Pramāṇa, Vol. 1, No. 2, 1973, Printed in India. © 1973

Stimulated emission of x-rays from plasmas generated by short-pulse-laser-heating of solid targets

SUDHANSHU S JHA
Tata Institute of Fundamental Research, Bombay 400005

MS received 26 February 1973

Abstract. The problem of heating of a solid target to generate a nonequilibrium plasma by subnanosecond laser pulses is considered. For an appreciable absorption of energy from a Nd-glass laser, the critical density of the electrons in the plasma turns out to be 10^{21} cm^{-3}. These electrons can be heated up to 10^7 K or more by using pulses of about 10 picosecond duration and absorbed energy flux of the order of 10^{21} erg cm^{-2} sec^{-1}. Starting from neutral atoms in a solid with a high atomic number, e.g., $Z=26$, for times in the picosecond regime the relevant rate equations are solved analytically to predict densities of the atoms at different ionization levels. It is shown that during such a short time the population density of the ions isoelectronic to neon builds up to a very large amount. This in turn leads to the population inversion in the $4s \rightarrow 3p$ soft x-ray laser transition, via the electron-impact excitation of the 4s level of the isoelectronic neon ion. For the effective pumping times of the order of 5 picoseconds, a gain of the order of 10^2 db cm^{-1} is predicted for the laser transition in Fe XVII, Co XVIII or Cu XX.

Keywords. X-ray laser; laser-induced plasmas; Nd-glass laser.

1. Introduction

Recently, there has been a revival of interest in exploring the possibility of fabricating high frequency lasers in the soft x-ray region. Apart from their usefulness in high resolution absorption spectroscopy, the availability of such sources is extremely important for filling the existing gap in studying elementary excitations of a system in the intermediate wavelength region (10^{-5} to 10^{-7} cm). At present, inelastic neutron scattering measurements can give information only on short wavelength excitations, whereas inelastic light scattering experiments probe only long wavelength excitations. Already, lasers in the vacuum ultraviolet region upto 1100 Å are available (Hodgson and Dreyfus 1972), and there is even a half-hearted claim of the observation of the x-ray laser action in $CuSO_4$—gelatin solution (Kepros *et al* 1972). More definite results in the soft x-ray region are expected in the near future.

Various physical processes involving intense electromagnetic waves and fast electrons or ions can be considered for pumping a laser transition in the x-ray region. However, two distinct methods involving laser induced nonequilibrium plasmas (Bristow *et al* 1972) seem to be most promising at present. In the first approach, one hopes to achieve a population inversion due to electron-impact excitations of an upper laser level of an ion in an extremely hot plasma, generated by heating a high Z-target (Mallozzi 1971). Alternatively, one could use such a plasma as an intense broad-band x-ray source to pump an x-ray laser transition either directly in another material or via the photo-ionization process. In this paper, we propose and investigate definite examples involving the first approach. The second method will be discussed elsewhere (Jha, to be published).

The time scales and the physical mechanism involved in the heating of solid targets like $_{26}$Fe, upto an electron temperature of the order of 10^7 K ($1{\cdot}16\times10^7$ K$=$1 keV), by using nanosecond or subnanosecond intense laser pulses will be discussed in Sec. 2. At such a temperature, the matter is in the state of a highly ionized plasma. Sec. 3 deals with the survey of basic physical transformations going on in a subnanosecond plasma. The relevant rate equations governing the densities of the atoms at different ionization levels will also be investigated. After a sufficiently long time, in the local thermal equilibrium case, the densities of these atoms are, of course, given by the well known Saha equations. The solution of the rate equations during very short pulse durations, will, however, be different. The possibilities of population inversion of levels of those ions in the plasma which have just ten bound electrons left are considered in Sec. 4. Since these are isoelectronic to neon in which it is relatively easy to obtain many pairs of lasing levels, such a consideration seems to be an excellent approach for obtaining an x-ray laser action in a high Z-plasma. With an effective pumping time of the order of 5×10^{-12} sec, numerical calculations of population inversions for 4s→3p transitions in $_{26}$Fe XVII, $_{27}$Co XVIII and $_{29}$Cu XX show that one can obtain gains of the order of 10^2 db cm^{-1}, with wavelengths in the region of 40 Å to 70 Å. The results are discussed in Sec. 5.

2. Laser Heating of Solid Targets

The interaction with a solid target of intense light pulses of 10^{-8} sec to 10^{-12} sec durations, having energy flux ϕ of the order of 10^{16} to 10^{23} ergs cm^{-2} sec^{-1}, is a fascinating subject in itself (Caruso and Gratton 1968; Dawson *et al* 1970; Engelhardt *et al* 1970; Basov and Kroklin 1964). Using, for example, Nd^{3+} glass laser ($\omega\simeq1{\cdot}7\times10^{15}$ sec^{-1}), one can obtain ϕ of the order of 10^{22} erg cm^{-2} sec^{-1} or more with pulsewidths t_p ranging from 10^{-11} sec to 10^{-12} sec. Such a high energy flux corresponds to an electric field amplitude $E=(8\pi\phi/c)^{\frac{1}{2}}$ of the order of 3×10^6 esu, i.e. 10^9 volt cm^{-1}, which is much higher than the breakdown field (Bloembergen, Private communication). An electric field of this magnitude implies a velocity $v=(eE/m\omega)$ for an electron of the order of 10^9 cm sec^{-1} and its energy ϵ of the order of 1 keV.

When the intense light pulse reaches the surface of a solid, the surface atoms get ionized initially due to the multi-photon ionization process or equivalently due to the tunneling of electrons. However, almost immediately the avalanche ionization due to electron-impact excitations takes over. In the beginning of the avalanche, this is governed by the rate equation for the density of singly ionized atoms, or equivalently for the free electron density n_e, as

$$\frac{dn_e}{dt}=\langle\sigma_{ie}v\rangle\,{}_1\mathcal{N}_1 n_e \tag{2.1}$$

where $\mathcal{N}_1$ is the density of neutral atoms, and v is the electron velocity. According to the classical expression (Rudge 1968), for the electron energy greater than the ionization energy I_1 of the neutral atom, the avalanche growth rate $\sigma_{ie}v\mathcal{N}_1$ is given by

$$\sigma_{ie}v\,\mathcal{N}_1=4\left(\frac{I_H}{I_1}\right)\left(1-\frac{I_1}{\epsilon}\right)\pi a_0^2 v\mathcal{N}_1\ \text{sec}^{-1} \tag{2.2}$$

where a_0 is the Bohr radius, and I_H is the ionization energy of the hydrogen atom. Thus, almost instantaneously $(1/\sigma_{ie}v\mathcal{N}_1 \simeq 10^{-15}$ sec), a high density plasma is created near the surface, and the multi-ionization processes and heating start. However, the plasma frequency ω_p for such a dense system, with electron density corresponding to the bulk atomic density, will be much larger than the laser frequency $\omega = 1{\cdot}7 \times 10^{15}$ sec^{-1}. The light wave is, therefore, expected to be totally reflected back within a distance of the order 10^{-5} cm to 10^{-6} cm from the surface. To solve this problem of very little absorption of the energy, at least partially, one should use a weak prepulse to produce a low density plasma first, before the main pulse arrives. Although the electron-electron collision is still not important, a substantial amount of light can, nevertheless, be absorbed by the plasma because of highly turbulent motion of the electrons (Mulser *et al* 1973), if ω is close to ω_p. For the Nd-glass laser, this implies a critical plasma density of 10^{21} cm^{-3}.

In order to understand the time development of heating of the electrons and the ions, one has to consider the dynamic equations for the plasma in detail. Heat transfer from the electrons to the ions in a system containing deuterium is particularly relevant for studying the rate of thermonuclear reactions. However, we are concerned here mainly with the electron system, which we will describe by a local temperature T. The validity of the assumption of a local Maxwellian distribution of velocities of the electrons depends on the electron-electron collision time. One finds

$$t_{ee} \simeq \frac{m^2 \langle v \rangle^3}{e^4 n_e \ln\Lambda} \simeq 10^{-12} \text{ sec} \tag{2.3}$$

for 1 keV electrons of density 10^{-21} cm^{-3}, with the ‘ Coulomb logarithm ’ factor (Spitzer 1956) $\ln\Lambda \simeq 10$. Thus for very short pulses in the picosecond region, our description of the electron system characterized by a temperature T will not be a very good approximation during the pulse period. In any case, ions will never come into equilibrium in such a short time, since the electron-ion collision time (Spitzer 1956) is much longer ($t_{ie} \simeq 10^{-9}$ sec).

For the heat transfer in the electron system, starting from the absorption of the energy near the surface, we have to consider two important processes: (i) the thermal conduction, and (ii) the expansion of the plasma or the propagation of the shock wave. It follows from the work of Korobkin *et al* (1968) that during the pulse period of very short duration, we need not separately consider the energy loss due to ‘ thermal ’ radiation. While calculating the nett absorption rate at the surface, we assume that it has already been corrected for the radiation losses. The heat conduction in the direction x perpendicular to the surface is governed by the diffusion equation

$$\frac{\partial T}{\partial t} = -\frac{\partial}{\partial x}\left(\frac{K_T}{C_v n_e}\frac{\partial T}{\partial x}\right) \tag{2.4}$$

where K_T is the thermal conductivity of the electron system, and the specific heat C_v is 3/2 times the Boltzmann's constant k. Thus at time t, the thickness of the ‘ heated ’ layer is determined from the expression

$$x_H(t) = (2K_T t/3n_e k)^{1/2} \tag{2.5}$$

in which K_T can be assumed (Spitzer 1956) to be of the form

$$K_T \simeq \frac{6(2/\pi)^{3/2}(kT)^{5/2}k}{m^{1/2}e^4 z\ln\Lambda} \simeq K_1 T^{5/2} \tag{2.6}$$

with

$$K_1 = 10^{-6} \text{ cgs units} \tag{2.7}$$

for the average ionic charge $z \simeq 10$ and $\ln\Lambda \simeq 10$.

If ϕ_a is the flux of light energy absorbed by the plasma the energy balance equation gives

$$\phi_a t = \tfrac{3}{2} n_e k \langle T \rangle x_H(t) \quad (2.8)$$

so that, using Eqs. (2.5) and (2.6) one obtains

$$\langle T \rangle = K_1^{-2/9} (\tfrac{3}{2} n_e k)^{-2/9} \phi_a^{4/9} t^{2/9} \quad (2.9)$$

$$x_H(t) = k_1^{2/9} (\tfrac{3}{2} n_e k)^{-7/9} \phi_a^{5/9} t^{7/9} \quad (2.10)$$

The shock wave or the plasma expansion wave travels with the sound velocity $v_s = (2kT/M)^{1/2}$ where M is the ionic mass. At time t it is expected to be located at a distance

$$x_E(t) = (2kT/M)^{1/2} t$$
$$= (2k/M)^{1/2} K_1^{-1/9} (\tfrac{3}{2} n_e k)^{-1/9} \phi_a^{2/9} t^{10/9} \quad (2.11)$$

if the expression (2.9) is used for T. For $n_e = 10^{21}$ cm^{-3} and $\phi_a = 10^{21}$ erg cm^{-2} sec^{-1}, which are the typical values used in this paper, we obtain

$$\frac{x_H(t)}{x_E(t)} \simeq 10^{-3} t^{-1/3} \quad (2.12)$$

This implies that for very short pulses with $t_p \ll 10^{-9}$ sec, the expansion process during the absorption may be completely ignored, whereas for $t_p \gg 10^{-9}$ sec, the conduction process is unimportant. Since this paper deals with the main pulse of typical duration of 10^{-11} sec, one may determine the average electron temperature, to be called T from now on, by equations (2.7) and (2.9). Therefore,

$$kT = (10^{-6})^{-2/9} k (\tfrac{3}{2} n_e k)^{-2/9} \phi_a^{4/9} t_p^{2/9} \simeq 1 \text{ keV} \quad (2.13)$$

for the typical values of ϕ_a and n_e under consideration. In this case the thickness of the heated layer is of the order of

$$x_H(t_p) \simeq 3 \times 10^{-3} \text{ cm} \quad (2.14)$$

Further, if z is the average ionic charge in the plasma with $n_e = 10^{21}$ cm^{-3}, the total density of the atoms and ions $\mathcal{N}$ is $10^{21}/z$ cm^{-3}. At 10^7 K, $\mathcal{N}$ is expected to be of the order of 10^{20} cm^{-3}, for an element like Fe. These estimates for the plasma parameters are used in the following sections.

3. Basic transformations, rate equations, and the Saha equations

Important atomic processes occurring in a high temperature plasma have been reviewed extensively by Elton (1970) and by McWhirter (1965). Mainly, one has to consider the ionization and excitation processes due to the electrons and the photons in the plasma, together with the corresponding recombination and deexcitation processes. However, it can be shown that while the laser radiation is partially absorbed by the plasma, it is almost transparent to its own bound-bound, free-bound and free-free emissions of radiation, most of which lie in the high frequency region. Thus, one can consider the plasma to be optically thin, and assume the validity of the so called modified ‘ time-dependent ’ Corona model (McWhirter 1965). In this model, it is assumed that the ionization is by collision of an electron with an atom or ion in its ground state and that the recombination is by radiative recombination ($e + A^+ \rightarrow A + h\nu$). In the high density region, this is modified to take into account the collisional three body recombination ($e + e + A^+ \rightarrow e + A$) also. Further, in this model the spectral line radiation is assumed to be emitted when an ion in its ground level suffers an inelastic electron collision, and the excited level subsequently decays to lower levels. The validity of this model rests on the fact that for the plasma under consideration, the photo-ionization rates are indeed insignificant.

Let the different ionization stages of an atom in the plasma be labelled by $j=1, 2, \ldots, Z+1$, where $j=1$ denotes the neutral atom, and $j=Z+1$ denotes the bare nucleus. The electron-impact ionization coefficient from the ground state of an atom at any such ionization level may be obtained by averaging an expression of the type (2.2) over the Maxwellian distribution of the electrons. However, it is more accurate (Elton 1970) to use the empirical expression

$$\langle \sigma_{ie} v \rangle_j = \frac{2\cdot5\times10^{-6}}{I_j^{3/2}}\,\eta_j\,\frac{(kT/I_j)^{1/2}}{(1+kT/I_j)}\,e^{-(I_j/kT)} \tag{3.1}$$

where kT and the ionization energy I_j are in eV, and where η_j is the number of equivalent outer electrons. For $I_j \sim 100$ eV, this is of the order of 5×10^{-9} cm^3 sec^{-1} for a 1 KeV plasma. Alternatively, one may use the semi empirical calculations of Lötz (1969) for elements like Fe, Co and Cu.

The electron-impact excitation coefficient for allowed transitions between bound levels $n\rightarrow u$ is given by

$$\langle \sigma_e(n\rightarrow u)v \rangle = \frac{1\cdot6\times10^{-5}}{\Delta E(kT)^{1/2}}\,e^{-(\Delta E/kT)} f_{nu} \langle \bar{g} \rangle \text{ cm}^3 \text{ sec}^{-1} \tag{3.2}$$

where kT and the excitation energy $\Delta E = E_u - E_n$ are in eV, and where the product of the oscillator strength f_{nu} and the average Gaunt factor $\langle \bar{g} \rangle$ may be assumed to be of the order of 10^{-1}.

For the radiative recombination coefficient, one may use (Elton 1970) Seaton's expression:

$$\langle \sigma_{rr} v \rangle_j = 5\cdot2\times10^{-14}(j-1)(I_j/kT)^{\frac{1}{2}}\times \times\left[0\cdot429+\tfrac{1}{2}\ln\left(\frac{I_j}{kT}\right)+0\cdot469\left(\frac{kT}{I_j}\right)^{\frac{1}{2}}\right] \tag{3.3}$$

For $j=11$, and $I_j \sim 100$ eV, it is of the order of 10^{-13} cm^3 sec^{-1}. A similar expression obtained by Griem may be written for the three-body collisional recombination coefficient $\langle \sigma_{re} v \rangle$. With $n_e = 10^{21}$ cm^{-3}, for small j this is comparable in magnitude to the radiative recombination coefficient (3.3).

Finally, for the spontaneous radiative decay rate for allowed transitions, we may use the expression

$$\frac{1}{\tau_{sp}} \equiv A_{un} = \left(\frac{8\pi^2 e^2}{mc^3}\right)\nu_{un}^2 f_{un} = 4\times10^7(\Delta E)^2 f_{un} \text{ sec}^{-1} \tag{3.4}$$

where the photon energy ΔE is in eV. For $\Delta E \sim 100$ eV, this is in the range of 10^{11} sec^{-1}.

As an approximation, n_e can be considered to be constant in time, despite the production of electrons by ionization, thus restricting attention on the density of one particular highly ionized species only. One can define constant ionization and recombination rates for the jth species by positive numbers

$$X_j = n_e \langle \sigma_{ie} v \rangle_j \text{ sec}^{-1} \tag{3.5}$$

and

$$R_j = n_e \{ \langle \sigma_{rr} v \rangle_j + \langle \sigma_{re} v \rangle_j \} \text{ sec}^{-1}, \tag{3.6}$$

respectively. The rate equations for the fractional densities $Q_j \equiv N_j/N$ of the *j*th species are then given by

$$\frac{dQ_j}{dt} = X_{j-1} Q_{j-1} - (X_j + R_j) Q_j + R_{j+1} Q_{j+1} \tag{3.7}$$

where

$$X_0 = R_1 = X_{Z+1} = R_{Z+2} = 0. \tag{3.8}$$

Let $\mathbf{Q}$ be a column vector in the (Z+1)-dimensional space, with components Q_j. Consider the eigen value problem

$$\mathbf{M}\mathbf{Q} = \lambda \mathbf{Q} \tag{3.9}$$

where the square matrix $\mathbf{M}$ is given by

$$M_{ji} = -X_{j-1} \delta_{j-1,i} + (X_j + R_j) \delta_{j,i} - R_{j+1} \delta_{j+1,i} \tag{3.10}$$

If the eigenvalues and eigenvectors of $\mathbf{M}$ are denoted by λ_n and $\mathbf{Q}(\lambda_n)$ respectively, the general solution of the rate equations (3.7) is given by

$$\mathbf{Q}(t) = \sum_{n=1}^{Z+1} C_n \mathbf{Q}(\lambda_n) e^{-\lambda_n t} \tag{3.11}$$

The coefficients C_n are determined from the initial conditions, which may be assumed to be

$$Q_j(t=0) = \delta_{j,1} \tag{3.12}$$

According to the earlier estimates of ionization and recombination coefficients, for $n_e = 10^{21}$ cm^{-3}, the typical values of X_j and R_j are $10^{11} - 10^{12}$ sec^{-1} and $10^8 - 10^9$ sec^{-1}, respectively. This implies that for time $t \ll 10^{-9}$ sec, one may neglect the recombination processes and use the equations for the radiative decay to predict the population density of a highly ionized ion. The eigenvalues of $\mathbf{M}$ are now simply X_j, and the eigenvectors are also of a simple form. Thus for very short times, one obtains

$$Q_j = \sum_{i=1}^{j} d_{ji} \, e^{-X_i t} \tag{3.13}$$

with

$$d_{11} = 1, j = 1 \tag{3.14}$$

$$d_{ji} = \frac{X_1 X_2 \ldots X_{j-1}}{(X_1 - X_i)(X_2 - X_i) \ldots (X_{i-1} - X_i)(X_{i+1} - X_i) \ldots (X_j - X_i)} \tag{3.15}$$

For times of the order of 10^{-9} sec, characteristic of the approach to the steady state corona model, the densities Q_j differ considerably from that given by equations (3.13)–(3.15). These are determined by the steady state solution of Eq. (3.7):

$$Q_j^{(s)} = \frac{X_1 X_2 \ldots \ldots \ldots X_{j-1}}{R_2 R_3 \ldots \ldots \ldots R_j} Q_1^{(s)} \tag{3.16}$$

$$Q_1^{(s)} = \left[1 + \frac{X_1}{R_2} + \frac{X_1 X_2}{R_2 R_3} + \ldots\ldots + \frac{X_1 X_2 \ldots X_Z}{R_2 R_3 \ldots R_{Z+1}} \right]^{-1} \tag{3.17}$$

However, for time $t \gg 10^{-9}$ sec after the pulse duration, a complete thermal equilibrium is expected to be established. Then, we have the well known Saha equations (Saha 1921) for the densities in the ground state:

$$\mathcal{N}_{j+1}^{T} = \frac{g_{j+1}}{g_j} \mathcal{N}_j^T n_e 2 \left(\frac{2\pi mkT}{h^2}\right)^{3/2} \exp(I_j/kT) \tag{3.18}$$

where g_{j+1} and g_j are statistical weights of the ground levels. In the problem considered in this paper, the most relevant expressions for the population densities are given by equations (3.13)–(3.15).

4. Population inversion and gain in ions isoelectronic to neon

It is relatively easy to obtain significant population inversions for a large number of optical transitions in neon gas, with or without He (Allen and Jones 1967). Neon has $1s^2\ 2s^2\ 2p^6\ {}^1S_0$ configuration in the ground state. The excited levels with configurations of $2p^5\ 3s$, $2p^5\ 3p$, $2p^5\ 4s$, etc., are best described in the *j-l* coupling scheme (Allen and Jones 1967) of Racah, but some of the relevant laser levels may also be described in the L-S coupling scheme. In order not to go into these usual details, in this paper the groups of states will be denoted by their atomic configurations alone. In particular, the population inversion between a pair of levels involving the 4s→3p transition will be considered here, since such a laser operating at 1·15 μ has been obtained without adding any helium to the neon gas. In the L-S coupling scheme this is the $4s\ {}^1P_1 \rightarrow 3p\ {}^3P_2$ transition. The physical mechanism (Faust and McFarlane 1964) involved is the electron-impact excitation of the metastable 4s level from the ground state, with a subsequent laser transition to the 3p level, which very quickly decays to the 3s level. The electron-impact excitation of the 3p level is insignificant in the beginning because it involves a two-step process via the excitation of the 3s level from the ground state. Figure 1 shows the energy level scheme and important transitions

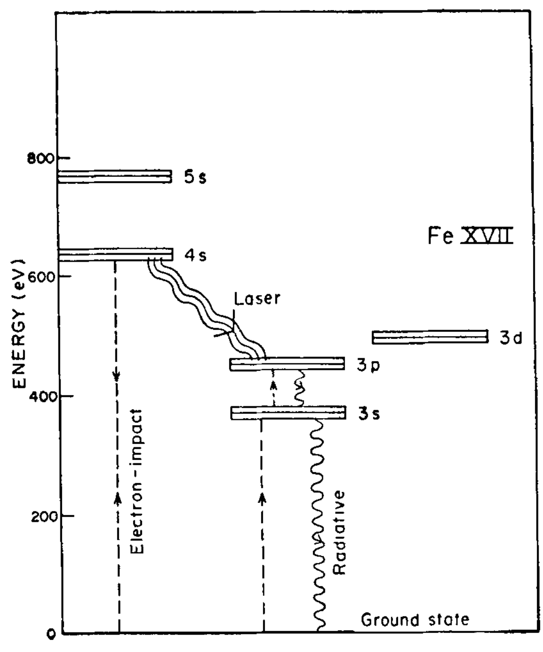

Figure 1. Average energies of several atomic configurations (Shadmi and Kastner 1971) in Fe XVII, isoelectronic to neon. Important electronic and radiative processes are also shown.

Table 1. Ionization rates in Fe ($n_e = 10^{21}$ cm^{-3})

Ionization stage j	Ionization energy I_j (eV)	X_j (10^{12} sec^{-1})	Ionization stage j	Ionization energy I_j (eV)	X_j (10^{12} sec^{-1})
1	7·87	300	10	262	3·0
2	16·20	100	11	290	2·0
3	30·7	60	12	331	1·5
4	54·8	30	13	361	1·0
5	75·0	20	14	392	0·6
6	99·0	10	15	457	0·4
7	125	8	16	489	0·2
8	151	6	17	1266	0·08
9	235	4			

for different configurations in Fe XVII. Since exact values for all the energy levels are not known, the approximate average values for different configurations as calculated by Shadmi and Kastner (1971) have been used.

In any ion in the plasma which is isoelectronic to neon, one expects a physical mechanism similar to the one discussed above for neon to be operative, leading to the population inversion between its 4s and 3p levels. If $\Delta\nu$ is the (Lorentzian) broadening of the spontaneous line, mostly the Stark broadening in the plasma, the gain coefficient α for a pair of laser levels $(2\rightarrow1)$ in the 4s$\rightarrow$3p configurations can be obtained from the well known expression

$$\alpha = 10(\ln e)\left[\frac{\lambda^2}{4\pi^2}\frac{A_{21}}{\Delta\nu}\left(n_2 - \frac{g_2}{g_1}n_1\right)\right] \text{db cm}^{-1} \tag{4.1}$$

where A_{21} is the spontaneous decay rate for the laser transition, n_2 and n_1 are the population densities in the upper and lower laser levels, and g_2 and g_1 are their statistical weights. In particular, for the 4s $^1P_1 \rightarrow$ 3p $^3P_2$ transition, $g_1 = 3$ and $g_2 = 5$.

For short times, neglecting the build up of the lower laser level, the rate equation for n_2 is given by

$$\frac{dn_2}{dt} = x_{g2}N_{Lg} - \tilde{A}_{21}n_2 \tag{4.2}$$

$$\tilde{A}_{21} \equiv A_{21} + X_{2g} = A_{21} + X_{g2}\exp(E_{2g}/kT) \simeq A_{21}, \tag{4.3}$$

where N_{Lg} is the ground state density of the ions isoelectronic to neon, labelled by $j = \mathrm{L}$, and where X_{g2} is the electron-impact excitation rate

$$X_{g2} = \langle\sigma_e(g\rightarrow2)v\rangle_L\, n_e \tag{4.4}$$

This excitation rate can be calculated from eq. (3.2). Since for short times, $N_{Lg}(t)$ is assumed to be given by equations (3.13)–(3.15), the solution of eq. (4.2) is

$$n_2 = X_{g2}N\sum_{i=1}^{L} d_{Li}\,\frac{[\exp(-X_it) - \exp(-\tilde{A}_{21}t)]}{(\tilde{A}_{21} - X_i)} \tag{4.5}$$

Based on Lötz's semiempirical results (Lötz 1969) X_j and I_j, for $j = 1$ to 17, are tabulated for Fe in table 1. This allows the calculation of $d_{17,i}$ from equations (3.14) and (3.15), representing the ladder ionization shown in figure 2. Similar numbers for $d_{L,i}$

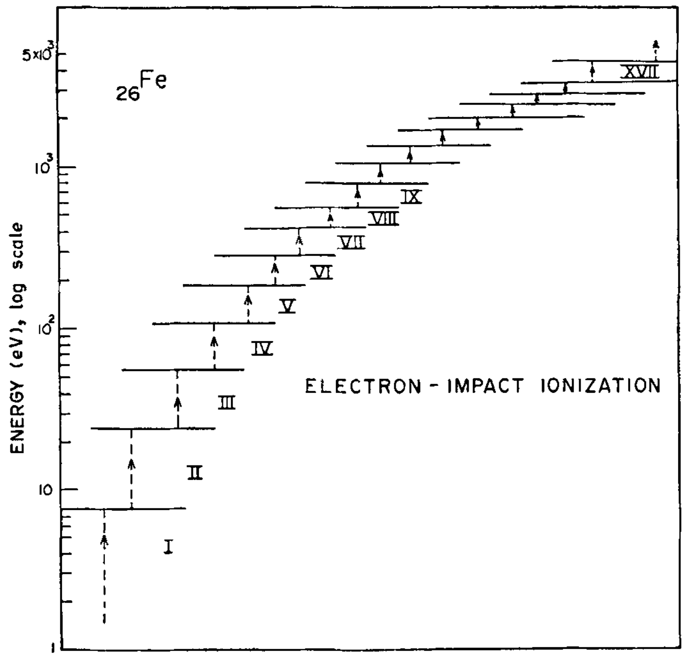

Figure 2. The ladder ionization process in Fe.

Table 2. Calculations of Gain/Inversion

Ion (Iso-electronic to neon)	Average $E_{4s}-E_g$ (eV)	Average $E_{4s}-E_{3p}$ (eV)	$\lambda(4s\to 3p)$ (Å)	X_{g2} (10^{12} sec^{-1})	A_{21} (10^{12} sec^{-1})	$\alpha\left[n_2-\frac{g_2}{g_1}n_1\right]^{-1}$ (db cm^2)
Fe XVII	639	188	66	0·04	0·7	$3{\cdot}5\times10^{-17}$
Co XVIII	729	204	61	0·03	0·8	$3{\cdot}0\times10^{-17}$
Cu XX	927	261	47	0·02	1·4	$3{\cdot}5\times10^{-17}$

are obtained for $_{27}$Co and $_{29}$Cu. Using equations (3.2), (4.4) and (3.4), we give in table 2 the values of X_{g2} and A_{21} for excitations in Fe XVII, Co XVIII and Cu XX, where the average excitation energies of the different configurations in each ion are taken from Shami and Kastner (1971). If the Stark broadening $\Delta\nu$ is assumed (Griem 1964) to be 10^{15} sec^{-1}, and n_1 is neglected for short times, the gain coefficient α may be calculated from Eqs. (4.1), (4.3) and (4.5). With $\mathcal{N}=10^{20}$ cm^{-3} and the pumping time of the order of five picoseconds, for each of the elemental plasmas under consideration, one gets

$$\alpha \simeq 10^2 \text{ db cm}^{-1} \tag{4.6}$$

Much higher values for the pumping time are not very useful because of the inevitable build up of the lower laser level in the 3p configuration. In any case, Eq. (4.6) represents a significant gain in the soft x-ray region.

5. Conclusions

From the considerations in the preceding Sections it is clear that theoretically it is possible to obtain a population inversion in the soft x-ray region by rapidly generating enough ions isoelectronic to neon in a high Z-plasma. This can be done by first using a relatively weak laser pulse to produce a low density hot plasma followed by a strong shorter pulse of a few picoseconds duration. For the case of Nd-glass laser, the critical density of the electrons in the plasma which can be heated to a sufficient degree turns out to be 10^{21} cm^{-3}. For solid targets like $_{26}$Fe, $_{27}$Co and $_{29}$Cu, the gains for stimulated emissions of the 4s→3p isoelectronic neon transitions are of the order of 10^2 db cm^{-1}, if the absorbed energy flux is of the order of 10^{21} ergs cm^{-2} sec^{-1}, with the pumping time of the order of five picoseconds.

All our conclusions based on the short-time analytic solution of the relevant rate equations could be criticized on the grounds that the assumptions about the constant density of the electrons and the Maxwellian velocity distributions for them are not correct. That in the picosecond regime the electronic distribution must be non-Maxwellian, is a valid criticism. However, a highly peaked non-Maxwellian distribution is in fact more favourable for obtaining higher gains, and it only shows that the estimates made here are perhaps on the lower side. The density of the electrons being constant in time during the pumping time of several picoseconds is also not rigorously true. But this is not expected to change the main conclusions. More exact quantitative results may be obtained by solving the rate equaitons numerically. In any case, the proposal described in this paper lends itself to experimental verification if one can detect either directly or indirectly time-resolved x-ray emission spectrum from the plasma.

References

Allen L and Jones D G C 1967 *Principles of Gas Lasers* (Butterworths & Co. Ltd, London) Chap. 6

Basov N G and Kroklin O N 1964 *Sov. Phys.–JETP* **19** 123

Bristow T C, Lubin M J, Forsyth J M, Goldman E B and Soures J M 1972 *Opt. Commun.* **5** 315

Caruso A and Gratton R 1968 *Plasma Physics* **10** 871

Dawson J, Kaw P and Green B 1970 *Phys. Fluids* **12** 875

Elton R C 1970 in *Methods of Experimental Physics*, Vol. 9A ed H R Griem and R Lovberg (New York: Academic Press)

Engelhardt A G, George T V, Hora H and Pack J L 1970 *Phys. Fluids* **13** 212

Faust W L and McFarlane R A 1964 *J. Appl. Phys.* **35** 2010

Griem H R 1964 *Plasma Spectroscopy* (New York: McGraw Hill)

Hodgson R T and Dreyfus R T 1972 *Phys. Lett. A* **38** 213

Kepros J G, Eyring E M and Cagle F W 1972 *Proc. Nat. Acad. Sci. USA* **69** 1744

Korobkin V V, Mandel'shtam S L, Pashinin P P, Prokhindeev A V, Prokhorov A M, Sukhodov N K and Shchelev M Ya 1968 *Sov. Phys.–JETP* **26** 79

Lötz W 1969 *Z. Phys.* **220** 446

Mallozzi P 1971 Symposium on Fundamental and Applied Laser Physics, Esfahan, Iran (unpublished)

McWhirter R W P 1965 in *Plasma Diagonostic Techniques*, ed R H Huddlestone and S L Leonard (New York: Academic Press)

Mulser P, Sigel R and Witkowski S 1973 *Phys. Rep. C* **6** 3

Rudge M R H 1968 *Rev. Mod. Phys.* **40** 564

Saha M N 1921 *Proc. Roy. Soc. London* **99** 135

Shadmi K and Kastner S O 1971 *Tables for average energies of ground and singly and doubly excited configurations in highly ionized atoms*