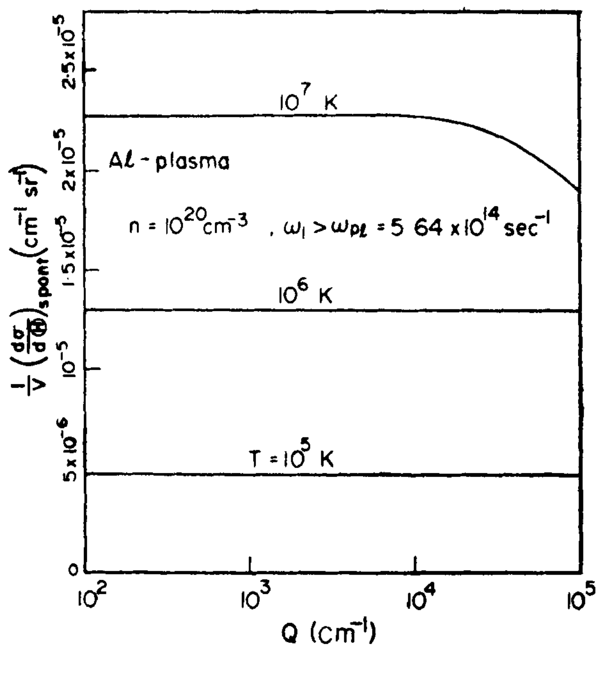

Figure 1. Integrated spontaneous scattering cross section as a function of Q, for an aluminium plasma of electron density $n=10^{20}$ cm^{-3}, at different temperatures

For the electron-ion potential, let us assume that we may describe it by the Nordheim-Bardeen model (Haug 1972) in both the solid state as well as gaseous plasmas:

$$v(r-R)=\begin{cases}\dfrac{Ze^2}{2r_i}\left(\dfrac{|r-R|^2}{r_i^2}-3\right) & |r-R|\le r_i\\ -\dfrac{Ze^2}{|r-R|} & |r-R|>r_i\end{cases}\tag{8}$$

where Ze is the effective charge of the ion and r_i its effective radius. With $\Omega \ll \omega_1$, ω_2, equations (5)–(8) lead to

$$\frac{1}{V}\left(\frac{d^2\sigma}{d\Theta d\Omega}\right)_{spont}=\left(\frac{e^2}{m^* c^2}\right)^2(N_\lambda+1)\frac{\hbar Q^2 n^2 p^2(Qr_i)}{2\rho\omega}|\hat{e}_1\cdot\hat{e}_2^*|^2\left|\frac{\epsilon_l(Q,\omega)-1}{\epsilon_l(Q,\omega)}\right|^2\times \times\delta(\Omega-\omega)\tag{9}$$

$$p(x)\equiv 3(\sin x-x\cos x)/x^3\tag{10}$$

where $\rho = MN/V$, is the ionic mass density. The effect of a finite lifetime for the phonon may be taken into account phenomenologically by replacing the δ-function in equation (9) by a Lorentzian of half width Γ. This procedure is valid as long as $\Gamma \ll \omega$.

At temperture T much less than the Fermi temperature T_F of the electrons, the integrated cross section (per unit volume per unit solid angle) in the case of Al metal is

$$\frac{1}{V}\left(\frac{d\sigma}{d\Theta}\right)_{spont.}=4\cdot 17\times 10^{-53}\left(\frac{m}{m^*}\right)^2\frac{Qn^2}{\rho v_s}(N_\lambda+1)\tag{11}$$

if $\epsilon_l(Q,\ \omega)\gg 1$ and $|\hat{e}_1.\hat{e}_2^*|=1$, and where v_s is the velocity of sound. Since phonon frequencies are small, the estimate (11) is expected to be valid in the degenerate case for Q smaller than the Thomos-Fermi wavevector $Q_{FT}=(6\pi ne^2/E_F)^{\frac{1}{2}}$. With $n=1\cdot 8\times 10^{23}\,\mathrm{cm}^{-3}$, $v_s=6\cdot 4\times 10^{+5}\,\mathrm{cm\,sec}^{-1}$ and $\rho=2\cdot 7\,\mathrm{gm/cm}^3$ for Al metal, this is of the order of $4\times 10^{-5}\,\mathrm{cm}^{-1}\mathrm{Sr}^{-1}$ at $T=300\,K$. For non-degenerate plasmas, equations (9) and (10) lead to integrated cross sections which depend on the temperature T and the plasma density n. We plot in figure 1 these cross sections at different temperatures as a function of Q, for an aluminium plasma of electron density $10^{20}\,\mathrm{cm}^{-3}$. For this purpose the finite temperature Lindhard function $\epsilon_l(Q,\ \omega\simeq 0)$ has been used. The variation of the cross section as a function of Q mainly comes from $\epsilon_l(Q,\ \omega\simeq 0)$ in equation (9). For $Q\ll 1/\lambda_D$ ($\lambda_D^2=k_BT/4\pi ne^2$ is the square of the Debye screening length), the cross section is independent of the momentum transfer $\hbar Q$. Thus, in this region, the integrated cross section is again given by equation (11). However, in this case the temperature dependent average atomic number Z, given by Shearer and Barnes (1971), is used to calculate the ionic mass density $\rho=nM/Z$ in the highly ionized hot plasma.

3. Stimulated Brillouin scattering

Next, let us consider the problem of stimulated Brillouin scattering in a metal or a plasma. In the slow varying envelope approximation (SVEA), the electric fields of the incident and the Stokes light waves, and the displacement of the longitudinal sound wave may be assumed to be

$$E_j(\mathbf{r}, t) = \hat{e}_j \mathscr{E}_j(\mathbf{r}, t) \exp \mathrm{i}(\mathbf{q}_j \cdot \mathbf{r} - \omega_j t) + \mathrm{c \cdot c}; \quad j = 1, 2 \tag{12}$$

$$U(\mathbf{r}, t) = \hat{\xi}\, U_s(\mathbf{r}, t) \exp \mathrm{i}(\mathbf{Q}.\mathbf{r} - \omega t) + \mathrm{c \cdot c}. \tag{13}$$

where $\mathscr{E}_{1,2}(r, t)$ and $U_s(r, t)$ are slowly varying functions of r and t. Here the coupling between the incident and stokes light waves, and the sound wave is through the effective interaction Hamiltonian

$$\mathscr{H}_{int}(\mathbf{r}, t) = \gamma_{12\lambda} \mathscr{E}_1(\mathbf{r}, t)\, \mathscr{E}_2^*(\mathbf{r}, t)\, U_s^*(\mathbf{r}, t) \tag{14}$$

where $\gamma_{12\lambda}$ is expressed in terms of the amplitude $A_{12\lambda}$ of equation (7) as

$$\gamma_{12\lambda} = \left(\frac{\epsilon^2}{m^* \omega_1 \omega_2}\right) \left(\frac{2MN\omega}{\hbar}\right)^{\frac{1}{2}} \frac{1}{V} A_{12\lambda} \tag{15}$$

We have, of course, neglected small dispersion effects in $\gamma_{12\lambda}$. It has to be noted that in the quantized theory this interaction exactly reproduces our old result for the spontaneous cross-section in section 2. In SVEA, the coupled Maxwell's equations and the wave equation for sound wave lead to (neglecting small linear dispersion effects)

$$\left\{\hat{\mathbf{q}}_1 \cdot \nabla + \alpha_1 + \frac{\sqrt{\epsilon(\omega_1)}}{c} \frac{\partial}{\partial t}\right\} \mathscr{E}_1(\mathbf{r}, t) = -\frac{2\pi \mathrm{i} \omega_1^2}{q_1 c^2} \gamma^*_{12\lambda}\, \mathscr{E}_2 U_s \tag{16}$$

$$\left\{\hat{\mathbf{q}}_2 \cdot \nabla + \alpha_2 + \frac{\sqrt{\epsilon(\omega_2)}}{c} \frac{\partial}{\partial t}\right\} \mathscr{E}_2(\mathbf{r}, t) = -\frac{2\pi \mathrm{i} \omega_2^2}{q_2 c^2} \gamma_{12\lambda}\, \mathscr{E}_1 U_s^* \tag{17}$$

$$\left\{\hat{\mathbf{Q}} \cdot \nabla + \alpha_s + \frac{1}{v_s} \frac{\partial}{\partial t}\right\} U_s^*(\mathbf{r}, t) = \frac{\mathrm{i}\gamma^*_{12\lambda}}{2\rho Q v_s^2} \mathscr{E}_1^* \mathscr{E}_2 \tag{18}$$

The attenuation constants α_s, α_1 and α_2 for the sound wave and the two light waves are introduced to take care of the linear losses in the medium. In the parametric approximation, we can ignore the space and time variation of the incident pump field $\mathscr{E}_1$, and study the solutions of equations (17) and (18) only. Such equations have been investigated earlier by Kroll (1965), and more recently by Starunov and Fabelinskii (1969).

For the backward Brillouin scattering, the unit propagation vectors $\hat{\mathbf{q}}_2$ and $\hat{\mathbf{Q}}$ are related by $\hat{\mathbf{q}}_2 = -\hat{\mathbf{Q}}$. With $\hat{\mathbf{q}}_2 \cdot \mathbf{r} = \zeta$, for the backward scattering equations (17) and (18) reduce to

$$\left\{\frac{\mathrm{d}}{\mathrm{d}\zeta} + \alpha_2 + \frac{\sqrt{\epsilon(\omega_2)}}{c} \frac{\partial}{\partial t}\right\} \mathscr{E}_2(\zeta, t) = -\frac{2\pi \mathrm{i} \gamma_{12\lambda} \omega_2^2}{q_2 c^2} \mathscr{E}_1 U_s^* \tag{19}$$

$$\left\{-\frac{\mathrm{d}}{\mathrm{d}\zeta} + \alpha_s + \frac{1}{v_s} \frac{\partial}{\partial t}\right\} U_s^*(\zeta, t) = \frac{\mathrm{i}\gamma_{12\lambda}}{2Q\rho v_s^2} \mathscr{E}_1^* \mathscr{E}_2 \tag{20}$$

The solution of equations (19) and (20) is sought in the form $\mathscr{E}_2, U_s^* \sim \exp(\mathrm{i}\beta t + G\zeta/2)$. The specification of the boundary conditions (Starunov and Fabelinskii 1969) in the form

$$\mathscr{E}_2(\zeta = 0, t) = \mathscr{E}_2(0)\, \mathrm{e}^{\mathrm{i}\beta t}; \; U_s^*(\zeta = L, t) = 0 \tag{21}$$

leads to the solutions of (19) and (20) as

$$\mathscr{E}_2(\zeta, t) = \frac{\mathscr{E}_2(0)\, e^{i\beta t + G_-\zeta/2} \left\{ 1 - \frac{2a_2 + G_-}{2a_2 + G_+} e^{(G_- - G_+)(L-\zeta)/2} \right\}}{\left\{ 1 - \frac{2a_2 + G_-}{2a_2 + G_+} e^{(G_- - G_+)L/2} \right\}} \tag{22}$$

$$U_s^*(\zeta, t) = \frac{i\gamma_{12\lambda}\, \mathscr{E}_1^* \mathscr{E}_2(0)\, e^{i\beta t + G_-\zeta/2} \left\{ 1 - e^{(G_- - G_+)(L-\zeta)/2} \right\}}{Q\rho v_s^2 (2a_2 + G_+) \left\{ 1 - \frac{2a_2 + G_-}{2a_2 + G_+} e^{(G_- - G_+)L/2} \right\}} \tag{23}$$

where L is the length of the sample along the ζ-direction and

$$G_\pm(\beta) = \left(a_s - a_2 + \frac{i\beta}{v_s} \right) \pm \sqrt{\left(a_s + a_2 + \frac{i\beta}{v_s} \right)^2 - \frac{4\pi\omega_2^2 |\gamma_{12\lambda}|^2 |\mathscr{E}_1|^2}{\rho Q q_2 v_s^2 c^2}} \tag{24}$$

To obtain the above expression, we have utilized $v_s \ll c$. For the incident field amplitude $|\mathscr{E}_1|$ less than a critical field $|\mathscr{E}_{1c}|$ defined by

$$|\mathscr{E}_{1c}| = \left[\frac{\rho Q q_2 v_s^2 c^2 (a_s + a_2)^2}{4\pi\omega_2^2 |\gamma_{12\lambda}|^2} \right]^{\frac{1}{2}} \tag{25}$$

one has a stable gain in the entire region of interaction. Further, in this case, for $L \gg [\mathrm{Re}(G_+ - G_-)]^{-1}$, equations (22) and (23) reduce to

$$\mathscr{E}_2(\zeta, t) = \mathscr{E}_2(0)\, e^{i\beta t + G_-\zeta/2} \tag{26}$$

$$U_s^*(\zeta, t) = \frac{i\gamma_{12\lambda}\, \mathscr{E}_1^* {}_2\mathscr{E}(0)\, e^{i\beta t + G_-\zeta/2}}{\rho Q v_s^2 (2a_2 + G_+)} \tag{27}$$

The SBS gain, which is determined by $\mathrm{Re}G_-(\beta)$, is maximum for $\beta = 0$, It is given by

$$\mathrm{Re}G_-(\beta)|_{\beta=0} = G_-(\beta = 0) = (a_s - a_2) - \sqrt{(a_s + a_2)^2 - \frac{4\pi\omega_2^2 |\gamma_{12\lambda}|^2 |\mathscr{E}_1|^2}{\rho v_s^2 c^2 Q q_2}}. \tag{28}$$

In the above expression $G_-(\beta = 0) = 0$ determines the threshold value for the incident field, for a positive gain. Thus, the threshold field is determined from the relation

$$|\mathscr{E}_{1th}| = \left[\frac{\rho Q q_2 v_s^2 c^2\, a_2 a_s}{\pi\omega_2^2 |\gamma_{12\lambda}|^2} \right]^{\frac{1}{2}} \equiv \frac{2(a_2 a_s)^{\frac{1}{2}}}{(a_s + a_2)} |\mathscr{E}_{1c}| \tag{29}$$

Note when $|\mathscr{E}_1|$ is greater than $|\mathscr{E}_{1c}|$, the denominators of equations (22) and (23) start oscillating as a function of L. For every L greater than or equal to a critical length L_c, there exists a real positive quantity $\beta_c'(L) = i\beta_c(L)$ for which these denominatons vanish. This implies that the amplitudes of the Stokes and the sound waves take extremely large values inside the medium for such values of L. In the special case of great practical importance in which $L \to \infty$ and $i\beta = i\beta_c(\infty)$, equations (22) and (23) reduce to simpler forms:

$$\mathscr{E}_2(\zeta, t) = \mathscr{E}_2(0) \exp\left[v_s t \left\{ \sqrt{\frac{4\pi\omega_2^2 |\gamma_{12\lambda}|^2 |\mathscr{E}_1|^2}{\rho Q q_2 v_s^2 c^2}} - (a_s + a_2) \right\} + \right.$$
$$\left. + \frac{\zeta}{2} \left\{ \sqrt{\frac{4\pi\omega_2^2 |\gamma_{12\lambda}|^2 |\mathscr{E}_1|^2}{\rho Q q_2 v_s^2 c^2}} - 2a_2 \right\} \right] \tag{30}$$

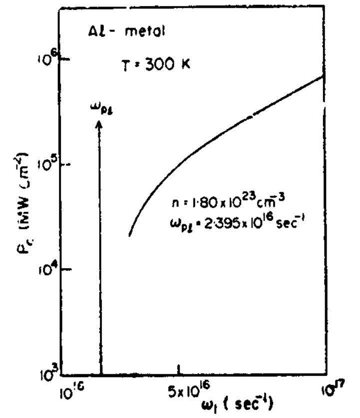

Figure 2. Critical power P_c as a function of incident laser frequency ω_1 for aluminium metal at $T = 300$ K.

$$U_s^* (\zeta, t) = \frac{i\gamma_{12\lambda}\mathscr{E}_1^*}{\rho Q v_s^2} \sqrt{\frac{\rho Q q_2 v_s^2 c^2}{4\pi\omega_2^2 \,|\, \gamma_{12\lambda} \,|^2\, |\, \mathscr{E}_1 \,|^2}}\, \mathscr{E}_2 (\zeta, t) \tag{31}$$

This is the well-known temporal instability in the medium (Kroll 1965). One has, therefore, anomalously larger conversion of the incident wave into the scattered wave. Exact calculation of this conversion necessarily requires the consideration of this problem beyond the parametric approximation used here.

To calculate the threshold and critical powers necessary for the incident wave, the knowledge of the linear attenuation coefficients α_2 and α_s for the Stokes wave and the sound wave, respectively, is essential. In the case of a metal or a degenerate plasma, for phonon wavelengths much less than the mean free path of the electrons, one has (Kittel 1963)

$$\alpha_2 = \frac{\omega_c}{2c\left(1 - \dfrac{\omega_{pl}^2}{\omega_2^2}\right)^{\frac{1}{2}}} \left(\frac{\omega_{pl}}{\omega_2}\right)^2; \qquad \omega_c \ll \omega_2 \tag{32}$$

$$\alpha_s = \frac{E_F^2\,(m^*)^2\,\omega}{9\pi\rho v_s^2} \tag{33}$$

where E_F, m^*, ω_{pl} and ω_c are the Fermi energy, the effective-mass, the plasma frequency and the collision frequency for the electrons, respectively. At room temperature, for the aluminium metal, with $\omega_c \simeq 10^{14}\,\mathrm{sec}^{-1}$, $\omega_2 \simeq \omega_1 = 6 \times 10^{16}$ sec^{-1} and $4\alpha_2/\alpha_s \simeq 5 \times 10^{-2}$, one finds the threshold power P_{th} is of the order of 5×10^3 MW cm^{-2}, and the critical power is of the order of 10^5 MW cm^{-2}. As a function of the incident frequency the critical power P_c is plotted in this case in figure 2.

For a nondegenerate plasma, we have (Palmer 1972, Shearer and Barnes 1971)

$$\alpha_2 = \frac{\langle \tau \rangle^{-1}}{2c\left(1 - \dfrac{\omega^2_{pl}}{\omega_2^2}\right)^{\frac{1}{2}}} \left(\frac{\omega_{pl}}{\omega_2}\right)^2; \qquad \omega_2 \langle \tau \rangle \gg 1 \tag{34}$$

$$\alpha_s = \frac{\Gamma_B}{2v_s} = \frac{Q^2\eta}{2\,v_s} \tag{35}$$

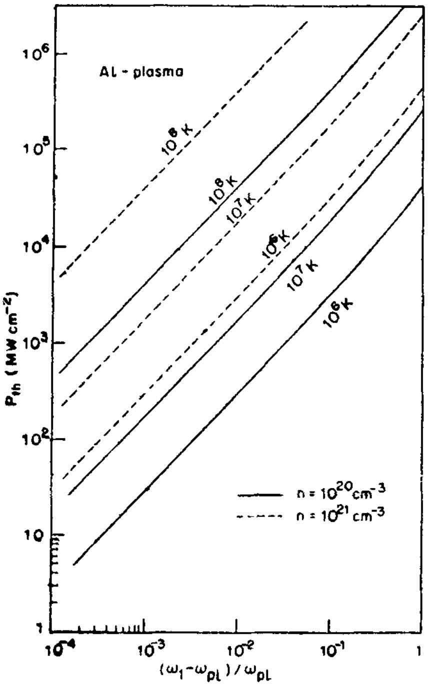

Figure 3. Threshold power P_{th} as a function of $(\omega_1 - \omega_{pl})/\omega_{pl}$ at different temperatures, for Al-plasmas of electron densities $n = 10^{20}\,cm^{-3}$ and $n = 10^{21}\,cm^{-3}$

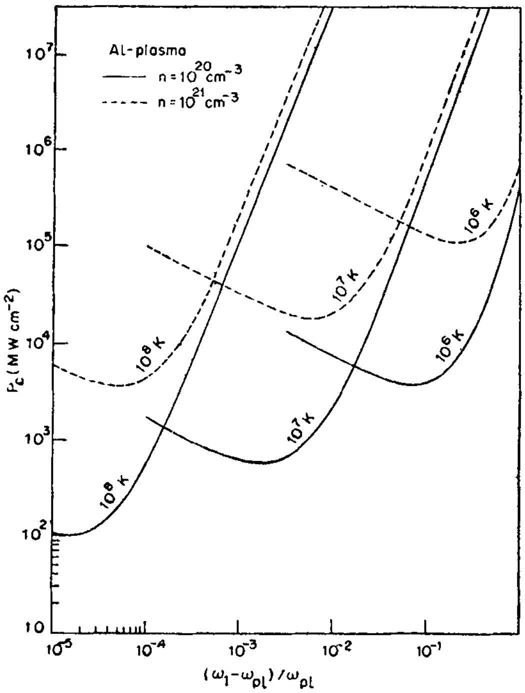

Figure 4. Critical power P_c as a function of $(\omega_1 - \omega_{pl})/\omega_{pl}$ at different temperatures for Al-plasmas of electron densities $n = 10^{20}\,cm^{-3}$ and $n = 10^{21}\,cm^{-3}$.

where Γ_B, η and $\langle \tau \rangle$ are, respectively, the spontaneous width of the phonon line, the ionic viscosity and the relaxation time for electron-ion system. The viscosity η and relaxation time $\langle \tau \rangle$ as functions of temperature T are determined by using the expressions given by Shkarofsky *et al.* (1966). We plot P_{th} and P_c as functions of $(\omega_1 - \omega_{pl})/\omega_{pl}$ in figures 3 and 4, at different temperatures for $n = 10^{20}cm^{-3}$ and $n = 10^{21}\,cm^{-3}$. The critical power passes through a minimum value at a frequency very close to the plasma frequency ω_{pl}. It should be emphasized here that actual numerical values of P_{th} and P_c critically depend on assumptions made regarding the nature of relaxation time $\langle \tau \rangle$.

4. Conclusions

From the preceding calculations of the spontaneous and stimulated Brillouin cross sections in metals and gaseous plasmas, it is clear that this inelastic scattering process should play an important role in the absorption of high power beams in a medium. This becomes specially important if we consider the backward wave scattering in a medium having the plasma frequency close to the incident lasser frequency. Although, the critical power for the temporal instability, and hence for the anomalous absorption, increases with rising temperaure, it can still become quite small for the incident frequency $\omega_1 \simeq \omega_{pl}$. It passes through a minimum at an incident frequency very close to the plasma frequency. In this analysis, the effect of a finite pulse-width of the incident laser beam has, however, not been taken into account. This is not expected to change our conclusions. provided the width is not shorter than about 10^{-10}–10^{-11} sec.

It has to be emphasized that our calculations for metals are based on the one band effective-mass approximation for the electronic motion. Since the high-frequency Brillouin scattering in metals is of considerable experimental interest, one may argue that the effect of other fully occupied bands should also be calculated. This gives rise to the usual type of photoelastic coupling, considered earlier in the low frequency region by Bennett *et al.* (1972). We however, believe that our approach, even in the case of metals, is sufficiently accurate for calculating the dominant contributions to the cross sections at high incident frequencies.

As far as the actual application of our calculation to the plasma heating is concerned, one has to be careful in interpreting our numerical results. To explain the anomalous heating one must simultaneously consider the laser mode structure in the finite plasma as well as other types of non-linearities in the system. This is now under investigation. Also, initial plasma parameters must be known very accurately, since final numerical results are very sensitive to these. Nevertheless, it may be appropriate to present a typical number of the depth of the penetration of the incident radiation into the plasma, if only SBS was taken into account. For the plasma length $L = 1 \cdot 5 \times 10^{-2}$ cm, (density $n = 10^{21}\text{cm}^{-3}$), incident electric field $\mathscr{E}_1 = 10^5$ esu focussed in 10^{-5} radians, the penetration depth is of the order of 10^{-3} cm. If this is compared with the absorption length $\alpha_1^{-1} \sim 10^{-3}$ cm at $T = 10^6$ K, one can say that the backward SBS should not be ignored in any detailed theory for laser plasma heating.

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