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A model for the multiplicity distribution in pp-collisions from 5 to 300 GeV/c

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Abstract. The multiplicity distribution of charged particles in pp-collisions in the range 5–300 GeV/c is explained on the basis of a dynamical model which leads to a specific mixture of Poisson distributions.

Keywords. Multiplicity distribution; pp-collisions; Poisson distributions; leading particles.

1. Introduction

Recently several authors have studied the problem of prong multiplicity distribution in high energy pp-collisions, using different approaches. In the simplest approach (Lach and Malamud 1973; Le Bellac *et al* 1973) one tries to fit the observed multiplicity distribution to some empirical form such as a Poisson distribution or a mixture of two Poisson distributions, etc. A somewhat different approach (Koba *et al* 1972; Buras and Koba 1973; Rama Rao and Sarma 1973; Slattery 1972, 1973) is to be found in the proposal of a possible scaling behaviour in some variable like $n/\langle n \rangle$ which seems to be valid in pp-collisions in the range $P_L = 50$ GeV/c to 300 Gev/c and in $\bar{p}$p-collisions at momenta as low as 3–7 GeV/c. In addition, (Nielsen and Olessen 1973; Harari and Rabinovici 1973; Van Hove 1973; Fialkowski and Miettinen 1973), one has models such as two component models, with some dynamical content in them, to explain the multiplicity distribution. However, in all the approaches, the analysis has so far been restricted to the high energy range 50–300 GeV/c.

Though the prong multiplicity distribution is only a gross feature of the multiparticle production, it is capable of giving considerable insight into the dynamics of particle production provided one attempts to understand all the available data both at low and high energies, from 5–300GeV/c. A striking feature of the multiplicity distribution is that it changes from one narrower than a Poisson distribution at energies less than 30 GeV to one broader than a Poisson distribution at energies greater than 50 GeV. A more quantitative index is provided by the correlation integral $f_2 = \langle n(n-1) \rangle - \langle n \rangle^2$ which changes from negative values below 30 GeV to progressively increasing positive values for energies above 50 GeV. Empirical fits or models which analyse data in a limited energy range cannot hope to elucidate the changing dynamical pattern from low to high energies apart from the fact that the success of such fits could sometimes be fortuitous. In this paper the complex pattern of the multiplicity distribution, over the whole range 5–300 GeV, is explained by a specific mixture of Poisson distributions which is based on a well-defined dynamical model. We first present the physical picture on which our model is based and give the expressions for the probability P_n for having n charged particles, the correlation integral f_2, etc. Calculations based on the model are then presented for different parametrizations.

2. The model

We assume that the incident protons after collisions emerge as some excited objects which retain their incident charges. These objects may be baryon isobars or some other massive objects, possibly with a continuous distribution of mass. What we call as leading particles (charged) are supposed to arise in the decay of these two excited objects. At low energies the leading particles may be 2 but their number would increase progressively with energy. At any given energy the number of leading particles would have a variation between 2 and some maximum value $2R$, where R would be a function of energy. In addition to the leading particles there would be another group of particles which may be identified with pionization or the central component. We refer to them as non-leading particles even though all of them may not be separated from the leading particles in their longitudinal momenta. The only distinction we need to make between the leading and the non-leading particles is that they have different number distributions. We assume that the emission of leading and non-leading particles to be unrelated, so that the probability of having a given number of non-leading particles in a collision is independent of the number of leading particles present in it. We further assume that the emission of the non-leading particles takes place randomly in pairs of opposite charges so that their distribution is a Poisson in $(n-2r)/2$, where $2r$ is the number of leading particles in an event with n particles. Such an assumption corresponds roughly to one's intuitive understanding of the central component. One has, however, no such *a priori* feeling for the number distribution of the leading particles. Our analysis of the experimental data, based on an estimate of the relative numbers of the leading and the non-leading particles and that the latter follow a Poisson distribution, suggests that the leading particles have a rather broad distribution particularly at high energies. We find that the one-parameter geometric distribution works fairly well. Further, the number of terms used in the geometric distribution is taken to be finite at a given energy. We denote the average multiplicities of the leading, non-leading and all charged particles by $\langle n_L \rangle$, $\langle n_{NL} \rangle$ and $\langle n \rangle$ respectively, so that

$$\langle n \rangle = \langle n_L \rangle + \langle n_{NL} \rangle \tag{1}$$

$$\langle n_L \rangle = \sum_{r=1}^{R} 2r\alpha_r, \qquad \langle n_{NL} \rangle = \sum_{m=0}^{\infty} 2m\bar{P}_m \tag{2}$$

where α_r and $\bar{P}_m$ are the probabilities of having $2r$ leading and $2m$ non-leading particles respectively. For the geometric distribution α_r with R terms and the Poisson distribution $\bar{P}_m$ one can write

$$a_r = \frac{(1-q)}{1-q^R} q^{r-1}, \qquad q \geqslant 0 \tag{3}$$

$$\bar{P}_m = e^{-\Delta} \frac{\Delta^m}{m!} \tag{4}$$

Using (3) and (4) one gets

$$\langle n_L \rangle = \frac{2}{1-q} - \frac{2Rq^R}{1-q^R}, \quad \langle n_{NL} \rangle = 2\Delta \tag{5}$$

By the assumption that the leading and non-leading particles are emitted independently, the probability of having a total of $2m$ particles ($m = 1, 2, 3, \ldots$) in an event is

$$\begin{aligned} P_{2m} &= \sum_{r=1}^{m} a_r \bar{P}_{m-r} \quad \text{if} \quad m \leqslant R \\ &= \sum_{r=1}^{R} a_r \bar{P}_{m-r} \quad \text{if} \quad m > R. \end{aligned} \tag{6}$$

The correlation integral, defined earlier, can be shown to be given by

$$f_2 = \langle n_{NL} \rangle + 4 \sum_{r=1}^{R} r^2 a_r - \langle n_L \rangle - \langle n_L \rangle^2 \tag{7}$$

The distribution given by (6) is a weighted sum of R Poisson distributions with the same average but the maximum of each being displaced progressively to the right by one unit in m, such that P_2 consists of only one term, P_4 a sum of two terms and so on while P_{2m}, for all $m \geqslant R$ consists of a sum of R terms.

3. Results and parametrization

We first treat $\langle n_L \rangle$, $\langle n_{NL} \rangle$ and R as free parameters to be fixed at each energy, to obtain the best χ^2-fit of the calculated values of P_{2m} to the experimental data. The results of the calculation are presented in table 1 along with the calculated values of f_2. The overall χ^2 is 86 for 73 data points while for energies above 50 GeV/c, χ^2 is 43 for 50 data points. The agreement with experiment shows that on the basis of our model one can understand the behaviour of the multiplicity distribution over the entire energy range from 5·5 to 303 GeV/c.

As the prong multiplicity distribution suggested in this paper is based on a dynamical model, the changes in the multiplicity distribution can be related to changes in the dynamical pattern of particle production. For energies below 30 GeV, the number of non-leading particles is small and the leading particles are either 2 or 4 only. It can be seen from (7) that, for $R = 2$ and small $\langle n_{NL} \rangle < \langle n_L \rangle$, f_2 would be negative. The resulting distribution is actually narrower than a single Poisson distribution in $(n-2)/2$ with an average equal to $(\langle n \rangle - 2)/2$. As the number of non-leading particles as well as the value of R increases with energy, the distribution becomes progressively broader than a single Poisson distribution. The manner in which the individual Poisson distributions in (6) add up to give the observed distribution is shown in figures 1 and 2 for 19 GeV/c and 205 GeV/c

Table 1. Fits for individual energies. The data have been taken from Alexander *et al* (1967), Smith *et al* (1969, 1971), and Boggild *et al* (1971). Ammosov *et al* (1972), Chapman *et al* (1972), Charlton *et al* (1972) and Dao *et al* (1972).

P_{lab} (GeV/c)	R	$\langle n_L \rangle$	$\langle n_{NL} \rangle$	$\langle n \rangle$	$\langle n \rangle$ Expt.	f_2	f_2 Expt.	χ^2/data points
5·5	2	2·61	·096	2·70	2·70	−1·67	− 1·673 ± ·009	·05/3
12·8	2	2·89	·602	3·50	3·53 ± ·05	−1·27	− 1·262 ± ·05	14· 0/6
19·0	2	2·98	1·02	4·00	4·02 ± ·03	−0·93	− 0·954 ± ·05	10· 7/7
28·4	2	3·05	1·51	4·56	4·56 ± ·04	−0·54	− 0·22 ± ·09	19· 0/7
50	3	3·24	2·12	5·36	5·32 ± ·13	1·11	1·37 ± 0·31	4· 7/8
69	4	3·37	2·53	5·91	5·81 ± ·15	2·33	2·42 ± 0·20	5· 6/9
102	5	3·87	2·64	6·32	6·36 ± ·12	3·88	3·9 ± 0·4	4· 6/9
205	6	4·87	2·87	7·75	7·65 ± ·17	6·78	7·44 ±0·72	9· 2/11
303	7	5·19	3·65	8·85	8·86 ± ·25	9·74	10·31 ± 0·91	18· 5/13

respectively. It may be noted in this context, that when a single Poisson distribution is written as weighted sum of two or more Poisson distributions (undisplaced) the latter would always have a broader dispersion than the former.

If we could parametrize $\langle n_L \rangle$, $\langle n_{NL} \rangle$ and R as functions of energy then (3), (4), (5) and (6) can be used to predict the multiplicity distribution at any given energy, Further, such a parametrization, confined to energies above 50 GeV/c, may hopefully give some indication of the asymptotic behaviour of the average multiplicity which is a problem of great interest. To make the parametrization more meaningful it is desirable to limit the wide choice of possible functions by invoking other constraints. In this context, we note that at high energies the leading particles are expected to correspond to the fragments of the target and the projectile and as such would be expected to obey the scaling hypothesis of Feynman and Yang. This suggests that $\langle n_L \rangle$ should have a logarithmic dependence on s, the square of the CM energy. However, experimentally the validity of scaling in the central region is still an open question. As a result we have tried both a logarithmic as well as a power dependence for the non-leading particles. The particulars of the two parametrizations for energies above 50 GeV/c are the following:

(i) $$\langle n_L \rangle = a + b \log s \tag{8a}$$

$$\langle n_{NL} \rangle = a' + b' \log s \tag{8b}$$

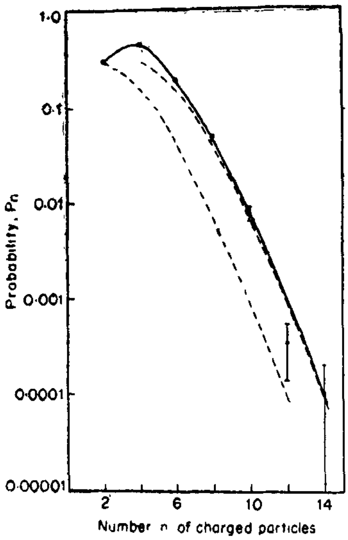

Figure 1. Multiplicity distribution at 19 GeV/c. The solid curve is the calculated multiplicity distribution. It is the sum of the dotted curves which are Poisson distributions $\bar{P}_m = e^{-\Delta} (\Delta^m/m!)$ where $m = (n-2)/2$ and $2\Delta = \langle n_{NL} \rangle$; further they are weighted as in eq. (6) and have their maxima displaced successively by two units along the abscissa.

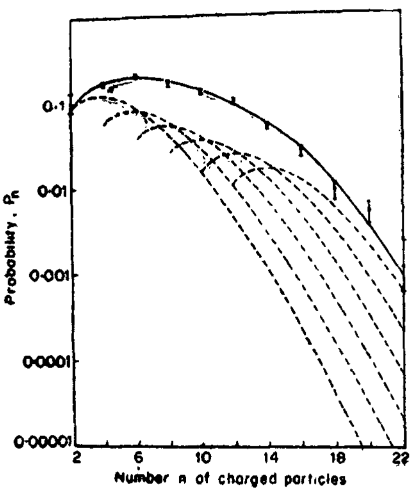

Figure 2. Multiplicity distribution at 205 GeV/c. The solid curve and the dashed curves have the same significance as in figure 1.

where s is in (GeV)^2. The values of the parameters which give the best fit to the values of multiplicity distribution $\langle n_L \rangle$ and $\langle n_{NL} \rangle$ in table 1 are $a = -1 \cdot 2$, $b = 0 \cdot 97$, $a' = -1 \cdot 87$ and $b' = 0 \cdot 863$. For the values of P_n calculated according to (8 a) and (8 b) and the values of R given in table 1, we get $\chi^2 = 51$ for 50 data points

(ii) $$\langle n_L \rangle = a + b \log s \tag{9 a}$$

$$\langle n_{NL} \rangle = a' + b's^{\alpha} \tag{9 b}$$

The values for the best fit are $a = -2 \cdot 58$, $b = 1 \cdot 24$, $a' = 0 \cdot 368$, $b' = 0 \cdot 598$ and $\alpha = 0 \cdot 25$. The value of χ^2 is now 46·5 for 50 data points. An approximate parametrization of R, which reproduces the values in table 1, is given by

$$R = \text{nearest integer of } [1 + 0 \cdot 47 (s/4M^2)^{\frac{1}{4}}] \tag{10}$$

where M is the proton mass. The predicted values of P_n at 400 GeV/c according to (9) and (10) are given in table 2.

Table 2. Predicted results for 400 GeV/c

$R = 8$, $\langle n_L \rangle = 5 \cdot 6$, $\langle n_{NL} \rangle = 3 \cdot 6$, $\langle n \rangle = 9 \cdot 2$, $f_2 = 12 \cdot 2$

n ..	2	4	6	8	10	12	14
P_n ..	0·055	0·136	0·180	0·176	0·145	0·108	0·078
n ..	16	18	20	22	24	26	28
P_n ..	0·055	0·035	0·019	0·009	0·003	0·001	0·0003

The parametrizations (8) and (9) give different predictions for the asymptotic behaviour of $\langle n \rangle$. As far as the value of chi-square for the fit to P_n is concerned both the parametrizations are equally acceptable. However, the total charged multiplicity $\langle n \rangle$ and f_2 given by (9) are a little larger than those given by (8) at energies beyond 303 GeV/c though the value given by (9) is in better agreement with the value of $\langle n \rangle$ at 1500 GeV/c. The present errors on the available values at 1500 GeV/c from ISR and at 10^4 GeV/c do not enable one to choose between them. We are thus unable to make a definite prediction for the asymptotic behaviour of $\langle n \rangle$.

We would like to comment on the suggested scaling behaviour of the multiplicity distribution (Koba *et al.* 1972) as a function of $n/\langle n \rangle$. This scaling behaviour is

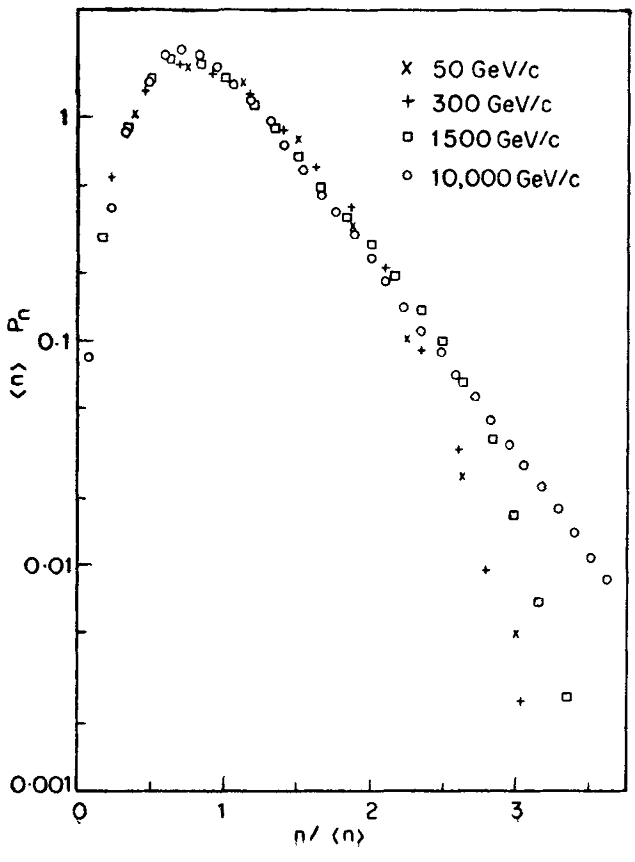

Figure 3. Calculated distribution for $\langle n \rangle$ P_n plotted as a function of $n/\langle n \rangle$ according to parametrization (9). The calculated distribution according to parametrization (8) also exhibits similar scaling behaviour.

established (Buras and Koba 1973, Slattery 1972, 1973) in the range 50–300 GeV/c. Though we have no such in-built scaling behaviour in our model, both the parametrizations (8) and (9) reproduce this behaviour in this range. Using the two parametrizations (8) and (9) together with (10) we have calculated the multiplicity distribution P_n at 1500 GeV/c and 10 TeV. In figure 3 the values of $\langle n \rangle P_n$ are plotted as a function of $n/\langle n \rangle$ for the energies 50 GeV/c and 303 GeV/c together with those for 1500 and 10 TeV. It is remarkable that for both the parametrizations the multiplicity distribution exhibits an approximate scaling behaviour up to 1500 GeV and for prong numbers $n < 40$ up to 10 TeV, though fairly marked deviations occur for $n > 40$ at the latter energy. It would thus seem that if this scaling is a transitory phenomena, then deviations from it may be detectable for very large prong numbers and at energies of the order of 10 TeV.

4. Concluding remarks

The dynamical content of the modle can be summarized saying that the events at a given energy can be divided into R groups. In the first group there are 2 leading particles, in the second 4 leading particles, etc., and these leading particles in a group do not fluctuate while the remaining particles would fluctuate according to a Poisson distribution in the number of pairs. The numbers of events in the groups are distributed according to a geometric distribution. Some of the dynamical features envisaged in the model, such as the existence of two components in each of the events, small numbers of leading particles at low energies, etc., may be amenable to experimental verification. However, the leading and non-leading particles may overlap in the momentum space, particularly at low energies, and their identification would then be difficult. Our model can be easily extended to other collisions and it would be interesting to see if a similar pattern of particle production is valid in them.

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