Scaling in prong-number distribution in $\bar{p}p$ collisions

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Abstract. The prong-number distributions in $\bar{p}p$ collisions at relatively low laboratory momenta ($\lesssim 7 \text{ GeV}/c$) exhibit the same type of scaling which has been observed in pp and $\pi^-\text{N}$ collisions at momenta $\gtrsim 50 \text{ GeV}/c$.

Keywords. Prong-number; scaling law; leading particle effects.

1. Introduction

Recently Koba, Nielsen and Olesen (1972) have argued that the prong-number distributions in high energy hadron collisions, should obey a simple scaling relation when expressed in terms of the variable $n/<n>$. Subsequently Slattery (1972, 1973) observed that the experimental data on pp inelastic collisions at laboratory momenta above 50 GeV/c in fact obey this scaling remarkably well. In this paper we point out that the $\bar{p}p$ inelastic collisions at relatively low values of laboratory momenta ($\lesssim 7\text{ GeV}/c$) exhibit this scaling, and that the points of $\bar{p}p$ data fall on the very same 'scaled' curve obtained for pp collisions at momenta $\gtrsim 50 \text{ GeV}/c$.

2. Evidence for Scaling

The normalized distribution function $Q_n(s)$ for creating $n$ charged particles when the total energy in the c.m. system is $\sqrt{s}$, is given by

$$Q_n(s) = \frac{\sigma_n}{\sigma_{in\ell}} \sum_n Q_n = 1$$

(1)

where $\sigma_n$ is the partial cross section for the production of n-prongs. The scaling relation envisaged by Koba et al (1972) is

$$Q_n(s) = \frac{1}{<n>} \Psi \left( \frac{n}{<n>} \right)$$

(2)

where $<n>$ is the average prong number,

$$<n> = \sum n \frac{Q_n(s)}{n}$$

As mentioned earlier, Slattery (1972, 1973) has shown that the available data on pp collisions follow the relation (2) at laboratory momenta starting from 50 GeV/c onwards. In addition, Buras and Koba (1973) noted that the recent data on $\pi^-p$
collisions at 40 GeV/c and 50 GeV/c and π⁻N data at 40 GeV/c also exhibit the above scaling relation and follow the same scaling curve as for pp. The following simple function* representing reasonably accurately the data on pp and π⁻N prong distributions above 40 GeV/c has been constructed by Buras and Koba (1973):

\[ \Psi_{BK} = az (1 + b z^2) \exp(-cz^2) \]  

where

\[ z = \frac{n}{\langle n \rangle}; \quad a = 2.926, \quad b = 0.417, \quad c = 1.028 \]

We find that the scaling observed in pp and π⁻N seems also to be valid in the case of \( \bar{p}p \) at surprisingly low momenta. Consider the prong-number distributions in \( \bar{p}p \) inelastic collisions of the type \( \bar{p}p \rightarrow n \) stable charged particles, where for \( n = 2 \) we exclude the elastic scattering events. The function \( \Psi \) defined by eq. (2) is plotted in figure 1 for the available data (table 1) at \( \rho_{lab} = 1.23, 1.54, 3.28, 3.66, 5.7, 6.90, \) and 6.94 GeV/c as a function of \( z \). At these momenta the observed values of the prong number \( n \) range from 0 to 8. The smooth curve plotted in figure 1 is the function given by eq. (3) representing the pp and π⁻N data. It is very significant that the \( \bar{p}p \) prong-number distributions appear to be already in the scaling region at laboratory momenta as low as 5.7 GeV/c and follow the smooth curve which is a good fit to the pp and π⁻N data.

A data point at \( z = 0 \) in figure comes from the zero-prong event in the case of \( \bar{p}p \) annihilations whereas in the case of pp this point will be reached only as \( \langle n \rangle \rightarrow \infty \) or \( s \rightarrow \infty \). For this reason, the \( z = 0 \) point should be excluded in comparing the \( \bar{p}p \) with the pp data. The approach to the scaling of the \( \bar{p}p \) data can be seen clearly from the chi square values obtained from the \( \bar{p}p \) data points as compared with the function given in eq. (3); \( \chi^2 = 190, 33, 27, 39, 19, 5 \) and 0.4 at \( \rho_{lab} = 1.23, 1.54, 3.28, \)

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* A function of this form for fitting the topological cross sections has been considered originally by Bozoki et al (1969).
Table 1. \(\bar{p}p\) data on prong distribution

<table>
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<tr>
<th>(\text{GeV/c} )</th>
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<tbody>
<tr>
<td>6.94</td>
<td>Dehne et al (1964)</td>
<td>6.94</td>
<td>Kitagaki et al (1968)</td>
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See also Enstrom et al (1972) for a general compilation of the data.

3.66, 5.7, 6.9 and 6.94 GeV/c, respectively (the number of data points is 3 for 1.23 and 1.54 GeV/c, and 4 for each of the higher momenta, excluding the \(z=0\) point).

These values of \(\chi^2\) are large for the following reasons: the available experimental data have large systematic errors and do not agree among themselves as can be seen for instance from the data at \(p_{\text{lab}}=6.9\) and 6.94 GeV/c. The values of \(\chi^2\) here represent merely how well the data agree with the simple function of Buras and Koba (1973), rather than with the \(pp\) data. The function \(\Psi_{\text{BK}}\) given in eq. (3), it should be remembered, reproduces the basic features of the \(pp\) and \(\pi^-N\) data rather than the details; e.g. the \(pp\) data at 303 GeV/c (Dao et al 1972) when compared with \(\Psi_{\text{BK}}\) yields \(\chi^2=19\) for 13 data points. Therefore the absolute values of \(\chi^2\) as such are not significant for our considerations but the systematic tendency for these values to decrease as we go to larger momenta is noteworthy.

3. Discussion

The early onset of scaling in \(\bar{p}p\) can be understood if we suppose that the mechanism of particle production, and hence \(Q_n(s)\), depends only on the effective energy \(W\) available for the creation of particles. At low momenta since annihilation is the dominant process in \(\bar{p}p\) collisions, all the cm energy is available for particle creation; \(W=\sqrt{s}\). In the \(pp\) case, on the other hand, only a certain fraction \(\gamma(s)\) of \((\sqrt{s}-2M)\) is available for particle production because the incident particles after collision emerge with a sizeable fraction of their initial cm energy; \(W(s)=\gamma(s)(\sqrt{s}-2M)\). The onset of scaling can then be viewed as the near independence of \(\langle n \rangle Q_n(s)\) on the variable \(W\). If we assume that \(p_{\text{lab}} \approx 7\) GeV/c is the momentum beyond which \(\bar{p}p\) scales, then \(W=\sqrt{s} \approx 4\) GeV; and if \(p_{\text{lab}} \approx 50\) GeV/c is the corresponding threshold for scaling in \(pp\), \(W=\gamma(\sqrt{s}-2M) \approx 8\gamma\) GeV. In order to have the same available energy \(W\) in both the collisions we require \(\gamma \approx 0.5\), which is a reasonable value for the inelasticity parameter. Further, the difference between the observed values \(\langle n \rangle =3.5 \pm 0.2\) for \(\bar{p}p\) at 7 GeV/c and \(\langle n \rangle =5.32 \pm 0.13\) for \(pp\) at 50 GeV/c (Ammosov et al 1972) can also be viewed (Fry et al 1973) as arising from the average number of leading particles (1.8 \pm 0.2). As we go to higher laboratory momenta (\(\approx 50\) GeV/c) leading particle effects will be present in \(\bar{p}p\) also, and we expect the scaling for \(\bar{p}p\) to follow that of \(pp\) in that region. It is probable that in a small range of laboratory momenta, above the momenta where scattering into annihilation channels dominates (\(\gtrsim 10\) GeV/c) and below the momenta (\(\lesssim 50\) GeV/c) where leading particle effects become prominent, there may be breakdown of scaling.

If \(W\) is the relevant variable for describing the prong-number distributions, then the scaling behaviour should also be evident in \(e^+e^-\rightarrow\text{hadrons}\) at \(\sqrt{s} \approx 4\) GeV. The
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available data from storage rings are, however, not sufficient to make a meaningful quantitative comparison, although the trend of the existing data on e+e− collisions (Ceradini et al 1972) at √s ≈ 2 GeV does show some evidence for the scaling of Ψ.

4. Conclusion

In conclusion, we have observed that the prong number distributions in pp inelastic reactions at relatively low laboratory momenta (≤ 7 GeV/c) follow the scaling law, equation (2) conjectured originally by Koba, Nielsen and Olesen for high energy hadron collisions, and verified in pp collisions and π−N collisions at laboratory momenta above 50 GeV/c. The observations of Slattery (1972, 73), and Buras and Koba (1973) combined with the evidence presented here for the pp collisions point towards the universality of scaling in multiple production of charged particles in high energy inelastic collisions.

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References