Pramāṇa, Vol. 2, No. 2, 1974, pp 80–83. Printed in India.

Comments on the high energy behaviour of total cross-sections based on light-cone algebra

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MS received 26 October 1973

Abstract. Based on considerations of the light-cone algebra of currents and pseudoscalar densities, it is suggested that either the high energy meson-nucleon cross-sections are constants (even though the pp-cross-section increases indefinitely) or that they deviate from constancy by logarithmic terms whose scale is set by a mass, of the order of at least tens of Gev, and which also characterises the consequent logarithmic violations of Bjorken scaling.

Keywords. Total cross-sections; light-cone algebra; Bjorken scaling.

Recent observations (Amaldi *et al* 1973, Amendolia *et al* 1973) of the proton-proton total cross-section rising with energy immediately raise the question of the corresponding behaviour of the meson-nucleon cross-sections. We would like to make some comments on this question based on the extended light-cone algebra of currents and pseudoscalar densities. In recent papers (Das *et al* 1973) we have obtained certain inequalities relating the scaled structure functions of deep inelastic lepton-nucleon scattering (in the limit of vanishing scale variable ξ) and the asymptotic meson nucleon total cross-sections. The major underlying assumptions made in that work are:

(1) (*a*) The closed light-cone algebra of currents and pseudoscalar densities, extending to the light-cone the equal-time commutators which form part of the $U(12)$ algebra. (*b*) The specific nature of the leading light-cone singularities as determined from formal quark-gluon field theory.

(2) The identification of the pseudoscalar meson fields ϕ_i with the pseudoscalar quark bilinear densities v^i: $\phi_i = Z_i v^i$.

(3) (*a*) Diffractive behaviour of the forward meson-nucleon amplitude both on and off the meson mass-shell. (*b*) The interchangeability of the Regge and light-cone limits for the leading terms of the said amplitude.

Additionally, we had to make certain plausible technical assumptions on the non-light-cone contributions and on the non-leading parts in the light-cone terms. These led to a number of inequalities, for instance in the pion case the following one:

$$\frac{16\sigma_{\pi^+(\pi^-)p}(\infty)}{9\pi(\mu_\pi Z_\pi)^2} \lesssim F_2^{\nu(\bar{\nu})p}(0) \lesssim \frac{2\sigma_{\pi^+(\pi^-)p}(\infty)}{\pi(\mu_\pi Z_\pi)^2}\left(1 - \frac{\mu_\pi^{\,2}}{2\cdot 5\ \mathrm{GeV}^2}\right) \qquad (1)$$

An important implication of these inequalities is that the validity of Bjorken scaling (in the limit $\xi \to 0$) is linked to the constancy of the asymptotic meson-nucleon total cross-sections. Hence, if the former continues to remain valid in future (deeper inelastic) experiments, the meson-nucleon cross-sections are expected, according to our previous considerations, to exhibit constant behaviour at high energies in spite of the observed rising trend of the p-p total cross-sections (which are outside the purview of our considerations). We may look forward to this contrasting high-energy behaviour of cross-sections in future experiments with meson beams at the new accelerators.

The possibility that, among the hadrons, the pseudoscalar mesons may behave differently from the others is not unexpected. It is suggested by many features of the body of ideas relating the role of currents, densities and symmetry breaking mechanisms. The pseudoscalar mesons are presumably goldstone bosons of the spontaneously broken chiral $SU_3 \otimes SU_3$ symmetry generated by the charges of vector and axial-vector currents. Further, in our picture, the mesons are more like "current-quark-antiquark" combinations, whereas the baryons have the simple 3-quark structure only in terms of the "constituent-quarks" and not in terms of "current-quarks" [the two pictures are presumably related by a complicated Melosh-like transformation (Melosh 1973)]. Even in quark-parton theory, considered by West (1973) and embodying Bjorken scaling and rising pp cross-section, it is possible to arrange the meson-nucleon cross-sections to be constants at high energy.

In the eventuality of the meson-nucleon cross-sections also increasing indefinitely with energy, our assumption (3*a*) will have to be relaxed. However, even in this case, we are able to maintain a connection with the deep-inelastic lepton-nucleon structure functions so long as the underlying light-cone algebra (assumption 1*a*) is retained. But we must correspondingly pay the price of modifying the assumed light-cone singularity structure of assumption (1*b*). Of course, in order to achieve such possibly interesting relations, our remaining assumptions must continue to hold. The result that emerges in this case is that Bjorken scaling must be violated. *The functional form of the energy-dependence (for $\xi = 0$) in the terms that cause this violation is forced, by the universality of the underlying algebraic structure, to be the same as that of the indefinitely increasing parts of the meson-nucleon cross-sections.*

The part of the universal algebraic structure implied in assumption (1*a*), relevant for us, may be stated as follows*:

$$[J_\mu^i(z), J_\nu^j(0)]$$
$$\hat{=} \partial^\alpha \Big[\sum_a D_a(z)\, S_{\mu\nu\alpha\beta} \Big\{ i f^{ijk} S_a^{\beta,k}\Big(\frac{z}{2}, -\frac{z}{2}\Big) + d^{ijk} A_a^{\beta,k}\Big(\frac{z}{2}, -\frac{z}{2}\Big)\Big\}$$
$$+ \text{axial terms}\Big] \qquad (2)$$

* We have put the gradient ∂^α outside in the right hand sides, so that these commutators are in conformity with the structure implied by light-plane quantisation (Cornwall *et al* 1971). The notations are similar to Das *et al* (1973).

$$[v^i(z), v^j(0)]$$
$$\hat{=} \partial_\mu \left[\sum_a D_a(z) \left\{ if^{ijk} S_a^{\mu,k}\left(\frac{z}{2}, -\frac{z}{2}\right) + d^{ijk} A_a^{\mu,k}\left(\frac{z}{2}, -\frac{z}{2}\right) \right\} \right] \quad (3)$$

Here $D_a(z)$ are a set of C-number functions singular on the light-cone $z^2 = 0$, and the bilocal operators $S_a^{\mu,k}$ and $A_a^{\mu,k}$ are generalisations of the standard bilocal operators (*see* Das *et al* 1973). When, as done earlier (Das *et al* 1973), we also adopt assumption (1*b*), then

$$D_a(z) = \delta_{a0} D_0(z),$$
$$D_0(z) = \frac{1}{2\pi}\, \epsilon(z_0) \cdot \delta(z^2),$$

and $A_0^{\mu,k}$ and $S_0^{\mu,k}$ are the usual antisymmetric and symmetric bilocal quark current densities having regular matrix elements on the light-cone. We now implement the modification of assumption (1*b*), alluded to in the previous paragraph. As an illustration consider the possibility that apart from the term $a = 0$ there is an additional term $a = 1$ in eqns (2) and (3). Taking the forward spin averaged nucleon matrix element of these equations we then have:

$$\langle p \mid [J_\mu^i(z), J_\nu^j(0)] \mid p\rangle$$
$$\hat{=} \partial^\rho \left[S_{\mu\nu\rho\sigma} \left\{ \frac{1}{2\pi}\, \epsilon(z_0)\delta(z^2) F_{0,\sigma}^{ij}(z \cdot p) + \epsilon(z_0)\theta(z^2) z^{-2} \times \right.\right.$$
$$\left.\left. \times \ln(\mu^2 \mid z^2 \mid) F_{1,\sigma}^{ij}(z \cdot p) \right\} + \text{axial part} \right] \quad (4)$$

$$\langle p \mid [v^i(z), v^j(0)] \mid p\rangle$$
$$\hat{=} \partial^\rho \left[\frac{1}{2\pi}\, \epsilon(z_0)\, \delta(z^2)\, F_{0,\rho}^{ij}(z \cdot p) + \epsilon(z_0)\, \theta(z^2)\, z^{-2} \ln(\mu^2 \mid z^2 \mid) \right.$$
$$\left. \times F_{1,\rho}^{ij}(z \cdot p) \right] \quad (5)$$

In eqs (4) and (5) the specific form of the additional light-cone singularity has been illustratively chosen to be $\epsilon(z_0)\,\theta(z^2)\, z^{-2} ln(\mu^2 \mid z^2 \mid)$, where μ is some characteristic mass (perhaps the gluon mass). Further,

$$F_{a,\mu}^{ij}(z \cdot p) \propto \langle p \mid if^{ijk} S_{a,\mu}^{k}(\tfrac{1}{2}z, -\tfrac{1}{2}z) + d^{ijk} A_{a,\mu}^{k}(\tfrac{1}{2}z, -\tfrac{1}{2}z) \mid p\rangle \quad (6)$$

and the $F_{a,\rho}^{ij}(z \cdot p)$ are assumed to be non-singular. Note that the modifications introduced are adopted so as to leave the usual equal-time commutators unaffected. These modifications would consequently imply, in the standard manner, a violation of the Bjorken scaling* such as:

$$F_2^{\nu p}(\nu, \xi) \underset{\substack{\nu \text{ large} \\ \xi \text{ fixed}}}{\frown} f_2^{\nu p}(\xi) \left[1 + \alpha^{\nu p}(\xi) \ln^2 \frac{\nu}{\mu} + \beta^{\nu p}(\xi) \ln \frac{\nu}{\mu} \right] \quad (7)$$

The corresponding equation (5) implies a similar high energy behaviour for the highly off-shell meson-nucleon amplitudes (the light-cone limit). Here $\alpha^{\nu p}(\xi)$ and $\beta^{\nu p}(\xi)$ are related to the Fourier-transforms of the bilocal operators S_0, A_0 and S_1, A_1. Thus we can obtain again relations, analogous to our earlier work,

* However the non-leading behaviour of the longitudinal structure function W_L in the scaling limit continues to hold.

if we modify our assumption (3*a*) correspondingly. For example, in the case of the $\pi^+(\pi^-)$ p-scattering amplitude $T_{\pi^+(\pi^-)\mathrm{p}}$, it should read:

$$\frac{-\,\mathrm{i}T_{\pi^+(\pi^-)\mathrm{p}}\,(q^2,\,\nu)}{\nu\left[1+\alpha^{\nu(\bar\nu)p}\,(0)\ln^2\frac{\nu}{\mu}+\beta^{\nu(\bar\nu)p}\,(0)\ln\frac{\nu}{\mu}\right]}\xrightarrow[\nu\ \text{large}]{}\text{function}\,(q^2) \tag{8}$$

We repeat that the modified assumption is motivated by our earlier relations (Das *et al* 1973), which do seem to imply some interesting connections. The specifics of the modification do of course follow from the particular form chosen for the matrix-element of the $a = 1$ term.

Then with the implied behaviour of the cross-sections in the form

$$\sigma_{\pi^+(\pi^-)\mathrm{p}}\,(\nu)\xrightarrow[\nu\ \text{large}]{}\sigma^0{}_{\pi^+(\pi^-)p}\left[1+\alpha^{\nu(\bar\nu)p}\,(0)\ln^2\frac{\nu}{\mu}+\beta^{\nu(\bar\nu)p}\,(0)\ln\frac{\nu}{\mu}\right], \tag{9}$$

we obtain the inequality of eq. (1), in which $\sigma_{\pi^+(\pi^-)p}\,(\infty)$ is replaced by $\sigma^0{}_{\pi^+(\pi^-)p}$ and $F_2^{\nu(\bar\nu)p}$ by $f_2^{\nu(\bar\nu)p}$. Similar inequalities follow also for $\sigma^0{}_{KN}$ and $f_2^{eN}\,(0)$, etc.

It should perhaps be emphasised that our illustrative example of the additional singularity, which has been chosen to give the $ln^2\nu$ behaviour, does not come from perturbative calculations of simple field theoretic models to the lowest non-trival orders in which they have been preformed (Jackiw *et al* 1972, Lee 1972).

Concerning our modified inequalities we may note the following points. (i) In view of the consistency of the present experiments with Bjorken scaling, $\alpha\,(\xi)$ and $\beta\,(\xi)$ are expected to be small. Taking for $\sigma^0{}_{\pi N}$, $\sigma^0{}_{KN}$ the present values of the cross-sections at the highest energies available, and for $f_2^{lN}\,(0)$ the scaled structure functions measured at *SLAC* (for ξ small), our earlier estimates of the bare current-quark masses remain essentially unchanged. (ii) Our earlier conclusion regarding the nature of chiral symmetry breaking and the Llewellyn-Smith inequality continues to hold (with the above substitutions). (iii) The characteristic mass μ, that sets the scale of the logarithmic terms, is expected to be of the order of the gluon mass if we take the cue from certain interacting field theory models (Jackiw *et al* 1972). Present data on Bjorken scaling suggest μ to be at least in the region of tens of GeV.

Our considerations thus lead us to expect that, either the high energy meson-nucleon cross-sections show constant behaviour (even though the pp cross-section increases indefinitely), or that they deviate from constancy by logarithmic terms whose scale is set by a mass of the order of at least tens of Gev, which also characterises the consequent logarithmic violations of Bjorken scaling.

References

Amaldi U, Biancastelli R, Bosio C *et al* 1973 *Phys. Lett.* **B 44** 112
Amendolia S R, Belletini G, Braccini P L *et al* 1973 *Phys. Lett.*, **B44** 119
Cornwall J and Jackiw R 1971 *Phys. Rev.* **D4** 367
Das T, Pandit L K and Roy P 1973 *Phys Lett.* **B 44** 76; 1973 *Nucl. Phys.* **B 53** 567
Jackiw R and √altz R E 1972 *Phys. Rev.* **D6** 702
Lee T D 1972 *Phys. Rev.* **D6** 1120
Melosh IV H J 1973 *Ph.D. Thesis*, California Institute of Technology
West G B 1973 *Phys. Rev. Lett.* **31** 798