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Scaling in prong-number distribution in $\bar{p}p$ collisions

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Abstract. The prong-number distributions in $\bar{p}p$ collisions at relatively low laboratory momenta ($\lesssim 7$ GeV/c) exhibit the same type of scaling which has been observed in pp and π^-N collisions at momenta $\gtrsim 50$ GeV/c.

Keywords. Prong-number; scaling law; leading particle effects.

1. Introduction

Recently Koba, Nielsen and Olesen (1972) have argued that the prong-number distributions in high energy hadron collisions, should obey a simple scaling relation when expressed in terms of the variable $n/\langle n\rangle$. Subsequently Slattery (1972, 1973) observed that the experimental data on pp inelastic collisions at laboratory momenta above 50 GeV/c in fact obey this scaling remarkably well. In this paper we point out that the $\bar{p}p$ inelastic collisions at relatively low values of laboratory momenta ($\lesssim$ 7GeV/c) exhibit this scaling, and that the points of $\bar{p}p$ data fall on the very same 'scaled' curve obtained for pp collisions at momenta $\gtrsim$ 50 GeV/c.

2. Evidence for Scaling

The normalized distribution function $Q_n(s)$ for creating n charged particles when the total energy in the C M system is $\sqrt{s}$, is given by

$$Q_n(s) = \frac{\sigma_n}{\sigma_{inel}}, \quad \sum_n Q_n = 1 \tag{1}$$

where σ_n is the partial cross section for the production of n-prongs. The scaling relation envisaged by Koba *et al* (1972) is

$$Q_n(s) = \frac{1}{\langle n\rangle} \Psi\left(\frac{n}{\langle n\rangle}\right) \tag{2}$$

where $\langle n\rangle$ is the average prong number,

$$\langle n\rangle = \sum_n n\, Q_n(s)$$

As mentioned earlier, Slattery (1972, 1973) has shown that the available data on pp collisions follow the relation (2) at laboratory momenta starting from 50 GeV/c onwards. In addition, Buras and Koba (1973) noted that the recent data on π^-p

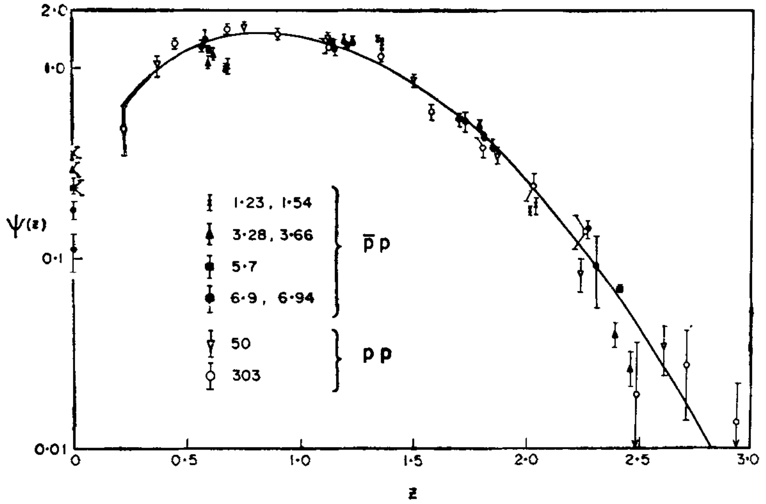

Figure 1. Plot of the function Ψ, defined by equation (2), versus $z=n/<n>$ for $\bar{p}p$ and pp inelastic reactions at various laboratory momenta. The data points of $\bar{p}p$ are taken from the literature (table 1). The continuous curve is the expression in equation (3) given by Buras and Koba (1973) as a fit to the high energy pp and π^-N data. The pp data at 50 GeV/c and 303 GeV/c are plotted for comparison only.

collisions at 40 GeV/c and 50 GeV/c and π^-N data at 40 GeV/c also exhibit the above scaling relation and follow the same scaling curve as for pp. The following simple function* representing reasonably accurately the data on pp and π^-N prong distributions above 40 GeV/c has been constructed by Buras and Koba (1973):

$$\Psi_{BK} = az\,(1+bz^2)\exp(-cz^2) \tag{3}$$

$$z = \frac{n}{<n>};\ a=2{\cdot}926,\ b=0{\cdot}417,\ c=1{\cdot}028$$

We find that the scaling observed in pp and π^-N seems also to be valid in the case of $\bar{p}p$ at surprisingly low momenta. Consider the prong-number distributions in $\bar{p}p$ inelastic collisions of the type $\bar{p}p\to n$ stable charged particles, where for $n=2$ we exclude the elastic scattering events. The function Ψ defined by eq. (2) is plotted in figure 1 for the available data (table 1) at p_{lab} = 1·23, 1·54, 3·28, 3·66, 5·7, 6·90 and 6·94 GeV/c as a function of z. At these momenta the observed values of the prong number n range from 0 to 8. The smooth curve plotted in figure 1 is the function given by eq. (3) representing the pp and π^-N data. It is very significant that the $\bar{p}p$ prong-number distributions appear to be already in the scaling region at laboratory momenta as low as 5-7 GeV/c and follow the smooth curve which is a good fit to the pp and π^-N data.

A data point at $z=0$ in figure comes from the zero-prong event in the case of $\bar{p}p$ annihilations whereas in the case of pp this point will be reached only as $<n>\to\infty$ or $s\to\infty$. For this reason, the $z=0$ point should be excluded in comparing the $\bar{p}p$ with the pp data. The approach to the scaling of the $\bar{p}p$ data can be seen clearly from the chi square values obtained from the $\bar{p}p$ data points as compared with the function given in eq. (3); χ^2=190, 33, 27, 39, 19, 5 and 0·4 at p_{lab}=1·23, 1·54, 3·28,

*A function of this form for fitting the topological cross sections has been considered originally by Bozoki *et al* (1969).

Table 1. $\bar{p}p$ data on prong distribution

GeV/c	Reference	GeV/c	Reference
1·23 and 1·54	Cooper *et al* (1968, 1970)	5·7	Alles-Boreli *et al* (1966)
3·28 and 3·66	Ferbel *et al* (1965)		Boeckmann *et al* (1966)
	Dehne *et al* (1964)		Fridman *et al* (1968)
6·94	Ferbel *et al* (1968)	6·9	Kitagaki *et al* (1968)

See also Enstrom *et al* (1972) for a general compilation of the data.

3·66, 5·7, 6·9 and 6·94 GeV/c, respectively (the number of data points is 3 for 1·23 and 1·54 GeV/c, and 4 for each of the higher momenta, excluding the $z=0$ point).

These values of χ^2 are large for the following reasons: the available experimental data have large systematic errors and do not agree among themselves as can be seen for instance from the data at $p_{\text{lab}}=6{\cdot}9$ and 6·94 GeV/c. The values of χ^2 here represent merely how well the data agree with the simple function of Buras and Koba (1973), rather than with the pp data. The function Ψ_{BK} given in eq. (3), it should be remembered, reproduces the basic features of the pp and π^-N data rather than the details; e.g. the pp data at 303 GeV/c (Dao *et al* 1972) when compared with Ψ_{BK} yields $\chi^2=19$ for 13 data points. Therefore the absolute values of χ^2 as such are not significant for our considerations but the systematic tendency for these values to decrease as we go to larger momenta is noteworthy.

3. Discussion

The early onset of scaling in $\bar{p}p$ can be understood if we suppose that the mechanism of particle production, and hence $Q_n(s)$, depends only on the effective energy W available for the creation of particles. At low momenta since annihilation is the dominant process in $\bar{p}p$ collisions, all the CM energy is available for particle creation; $W=\sqrt{s}$. In the pp case, on the other hand, only a certain fraction $\gamma(s)$ of $(\sqrt{s}-2M)$ is available for particle production because the incident particles after collision emerge with a sizeable fraction of their initial CM energy; $W(s)=\gamma(s)(\sqrt{s}-2M)$. The onset of scaling can then be viewed as the near independence of $\langle n\rangle Q_n(s)$ on the variable W. If we assume that $p_{\text{lab}} \simeq 7$ GeV/c is the momentum beyond which $\bar{p}p$ scales, then $W=\sqrt{s} \simeq 4$ GeV; and if $p_{\text{lab}} \simeq 50$ GeV/c is the corresponding threshold for scaling in pp, $W=\gamma(\sqrt{s}-2M) \simeq 8\gamma$ GeV. In order to have the same available energy W in both the collisions we require $\gamma \simeq 0{\cdot}5$, which is a reasonable value for the inelasticity parameter. Further, the difference between the observed values $\langle n\rangle=3{\cdot}5\pm0{\cdot}2$ for $\bar{p}p$ at 7 GeV/c and $\langle n\rangle=5{\cdot}32\pm0{\cdot}13$ for pp at 50 GeV/c (Ammosov *et al* 1972) can also be viewed (Fry *et al* 1973) as arising from the average number of leading particles ($1{\cdot}8\pm0{\cdot}2$). As we go to higher laboratory momenta ($\sim$ 50 GeV/c) leading particle effects will be present in $\bar{p}p$ also, and we expect the scaling for $\bar{p}p$ to follow that of pp in that region. It is probable that in a small range of laboratory momenta, above the momenta where scattering into annihilation channels dominates ($\gtrsim$ 10 GeV/c) and below the momenta ($\lesssim$ 50 GeV/c) where leading particle effects become prominent, there may be breakdown of scaling.

If W is the relevant variable for describing the prong-number distributions, then the scaling behaviour should also be evident in $e^+e^- \rightarrow$ hadrons at $\sqrt{s} \gtrsim 4$ GeV. The available data from storage rings are, however, not sufficient to make a meaningful quantitative comparison, although the trend of the existing data on e^+e^- collisions (Ceradini *et al* 1972) at $\sqrt{s} \simeq 2$ GeV does show some evidence for the scaling of Ψ.

4. Conclusion

In conclusion, we have observed that the prong number distributions in $\bar{p}p$ inelastic reactions at relatively low laboratory momenta ($\lesssim 7$ GeV/c) follow the scaling law, equation (2) conjectured originally by Koba, Nielsen and Olesen for high energy hadron collisions, and verified in pp collisions and π^-N collisions at laboratory momenta above 50 GeV/c. The observations of Slattery (1972, 73), and Buras and Koba (1973) combined with the evidence presented here for the $\bar{p}p$ collisions point towards the universality of scaling in multiple production of charged particles in high energy inelastic collisions.

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References

Alles-Borelli V, French B, Frisk A and Michejda L 1966 *Nuovo Cimento A* **46** 438

Ammosov V V, Boitsov V N, Ermolov P F, Fenyuk A B, Gorichev P A, Kistenev E P, Klimenko S V, Manyukov B A, Moiseev A M, Sulyaev R M, Tarasevich S V, Vorobjev A P, Blumenfeld H, Derre J, Granet P, Jabiol M A, Leveque A, Loret M, Pauli E, Pons Y, Prevost J, Schener J C, Boratav M, Laberrique J, Nguyen H K, and Orenstein S, 1972 *Phys. Lett. B* **42** 519

Boeckmann K, Nellen B, Paul E, Wagini B, Borecka I, Diaz J, Heeren U, Liebermeister U, Lohrmann E, Raubold E, Soding P, Wolff S, Kidd J, Mandelli L, Mosca L, Pelosi V, Ratti S, and Tallone L 1966 *Nuovo Cimento A* **42** 954

Bozoki G, Gombosi E, Posch M and Vanicsek L 1969 *Nuovo Cimento A* **64** 881

Buras A J and Koba Z 1973 *Nuovo Cimento Lett.* **6** 629

Ceradini F, Santonico R, Conversi M, Paoluzi L, Grilli M, Iarocci E, Spillantini P, Valente V, Visentin R, and Nigro M 1972 *Phys. Lett. B* **42** 501

Cooper W A, Hyman L G, Manner W, Musgrave B, and Voyvodic L 1968 *Phys. Rev. Lett.* **20** 1059

Cooper W A, Hodges D, Hyman L, Jach T, Loken J, Manner W, Musgrave B, and Royston R 1970 *Nucl. Phys. B* **16** 155

Dao F T, Gordon D, Lach J, Malamud E, Meyer J, Poster R, and Slater W 1972 *Phys. Rev. Lett.* **29** 1627

Dehne H C, Lohrmann E, Ranbold E, Soding P, Teucher M W, and Wolf G 1964 *Phys. Rev. B* **136** 843

Enstrom J E, Ferbel T, Slattery P F, Wermer B L, Guiragossian Z G T, Sumi Y, and Yoshida T May 1972 *$\bar{N}N$ and $\bar{N}D$ Interactions—A Compilation*, Lawrence Berkeley Laboratory Report LBL–58

Ferbel T, Firestone A, Sandweiss J, Taft H D, Gailloud M, Morris T W, Bachman A H, Baumel P, and Lea R M 1965 *Phys. Rev. B* **137** 1250

Ferbel T, Johnson J A, Kraybill H L, Sandweiss J, and Taft H D 1968 *Phys. Rev.* **173** 1307

Fridman A, Maurer G, Michalon A, Oudet J, Schiby B, Strub R, Voltolini C, and Cuer P 1968 *Phys. Rev.* **167** 1268

Fry J R, Grossman P, Mason P, Muirhead M, Brankin C, Matthews R, Angelopoulos A, Apostolakis A, Michaelidis P, Stergiou A, Theocharopoulos P, Filippas T A, Simopoulou E, Tsilimigras P, Vayaki A, Zevgalatokos E, Hoedl J, Kuhn D, Markytan M, Schmid P, and Strauss J 1973 *Phys. Lett. B* **43** 146

Kitagaki T, Takahasi K, Tanaka S, Sato T, Yamaguchi A, Hasegawa K, Sugawara R, and Tamai K 1968 *Phys. Rev. Lett.* **21** 175

Koba Z, Nielsen H B, and Olesen P 1972 *Nucl. Phys. B* **40** 317

Slattery P 1972 *Phys. Rev. Lett.* **29** 1624; 1973 *Phys. Rev. D* **7** 2073