Proc. Indian Acad. Sci. (Math. Sci.) (2020) 130:45
https://doi.org/10.1007/s12044-020-00584-6

Dr C R Rao's contributions to the advancement of economic science

T KRISHNA KUMAR[1,*], H D VINOD[2] and SURESH DEMAN[3]

[1]Rockville Analytics, 13201 Carriage Court, Rockville, MD 20850, USA
[2]Fordham University, The Bronx, New York, NY 10458, USA
[3]Centre for Economics and Finance, P.O. Box 17517, London SE9 2ZP, UK
*Corresponding author.
E-mail: tkkumar@gmail.com; vinod@fordham.edu; s_deman2000@yahoo.co.uk

Abstract. In this paper, the authors review Dr C R Rao's contributions to statistical foundations in economic science, and the importance of his work in advancing econometric modelling and statistical inference in celebration of his birth centenary.

Keywords. Characterization of linear structural equations models; identification; estimability; asymptotic efficiency, information limit to variance and Rao–Cramér inequality; Rao score test; data analytics.

Mathematics Subject Classification. 00A06, 15A06, 60E10, 62F03, 62J05.

1. Introduction

Calyampudi Radhakrishna Rao (henceforth Dr Rao for brevity, the way several of his students had always addressed him) will be celebrating his birth centenary on September 10 of 2020. Dr Rao is an outstanding mathematician, statistician, computational methods expert, biometrician, psychometrician, and an econometrician. He is also popularly known as a "Living Legend in Statistics". The year 2020 also happens to be the 90th anniversary of the founding of the Econometric Society, the 75th year of the discovery of Cramér–Rao inequality, and the 60th anniversary of the Indian Econometric Society of which Dr Rao was the founding President.

We concentrate here on Dr Rao's contributions to the advancement of economic science through econometrics. Dr Rao followed the spirit and meaning of statistics as defined by P C Mahalanobis: "We are convinced that statistics represents a fundamental method of analysis of data …which is applicable to any science…., and we feel that it is desirable to emphasize this essential unity in the methodology of statistics" (Editorial in *Sankhya's* inaugural issue, 1933). Economics is to statistics like what physics is to mathematics; through its day-to-day problems economics poses such interesting questions for scientific analysis that it is a delightful playground for statisticians to apply their theories and methods. It is therefore not surprising that Dr Rao made contributions to econometrics. When Dr Rao received his US Medal of Science in 2002, the citation read: "…prophet of new age, for his pioneering contributions to the foundations of statistical theory and multi-

This article is part of the "Special Issue in Honour of Professor C R Rao on His Birth Centenary"

© Indian Academy of Sciences

Published online: 25 July 2020

variate statistical methodology and their applications, enriching the physical, biological, mathematical, economic and engineering sciences."

Dr C R Rao's contributions to the advancement of economic science are deep and wide, centered around statistical methods and theories used widely and extensively in econometrics. These contributions included providing an answer to a fundamental question in econometrics posed by Ragnar Frisch, the founding member of the Econometric Society. It is a question the answer to which provides the probabilistic foundations of linear econometric models. Those are the same type of models (linear structural equations) for building which Ragnar Frisch, Jan Tinbergen, Laurence Klein, and Trygve Haavelmo were awarded a Nobel Prize in economics more than two decades later. A system of linear structural equations is the workhorse of econometrics. Dr Rao was elected a Fellow of the International Econometric Society in 1972 in recognition of that work as well as his contributions in statistics on estimation and inference in statistical models, including major econometric models that emerged from the Cowles Foundation for Research in Economics (Economic Science). That problem posed by Frisch was solved by Dr Rao in 1943 [57] and published in *Econometrica*, the official journal of the International Econometric Society in 1947 [60].

That contribution of Dr Rao, created an entirely new branch of statistical theory, called the characterization of probability distributions. This way, Dr Rao has elevated the status of econometrics as a subject that not only applies statistical methods to economics but as a subject that contributed in advancing statistical theories and methods useful in other branches of science. The topic of characterization of probability distributions has a great potential for future research in econometric theory. Unfortunately, econometricians have not yet fully exploited that development initiated by Dr Rao. There are only a few applications of characterization theorems in economics, while mainstream economists remain unaware of them. (Vinod [89] and Krishnaji [36,37] are exceptions described later in this article.) Economists take for granted the probabilistic basis of econometrics, instead of crediting Dr Rao for this work. They simply assume that errors have a Gaussian distribution and the econometric models are linear.

Dr Rao's work on linear models, estimability of linear functions, maximum likelihood estimation, Fisher–Rao information and information-based limit to efficiency, Rao-Blackwell theorem, maximum likelihood estimation using the scoring method, Rao's score test, his contribution to orthogonal arrays in design of experiments, his contributions to multivariate statistical methods on dimension reduction through principal components, and grouping or clustering of data used in data analytics, and so forth, are all a part and parcel of a very wide spectrum of statistical tools used in econometrics. The economics profession does not give Dr Rao proper credit for all these contributions. The reason for this neglect seems to be that economists have simply assumed that all these statistical foundations used by econometricians constitute a common pool of statistical methods, long ago discovered by several statisticians. But, the fact of the matter is that Dr Rao single-handedly developed so many of these methods.

Apart from these fundamental contributions, Dr Rao has also done considerable legwork (Section 2 has further details) for promoting econometrics research by helping set up the *Indian Econometric Society* in the 1960s and co-editing many volumes of *Handbook of Statistics* focused on Econometrics. Only a small section of the economics profession specializing in econometrics has so far recognized Rao's work by honoring him on different occasions.

The statistical "design of experiments" tools play a crucial role in designing economic experiments (such as auction markets). for which Vernon Smith received a Nobel Prize in economics. The field experiments using randomized control treatments in development economics (for which a Nobel prize in economics was recently awarded to Abhijit Banerjee, Esther Duflo and Michael Kremer) have used Rao-inspired tools. An article in Forbes Magazine (March 11, 1969) refers to orthogonal arrays invented by Dr Rao as "a new mantra." That method provides an optimal combination of factors or treatments needed to isolate the main effects and interactions between the factors. It minimizes the number of experiments needed and thereby reduces the cost of conducting experiments. Orthogonal arrays developed by Dr. Rao were used and popularized in quality improvement by Taguchi [85] by achieving quality improvement at the design stage of production process itself. We cannot imagine any decent application of "design of experiments" that does not use Rao's orthogonal array designs to reduce the number of experiments and their costs.

To place Dr Rao's contributions in proper perspective, one may compare how deep and wide are the applications of the statistical procedures described above in advancing economic science, compared to single scientific tools whose applications to economics bestowed the Nobel Prize in economics to others. For example, compare Dr Rao's contributions described above with that of the application of game theory by John Nash, discrete choice and dichotomous dependent variable linear model of Daniel McFadden, specification bias problem of James Heckman [26] and randomized control trials of Abhijit Banerjee, Esther Duflo, and Michael Kremer. One therefore, wonders if giving the economics Nobel Prize to Dr Rao could enhance the status of economics Nobel Prize during this year of his birth centenary!

2. Rao's legwork for econometrics and Indian Econometric Society

Econometric society is an international body to promote econometrics. It was founded on December 29, 1930, in Cleveland, Ohio, during the joint meetings of the American Mathematical Society, American Statistical Association, and the American Economic Association. It is the first International Society in Economics, established to harness the fields of mathematics and statistics to advance economic science. Some of the founding members were Ragnar Frisch, Irving Fisher, Harold Hoteling, Walter Shewhart, Norbert Wiener, Joseph Schumpeter and Charles F Roos. Professor Ragnar Frisch of Norway was the chief architect of this activity, having already started his activities in this direction in Europe, and seeking support from Irving Fisher of Yale University through a visit to Yale, arranged by the Rockefeller Foundation. The econometric society started publishing its own journal, *Econometrica*, from 1933, with a generous grant from a very successful stockbroker, Alfred Cowles. Coincidentally, the Indian Statistical Institute (ISI) also started publishing *Sankhya* from 1933, and the very first issue of *Sankhya* had an article on economics.

The origins of Indian econometrics can be traced at least to December 1951, if not earlier, when the International Econometric Society held two joint sessions in India, one with the International Statistical Institute in New Delhi and the other with the Indian Economic Conference in Patna. A short session of the International Statistical Institute Conference was held at the Indian Statistical Institute in Calcutta from 16 to 18 December 1951 (by this time, Dr Rao had already published a paper in *Econometrica* [60] answering a question posed by Ragnar Frisch that would form the probabilistic foundations for deriving the autonomous or structural relations that are linear in parameters). The International Biometric Society, the International Econometric Society, the International Association for Research in Income and Wealth, the International Union for the Scientific Study of

Population, also held joint or associated meetings with the International Statistical Institute. The Indian Statistical Institute acted as the host for all these activities. Both Mahalanobis and Dr Rao were instrumental in bringing that activity to India.

In January 1960, the Indian Science Congress held its annual conference with the theme "The Impact of Society on Science" at Mumbai. Dr C R Rao chaired a special symposium on "Statistical Methods in Economics" at the Mathematics Division of Indian Science Congress. Pioneers of Indian econometrics such as P C Mahalanobis, C R Rao, K Nagabhushanam, N S R Sastry (the then Head of statistical research at the Reserve Bank of India), M Mukherjee and N Sreenivasa Iyengar attended this symposium and laid the foundation for the establishment of an organized programme devoted exclusively to econometrics. These Indian members of the Econometric Society organized the First Indian Econometric Conference at the Indian Statistical Institute, Calcutta, in January 1960. P C Mahalanobis, who was the first elected Indian member of the Executive Council, and the first elected Indian Fellow (1951) of the International Econometric Society, delivered the inaugural address. The Indian Econometric Society was established in 1960 (although not registered until 1970), with Dr C R Rao as the President. Dr Rao presided over the Indian Econometric Society for 15 years from 1960–1975 and shaped its activities. Dr. Rao [70,75] encouraged new research in economic development.

3. Dr Rao's further role in advancing econometrics within India

Statistics is both a mathematical and applied science. For its development, it requires problems of application as motivating factors, and for its applications, it requires such problems and data. Economics presents a variety of challenging situations and problems for statisticians to apply statistical methods and also to develop new statistical theories and methods. The collection of reliable economic data is always quite central to the development of econometrics. The Prime Minister of India, Jawaharlal Nehru, appointed Mahalanobis as the Statistical Advisor to him in 1949 and requested him to develop the statistical data infrastructure needed for development planning. Dr Rao joined ISI as a professor at the age of 28 in 1948 and soon became the Director of the Research and Training School at ISI. Mahalanobis was the man of ideas to generate funds for ISI, and Dr Rao was the person to help him develop and implement those schemes. Thus Dr Rao served as a member of several Government committees for the development of national statistical systems, statistical education, and research in India. Some of them are, Chairman of the Committee on Statistics (1962–1969), Chairman of the Committee on Demographic and Communication for Population Control (1968–1969). He served as an advisor to the Director of Statistics at the Reserve Bank of India. In association with Mahalanobis, Dr Rao was instrumental in developing a nation-wide network of statistical officers to develop economic statistics. The first doctoral research in econometrics conducted at the Indian Statistical Institute by N. Srinivas Iyengar, published in Econometrica, was jointly supervised by Dr Rao and Professor Mahalanobis.

Ragnar Frisch laid the foundations for econometric research (for a history of econometrics and mathematization of economics, one may see [7–11,14,21,25,27,38,39,49] and [56]). He did extensive data analysis of economic data in 1926–28, employing scatter diagrams and regressions. He wished to determine, from the observed data, the underlying pattern, the theoretical model. Frisch advocated both exploratory econometric modeling and confirmatory econometric modeling. But in order to fully exploit statistical theory, he posed the question: How does probability enter in the econometric model building? Frisch assumed that all economic “laws” are deterministic by definition, and they become statistical regression equations because the economic variables are measured with errors, and such errors have probability distributions with mean zero and a constant (and hopefully low) variance. Hence econometric modeling started with an “errors in variables” model.

Frisch observed that variables can be expressed as endogenous and exogenous, where endogenous variables are those that are determined or explained by the statistical model, while the exogenous variables are those variables that are determined outside the model. It is natural, as a first step, to express each of the endogenous variables as a linear function of all exogenous variables (the so-called reduced form). Likewise, it is also natural to assume that there would be as many linear (structural) equations as there are endogenous variables. It is this later notion that Frisch had in mind when he coined the word “multicollinearity”.

Suppose we represent the system linear structural economic relations as follows:

$$Ay + Bz = u, \tag{1}$$

where y is a $G \times 1$ vector of G endogenous variables being explained through K exogenous variables given by the $K \times 1$ vector z, by G structural economic relations described above. Now, A being a $G \times G$ matrix, B being a $G \times K$ matrix, u being a set of G errors in equations (assumed to be *normally* distributed) let us rewrite them so as to move all the endogenous variables on to the left hand side as follows:

$$y = -A^{-1}Bz + A^{-1}u, \tag{2}$$

$$y = \Gamma z + v, \tag{3}$$

where y is a $G \times K$ matrix. It can be easily verified that even a deterministic model with errors in variables can be expressed in this form.

Various issues that arise with this specification of an econometric model are:

(i) Under what conditions on the distributions of random errors are the underlying equations, both structural and reduced form, are linear in parameters?
(ii) How do we “discover” autonomous or structural relations (the relationship between endogenous variables) given the reduced form equations (where all endogenous variables are expressed as linear functions of all exogenous variables)?
(iii) How do we estimate the reduced form equations?
(iv) How do we estimate the parameters of the autonomous or structural equations?
(v) What are the properties of those estimators, both in small and in large samples?
(vi) How do we choose between two alternate models?
(vii) How do we test hypotheses on the parameters of the model?

This paper reviews Dr Rao’s contributions to economic science on all the issues listed above, and a few other issues such as the statistical design of economic experiments, reduction of dimensions of data through principal component analysis and other data analytic methods and advancing econometrics knowledge through a series of advanced research monographs in econometrics. Our review of his work also indicates a rich future potential of his contributions in advancing economic science, as some of his contributions have not been fully exploited by economists.

4. Probabilistic foundations of econometrics and the impact of Dr Rao's work

As shown above, the system of linear structural equations is the workhorse of an econometric model, and Dr Rao's contributions to its specification, efficient estimation, the estimability (identifiability) of parameters, and testing of hypotheses in such linear models had a significant impact on the evolution of econometric research. In order to justify the specification of a linear regression between quantity and price, as either a demand function or as a supply function, *Frisch* posed the following *question*:

There are two endogenous variables x_0 and x_1, and another exogenous variable ζ_1 satisfying the following reduced form equations:

$$x_0 = a_0\xi_1 + \eta_0, \tag{4}$$

$$x_1 = a_1\xi_1 + \eta_1, \tag{5}$$

where η_0 and η_1 are random variables. Does there exist a linear regression relation, linear in parameters, between x_0 and x_1?

Rao–Fix theorem [19,20,60], that addressed this question, is stated by Laha [41] as: If ξ_1, η_0, and η_1 are three mutually independent random variables with finite first moments, then the necessary and sufficient condition for the regression of x_0 on x_1 to be linear with $a_0 \neq 0$ and a_1 contained in a closed interval, is that η_0 and η_1 must have a 'stable law' with finite moments. Laha [41], working with Rao on his doctoral dissertation, proved the theorem by drastically weakening the assumptions of Rao–Fix theorem. He replaced the independence condition by noncorrelation of the unobserved variable and the corresponding equation errors and replaced independence between the observational and equation errors by allowing for some correlation between them. Laha [42] also extended the theorem when there are more than two measured variables giving rise to linear multiple regressions. The condition then he required was that the errors be normally distributed. Thus, Rao and his student Laha established probabilistic foundations for linear structural equation models in economics in 1943 and 1956 respectively.

Chikuse [13] extended Rao's approach to a multivariate setting where both dependent variables and independent variables of the linear regression are vectors. The condition she gets is that the random variables and errors should belong to the class of generalized stable laws. One might, therefore, say that Frisch's question and Dr Rao's answer provide the probabilistic foundations of a system of linear structural equations in economics. The characterization result of Dr Rao, that came from a problem arising in economics, led to an entire new sub-field in statistics on the characterization of probability distributions. Dr Rao co-authored a book on the subject published by the USSR Academy of Sciences [3]. The word characterization comes from Rao's application of a general method involving partial differential equations of characteristic functions.

Besides providing probabilistic foundations for linear structural economic relations, Rao [72] on the characterization of distributions is perhaps the most significant contribution for many research areas, including maximum entropy methods. The extensive literature on maximum entropy fails to cite Rao with few exceptions. Vinod [89] does cite Rao in describing his *maximum entropy density* leading to an *R* package called 'meboot' for easier statistical inference with ubiquitous nonstationary time series, especially in macroeconomics. Thus, characterization has been used to solve the question posed by Frisch and other foundational problems. (see [30,32,33] and [76]).

For a summary of work done by Rao on characterization problems, one may also see Chapter 6 of Prakasa Rao [55]. It is quite unfortunate that mainstream econometricians do not pursue this approach. Krishnaji [36,37], a founder member of the Indian Econometric Society, used the characterization approach to justify fitting a particular family of distributions to specific economic data. His work and Dr Rao's contributions remain obscure among econometricians and do not receive the credit they deserve. An application of this approach in the future could generate a whole new area of research in econometrics.

Let us briefly mention one lesson from Rao's characterization results relevant for future research in econometrics. Economic theory and reasoning often only suggest that certain variables appear in an equation with either a positive or a negative association with the dependent variable. It does not specify the functional form in which such variables should enter. Instead of blindly assuming a linear in variables model with normally distributed errors, researchers should, if possible, transform their variables until they follow a normal distribution or a distribution belonging to the generalized stable law. Then, they would satisfy the conditions of Rao's theorem and safely justify using a system of linear (in parameter) regression relations between the endogenous variables so transformed.

5. Rao's impact on inference for a system of linear structural economic relations

We note that the first set of equations (1) involves more than one stochastic variable in each equation, and hence any regression we write would have stochastic regressors. The second set of regressions (3) is called the reduced form equations (for original authentic discussion of linear structural economic relations constituting an econometric model, the reader may refer to [28,34,35] and [87]). Here each equation has one endogenous variable that is regressed on a set of exogenous or non-stochastic variables. The likelihood of y given Z can be written down, knowing that the u's are *normally* distributed. The first issue in econometric modeling is to make sure that the specification of the model is such that the parameters of the model are 'identifiable'. For a historic background of the notion of identification going back to Borel, see [51]. According to modern textbooks, identification means: 'given the reduced form parameter estimates we can arrive at the structural parameters uniquely.'

Multicollinearity is another major commonly encountered econometric problem under which the design matrix $X''X$ becomes singular. In 1954, Rao received some data from Japan to examine the long-term effects of radiation in Nagasaki on the victims. The design matrix, in that case, was singular, and he had to replace it with some other matrix that was not singular. This problem led Rao to develop the concept of generalized inverse in linear models [69]. Rao [77] further generalized the statistical inference results in the linear model when the design matrix is singular.

The issue of *estimability* of a linear function of a parameter vector is also related to the question of multicollinearity. If X is the design matrix of a regression, near-zero determinant of $(X'X)$ is called multicollinearity. According to Frisch, this problem is present in the population, and not due to sampling variation, per se. Dr Rao, on the other hand, defined that a linear function of the parameters is *not estimable* (usually due to collinearity) if its variance is infinity. Rao's formulation allows one to think of multicollinearity as a sampling problem while identification is the population problem, if one asks: 'what happens to the estimation of a parameter when the sampling variation is reduced to its minimum through an infinite sample?' Rao's answer brings in Fisher's information matrix in the picture and leads to the standard formulation of identification issue in econometrics.

Identification of a parameter is thus related to the asymptotic variance of the estimator of that parameter, and if that variance is infinite, the parameter is not identified. Rao provided econometricians the tools to address the issue of multicollinearity and identification by highlighting infinite variance issues in terms of the Fisher information matrix. While the identification in linear models is related to his notion of estimability of the parameter, general parametric identifiability in models that are nonlinear in parameters is related to the non-singularity of the information matrix. In that form, it is a vital component of parametric identification of Rothenberg [82], and partial identification of Phillips [54]. One may also see [40] for employing this concept to check the identification in nonlinear regressions, and [17] for conditions under which identification and estimability are one and the same. Thus, the concept of identification in linear models is a topic on which Dr Rao made important contributions (see [58,59]).

Once a linear structural equation system is justified the statistical inference can be carried out using a single equation approach, equation by equation, ignoring the possible correlations between equations. Thus, the unbiased estimation of parameters of Γ and testing hypotheses on them using maximum likelihood methods employed by econometricians depend extensively on Fisher [18] and its extensions by Dr Rao [61].

There are two approaches to doing this. Each of the G reduced form equations of (3) can be estimated separately. These reduced-form equations are estimated by using the standard linear estimation methods for which the small and large sample theories are developed by Rao and Fisher (see [67,68]). Most of the econometric software packages use Rao's method of scoring for the calculation of the maximum likelihood estimates (MLEs) (mentioned in [90]). From the estimated Γ matrix, the estimates of A and B have to be obtained. As the estimators of Γ are consistent, those of A and B also are consistent. One can also use the estimated variance–covariance matrix of the errors of these G separate equations and estimate the system of G equations using seemingly unrelated (but related) regressions [93]. This is again a variant of the standard linear model developed by Rao [73] with an estimated variance–covariance matrix.

Alternately, one can normalize the G equations in (1) and treat each of those as a structural economic equation and estimate each of them separately. But then some of the explanatory variables on the right-hand side are themselves determined by other structural equations and hence are stochastic, violating the assumptions of a standard linear model. Those variables can be replaced by their expected values by regressing them on all exogenous variables or X. That is called the Two-Stage Least Squares method of Basmann [1,2] and Theil [86].

Hood and Koopmans [28] and Koopmans [35] presented methods of estimating the linear structural equations employing the method of maximum likelihood. They used two different methods of maximum likelihood. In one method, each structural equation is estimated separately by maximizing a constrained likelihood function, the constraint being the constraints on its parameters needed to identify that equation. This is called the Limited Information Maximum Likelihood method (LIML). In the other method, all the structural equations are estimated simultaneously by maximizing the restricted likelihood function, the restrictions pertaining to all the constraints on the structural parameters needed for identification. This is called the Full Information Maximum Likelihood method (FIML).

The very well-known result in econometrics that the Full-Information Maximum Likelihood estimators have an asymptotic variance–covariance matrix that is dominated by the asymptotic variance–covariance matrix of the Limited-Information Maximum Likelihood estimator, follows from the basic results of Fisher and Rao on information limit to variance, and the dictum that 'the more the information, the less is the variance.'

As Bates and White [3] pointed out the search for minimum variance econometric estimators has benefitted from the work on minimum variance estimation by Fisher and Rao. It is, however, surprising that although the econometric models and methods use estimation and inference tools developed by Dr Rao, many of the concepts and tools developed by Dr Rao are not yet fully exploited in econometrics. For instance, we do not see any applications of the Rao–Blackwell theorem to improve the asymptotic efficiency of econometric estimators. Nor does one use the concepts of sufficiency to improve the efficiency of econometric estimators.

Statistical inference in econometric models is based on the contributions made by Rao on estimation and hypotheses testing [58,59,61,62,64,66–68,73]. Most of these articles are published in journals of great international repute and highly quoted in econometrics journals and books, for example, in an article published in *Econometrica*, Neyman and Scott [52] cited [58], and Neyman is the one who named the inequality Cramér–Rao inequality. The papers by Rao [58,59,61] cited above are reproduced in the publication, “Breakthroughs in Statistics”, 1889–1990 (100 years), edited by Johnson and Kotz [29].

While Cramér–Rao bound and Rao–Blackwellization are finite sample results, their full potential needs to be further exploited in small-sample econometrics. For example, there are several papers in econometric journals comparing the small-sample results of alternate econometric estimators such as OLS, indirect least squares, two-stage least squares, etc. As all of them refer to the same model with a specified distribution of error(s), it is possible to write down the likelihood function and estimate the small sample limit to the variance given by Rao and compare the distance of each estimator’s variance (or variance–covariance matrix) from Rao’s limit. Likewise, the Rao–Blackwellization contained in his 1945 paper also has not been exploited fully in econometrics to improve inefficient estimators, or to show that shifts to efficient estimators from inefficient ones are equivalent to some type of Rao–Blackwellization.

Likelihood ratio test (LR) (based on the ratio of the likelihood under the null hypothesis to the likelihood under the alternative), Wald test (W) (based on the statistical generalized distance of the estimate from the hypothesized value) and Rao’s score test (RS) based on the difference between the slope of the likelihood function from the hypothesized value of zero are three most commonly used large-sample tests in econometrics for testing of hypotheses.

Rao’s contribution to asymptotic tests includes the score test presented in his 1948 article [61]. It is the one most extensively used in econometrics. For a review and extensions of his score test, one may see [6]. The papers by Ghosh [22], Mukherjee [50], Bera and Bilias [5] have established the optimal properties of Rao’s score test compared to the Wald test, and Likelihood Ratio test. The Rao’s score test was rediscovered eleven years later by Silvey, which is known as the Lagrangean Multiplier (LM) test [83]. Many econometricians were using the RS test under the name of LM test. Its potential applications as a specification test or a model diagnostic test in econometrics were highlighted and popularized by Breusch and Pagan [12]. Such specification or model diagnostic tests play a very crucial role in statistical inference in economics as a result of a methodological debate between theoretic and a-theoretic models and the persuasive arguments made by Leamer [43] and Sims [84].

The year 1998 marked the Golden Jubilee of Rao’s score test that is extremely popular in econometrics as well as in other areas of application. To commemorate that occasion, a special invited paper session was organized by the joint meetings of the American Statistical Association and Institute of Mathematical Statistics at Anaheim, California in August 1997. Selecting a few papers from that conference, a special issue of the *Journal of Statistical*

Planning and Inference, Vol. 97, No. 1 (2001), was brought out devoted to Rao's score test. One of the editors of that special issue and several contributors to that issue are econometricians. This is what the preface to that special issue says:

"…*Rao's test will continue to have wide-ranging influence in years to come. Possibly it can be said that the eventual impact of this statistic will be comparable to that of Karl Pearson's goodness of fit test suggested in 1900*".

In that special issue of the journal, econometricians Gonzalez–Rivera and Ullah [23] showed how the Rao score test can be extended to the situation when the underlying distribution is nonparametric. As machine learning algorithms and nonparametric and semi-parametric methods are becoming increasingly important recently, we can expect Rao's score test to be quite useful for years to come.

The score test was developed by Rao with a numerical example for which he had to obtain the maximum likelihood estimates. It was motivated by a practical problem posed by Sir R A Fisher, such as what we face in econometrics. It is interesting that Rao was a respected problem-solver even to Sir R A Fisher. He developed a computational algorithm for computing maximum likelihood estimates that defined the concept of a score as the partial derivative of the likelihood function. This method is now popularly known as Rao's method of scoring for maximum likelihood estimation, and it is incorporated in most of the computer software programs used by econometricians. This method, being iterative, provides in the terminal iteration the matrix of the second partials of the likelihood function and the information matrix, which when inverted gives the asymptotic limit to the variance–covariance matrix of the maximum likelihood estimators. Thus, Rao's contributions enter into applied econometric software and econometric analysis in a big way with regard to maximum likelihood estimation.

Rao [71] introduced the notion of weighted distributions, which was originally used in various sampling situations with early applications in cell kinetics and disease detection. These methods help inject flexibility in traditional maximum likelihood methods and could generalize Heckman's [26] two-step econometrics dealing with sample selectivity. US unemployment statistics focus on the number actively seeking employment as the denominator and ignore the discouraged workers who have stopped looking for jobs. In recessions, this can cause serious bias in official statistics, but the bias is less serious in boom times. Vinod [88] showed that Rao's weighted distributions can help deal with this asymmetric data bias problem in econometrics, which can be updated using [53].

6. Rao's contributions to frontier areas of research and business analytics

This section shows that Rao has played a leadership role in advancing empirical research in frontier areas of research in economic science and in business analytics using large data sets. Experimental economics is a field developed by Vernon Smith for which he received economics Nobel in 2002. The 2019 Nobel prize in economics was awarded to Abhijit Banerjee, Esther Dufflo and Michael Kremer for their work on experimental development economics (2019). All these emerging areas of research in economic science use statistical designs of experiments that use orthogonal arrays for designs developed by Dr Rao.

In 2001, Rao delivered the C G Khatri memorial lecture at Pennsylvania State University. In that lecture, he anticipated the current developments in business analytics. He said then that the current statistical theory and practice are model-based and ignore the issue of model selection. That state of affairs was predicated on people having small data sets and hence being able to choose simple models. With the increasing availability of very large sample data, such as point of sale data, and data from social and business networks, no one single model would fit the entire data, and one must give greater emphasis to data mining techniques.

One of the most important parts of any modern data mining tool is that of data reduction and classification of data. We see in these modern data mining tools the footprints of Rao again. Rao [63] and, Rao and Slater [81] are the first research studies on classification based on multiple measurements and on factor analysis, principal component analysis, and cluster analysis. These topics have become so common in textbooks that people often do not realize that the foundations for these analyses were laid by Dr Rao. He has taken up the modern version of the classification, using bagging and boosting technique, and advocated its use in classification in bioinformatics with numerical illustrations [79]. These techniques are highly useful in business analytics requiring data mining for pattern recognition with a point of sale data [44].

In December 2008, Rao organized an International Conference on Business Data Mining at C R Rao Advanced Institute for Mathematics, Statistics and Computer Science in Hyderabad, India. Econometrics is receiving a shot in the arm through its applications in the business sector through what is now termed as "business analytics." Data warehousing and data mining are being exploited to squeeze the last drop of profit by the business world. Dr Rao edited a *Handbook of Statistics* on Big Data Analytics [24]. In this volume, Dr Rao co-authored an article on "Cognitive Computing: Theory and Applications". With his vision, he continues to extend the frontiers of econometrics by sensitizing economists and statisticians by opening these floodgates of applications of Big Data Analytics. In the inaugural (Presidential) addresses to the Indian Econometric Society, he made useful suggestions on the future directions of econometrics under the title "Information and Structure: Some Explorations in Econometrics" [80].

In addition to the above contributions that have had an immense impact on econometric research, he played direct as well as catalytic roles in promoting econometric research. As the editor of the series *Handbook of Statistics*, Dr Rao took a special interest in promoting econometric research through commissioning several *Handbooks on Econometrics* that are reference works in econometrics. The first among them is volume 11, "Handbook of Statistics, Econometrics" [48], which has six parts covering a wide range of econometric issues. The topics covered include: endogenous stratification sampling, nonparametric regression, limited dependent variables and panel data models (under rational expectations), multivariate Tobit models, nonlinear time series and micro-econometric models, a discussion of pseudo-likelihood methods and Rao's score test, alternatives to likelihood methods including generalized method of moments (GMM), testing for heteroscedasticity, computer-intensive bootstrap methods, continuous-time models in finance, calibration models and outlier detection methods. Rao took a personal interest in editing volume 11 with econometricians, Maddala and Vinod [48] and contributed a chapter proving strong consistency of M estimators with realistic assumptions for econometric applications.

In addition to the 1993, volume 11 on econometrics, Rao also brought out two other volumes of the Handbook in the 1990s, one on "Statistical Methods in Finance" [46], and another on "Robust Inference in Statistics" [47] that have several potential applications in econometrics. One of the most promising applications of robust regression is fractile regression. One important point to note here is that he contributed chapters to these volumes on multivariate statistical inference and robust estimation in linear models. Vinod and Rao [91] have recently coedited volume 41 entitled *Conceptual Econometrics using R*, and volume 42 entitled *Financial, Macro and Micro Econometrics Using R* of the *Handbook of Statistics* [92]. They showed Rao's abiding interest in practical econometrics and implementing software.

7. Dr Rao's influence on the econometrics profession

Major portions of Rao's professional work and his original knowledge and contributions to statistics have been beautifully compiled by him into two volumes; the first one is "Linear Statistical Inference", first published in English in 1965 and had been reprinted four times and translated into six foreign languages. This book is extensively cited in economics journals and appears on the bookshelves of most econometricians. The somewhat less cited 1989 book, "Statistics and Truth: Putting Chance to Work" is based on his series of Ramanujan lectures delivered in India under the auspices of the Indian Council of Scientific and Industrial Research. It gives an excellent motivation to all economists to learn and practice statistics to solve day-to-day problems and has been printed thrice and translated into six different foreign languages.

The *Journal of Quantitative Economics* (JQE) brought out a Special Issue in honor of Dr Rao. The author in the Preface of the JQE commented:

"*One of the purposes of this special issue is to recognize Dr Rao's own contributions to econometrics and acknowledge his major role in the development of econometric research …*"

In fact, his statistical methodology continues to inspire and guide mainstream research in econometrics ranging from generalized inverse of a singular matrix, unified theory of linear models with fixed and mixed effects, MINQUE theory for estimation of variance components [74], extension of principal component analysis using covariates for studying trends of economic variables, characterization of probability distributions, score tests, factor analysis [65,81]; etc. Further, weighting distribution, pooling and shrinkage, varying parameter methods, generalized spatial median and majorization are all examples of new applications in econometrics.

The length, breadth and depth of his contributions have a long way to go. We perhaps see no other concept used in econometrics as much as the asymptotic theory of statistical inference, the foundations of which were laid by Rao [61]. There is another related area, the inefficiency of least squares, that could generate a lot of interest among econometricians, as the least squares method is most widely used by econometricians [78].

On the occasion of his 70th birthday, a survey of the literature was published in the *Journal of Quantitative Economics* (1991) (one of us, H D Vinod, did most of the editorial work in bringing out this special issue of the journal). Three prominent economists brought out, on his 75th birthday, a special publication as a tribute to his influence on the advancement of economic science [45]. Vinod [88] showed that Rao [71] has a great potential in economics where observed data do not exactly match the underlying economic concept. He illustrates with the missing information on discouraged workers from official unemployment statistics. The topic has become more urgent in the current world economy. This special issue showed that Rao's seminal paper on scoring was cited more than 130 times by leading econometricians and economists like T Amemiya, E R Brandt. T S Breusch, A D Chesher, G C Chow, D R Cox, R Davidson, J Durbin, R F Engle, L G Godfrey, W E Griffith, A Harvey, J J Hausman, D Hendry, M L Higgins, G C Judge, C G Khatri, R Koenker, J Kmenta, L F Lee, J G MacKinnon, G S Maddala, W Newey, J Neyman, A R

Pagan, M H Pearson, T J Rothenberg, J D Sargan, N E Savin, G Tauchen, Aman Ullah, H D Vinod, A Wald, K F Wallis, H White, J M Woodridge, and others. The citations to Rao's work by these authors are in journals of great international repute such as *Econometrica*, *Journal of Econometrics, Advanced Econometrics, Australian Economic Papers, Review of Economic Studies, International Economic Review, Handbook of Econometrics, Econometrics Reviews, Journal of Economic Literature, Econometrics Theory, Econometrica, and Quantitative Economics, Proceedings of Econometric Society, Time Series Econometric Modeling*, etc. In fact, many volumes of Handbooks on Econometrics published by North-Holland, econometrics journals, and interviews attest to the intellectual debt to Rao.

In addition, there are several other special issues of journals, as well as published interviews with Rao that one could consult. See, for example, interviews with DeGroot [15] and Bera [4]. There are unique books brought out in his honor ([16,31]). Rao has a very inspiring biography and has received numerous honors, awards from various heads of state, including the US, India, Russia, and others from around the world.

Acknowledgements

The authors thank P C B Phillips, B L S Prakasa Rao, M Bhaskar Rao, Anil Bera, Kalyan Singhal and the late John Nash and late T N Srinivasan for helpful comments on an earlier version of this article. The authors thank Olav Berkholtz for providing them, from Frisch archives in Oslo, the rare notes between Frisch and Neyman that took place at Oxford and London in 1936. Those notes formed the basis for Dr Rao's 1947 *Econometrica* article. The authors alone are responsible for the contents of this paper.

References

[1] Basmann R, A generalized classical method of linear estimation of coefficients in structural equation, *Econometrica* **25** (1957) 77–83

[2] Basman R L, Remarks concerning the application of exact finite sample distribution function of GCL estimators in econometric statistical inference, *J. Amer. Stat. Assoc.* **58(304)** (1963) 943–976

[3] Bates C E and White H, Determination of estimators with minimum asymptotic variance covariance matrices, *Econometric Theory* **9** (1993) 633–648

[4] Bera A K, The ET interview: Prof. C. R. Rao, *Econometric Theory* **19** (2003) 331–400

[5] Bera A K and Bilias Y, Rao's score, Neyman's C and Silvey's LM test: An essay on historical development and some new results, *J. Stat. Planning and Inference* **97(1)** (2001) 9–44

[6] Bera A K and Ullah A, Rao's score test in econometrics, *J. Quantitative Economics* **7** (1991) 189–220

[7] Bjerkholt O, Frishch's econometric laboratory and the rise of Trygve Haavelmmo's probability approach, *Econometric Theory* **21(3)** (2005) 491–53

[8] Bjerkholt O, Writing the "probability approach" with nowhere to go: Haavelmo in the United States 1939–1944, *Econometric Theory* **21(5)** (2007) 775–83

[9] Bjerkholt O and B Dupont-Kieffer A, Problems and Methods of Econometrics: The Poincare Lectures of Ragnar Frisch 1933 (2009) (London: Routledge)

[10] Bjerkholt O and Dupont A, Ragnar Frisch's conception of econometrics, *History of Political Economy* **42(1)** (2010) 21–73

[11] Bjerkholt O and Qin D, Teaching Economics as a Science: The 1930 Yale Lectures of Ragnar Frisch, Working Papers 665 (2010) (Queen Mary: University of London)

[12] Breusch T and Pagan A, The Lagrangian multiplier test and its applications to model specification in econometrics, *Rev. Economic Studies* **37** (1980) 239–253

[13] Chikuse Y, Functional forms of characteristic functions and characterization of multivariate distributions, *Econometric Theory* **6** (1990) 445–458
[14] Debreu G, Mathematization of economic theory, *Amer. Economic Rev.* **81** (1991) 1–7
[15] DeGroot M H, A conversation with C R Rao, *Stat. Sci.* **2(1)** (1987) 53–67
[16] Dodge Y, Statistical Data Analysis and Inference, Papers Presented at the International Conference on Recent Developments in Statistical Data Analysis and Inference in Honor of C. R. Rao (1989) (Switzerland: Neuchâtel–Amsterdam: North-Holland)
[17] Dufour J M and Hsiao C, Identification, in: Microeconometrics, edited by S N Durlauf and L E Blume, The New Palgrave Economics Collection (2010) (London: Palgrave Macmillan)
[18] Fisher R, On the mathematical foundations of theoretical statistics, *Philos. Trans. Roy. Soc. Series A* **222** (1922) 309–368
[19] Fix E, Distributions which lead to linear regressions, Doctoral Thesis (1948) (Berkley: University of California)
[20] Fix E, Distributions which lead to linear regressions, Proceedings of the First Berkeley Symposium on Mathematical Statistics and Probability (1949) vol. 1, pp. 79–91
[21] Fox K A, Agricultural economists in the econometric revolution: Institutional. background, literature and the leading figures, *Oxford Economic Papers* **41(1)** (1989) 53–70
[22] Ghosh J, Higher order asymptotics for the likelihood ratio, Rao's and Wald's tests, *Stat. Probab. Lett.* **12** (1991) 505–509
[23] Gonzalez-Rivera G and Ullah A, Rao's score test with nonparametric density estimators, *J. Stat. Planning and Inference* **97** (2001) 85–100
[24] Govindaraju V, Raghavan V and Rao C R (eds), Handbook of Statistics: Big Data Analytics (2015) (Amsterdam: North-Holland Elsevier) vol. 33
[25] Haavelmo T, The probability approach to econometrics, *Econometrica, Suppl.* **12** (1944) 1–117
[26] Heckman J J, The common structure of statistical models of truncations, sample selection and limited and limited dependent variables and a simple estimator for such models, *Ann. Economic and Social Measurement* **5** (1976) 363–571
[27] Hendry D F and Morgan M S, The Foundations of Econometric Analysis (1995) (Cambridge: Cambridge University Press)
[28] Hood W and Koopmans T, Studies in Econometric Method (1953) (New York: John Wiley and Sons–London: Chapman and Hall)
[29] Johnson N L and Kotz S, Breakthroughs in Statistics (1991) (New York: Springer-Verlag) vols. 1–2
[30] Kagan A, Linnik Y V and Rao C, Characterization problems in mathematical statistics, *USSR Acad. Sci.* (English translation) (1973) (New York: Wiley)
[31] Kallianpur G, Krishnaiah P and Ghosh J, Statistics and Probability: Essays in Honor of CR Rao (1982) (Amsterdam: North Holland)
[32] Khatri C, Characterizations of multivariate normality-ii through linear regression, *J. Multivariate Anal.* **9** (1979) 589–598
[33] Khatri C and Rao C, Characterization of multivariate normality-i through independence of some statistics, *J. Multivariate Anal.* **6** (1976) 81–94
[34] Koopmans T C, Linear Regression Analysis of Economic Time Series (1937) (Amsterdarm: Haarlem, De Erven, F. Bohn, NV)
[35] Koopmans T, Statistical Inference in Dynamic Economic Models: Cowles Commission Monograph (1950) (New York: John Wiley and Sons–London: Chapman and Hall)
[36] Krishnaji N, Characterization of the Pareto distribution through a model of under-reported incomes, *Econometrica* **38** (1970) 251–255
[37] Krishnaji N, A note on characterizing property of the exponential distribution, *Ann. Math. Stat.* **42(1)** (1971) 361–362
[38] Kumar T K, The probabilistic approaches in econometrics, *J. Quantitative Economics* **9(1)** (1993) 1–16
[39] Kumar T K, Co integration and error-correction models: A historical and methodological perspective, *J. Quantitative Economics* **11(1)** (1995) 143–154

[40] Kumar T K and Gapinski J, Nonlinear estimation of the CES production parameters: A Monte Carlo study, *Rev. Economics and Stat.* **56(4)** (1974) 563–67

[41] Laha R, On the characterization of the stable law with finite expectations, *Ann. Math. Stat.* **27** (1956) 187–195

[42] Laha R, On some characterization problems connected with linear structural relations, *Ann. Math. Stat.* **28(1)** (1957) 126–139

[43] Leamer E, Specification searches: Ad-hoc inference with non- experimental data (1978) (New York: John Wiley)

[44] Lemmens A and Croux C, Bagging and boosting classification trees to predict churn, *J. Marketing Res.* **43(2)** (2006) 276–286

[45] Maddala G, Phillips P C and Srinivasan T, Advances in Econometrics and Quantitative Economics – Essays in Honor of Professor C. R. Rao (1995) (New Haven, CT: Basil Blackwell Ltd)

[46] Maddala G and Rao C, Handbook of Statistics: Finance (1996) (Amsterdam: North-Holland)

[47] Maddala G and Rao C, Handbook of Statistics: Robust Inference (1997) (Amsterdam: North-Holland)

[48] Maddala G, Rao C and Vinod H, Handbook of Statistics: Econometrics (1993) (Amsterdam: North-Holland) vol. 11

[49] Morgan M S, The History of Econometric Ideas (1990) (Cambridge, England: Cambridge University Press)

[50] Mukherjee R, Rao's score test: Recent asymptotic tests, in: Handbook of Statistics: Finance (1993) (Amsterdam: North-Holland) vol. 11

[51] Neyman J, Current problems of mathematical statistics, Proceedings of the International Congress of Mathematicians (1954)

[52] Neyman J and Scott E, Consistent estimates based on partially consistent observations, *Econometrica* **16(1)** (1948) 1–32

[53] Patil G P, Weighted distributions, *Encyclopedia of Environmetrics* **4** (2002) 2369–2377

[54] Phillips P, Partially identified economic models, *Econometric Theory* **5(2)** (1989) 181–240

[55] Prakasa Rao B, Identifiability in Stochastic Models: Characterization of Probability Distributions (1992) (Boston: Academic Press)

[56] Quin D, Foundations of Econometrics: A Historical Perspective (1993) (London: Oxford, Clarendon Press)

[57] Rao C R, Research on the theory of design of experiments and distribution problems connected with univariate and multivariate populations, Master's Thesis (1943) (Calcutta University, Statistics Department)

[58] Rao C R, Informational accuracy attainable in the estimation of statistical parameters, *Bull. Calcutta Math. Soc.* **37** (1945a) 81–91

[59] Rao C R, Generalization of Markoff's theorem and tests of linear hypotheses, *Sankhya: The Indian J. Stat.* **7(1)** (1945b) 9–16

[60] Rao C R, Note on a problem of Ragnar Frisch, *Econometrica* **15(3)** (1947) 245–249

[61] Rao C R, Large sample tests of statistical hypotheses concerning several parameters with applications to problems of estimation, *Proc. Cambridge Philos. Soc.* **44** (1948a) 50–57

[62] Rao C R, Sufficient statistics and minimum variance estimates, *Proc. Cambridge Philos. Soc.* **45(2)**(1948b) 213–218

[63] Rao C R, The utilization of multiple measurements in problems of biological classification, *J. R. Stat. Soc.* **10** (1948c) 159–203

[64] Rao C R, Method of scoring linkage data giving simultaneous segregation of three factors, *Heredity* **4** (1950) 37–59

[65] Rao C R, Estimation and tests of significance in factor analysis, *Psychometrika* **20** (1955) 93–111

[66] Rao C R, Asymptotic efficiency and limiting information, Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability (1961) vol. 1, pp. 531–546

[67] Rao C R, Efficient estimates and optimum inference procedures in large samples (with discussion), *J. R. Stat. Soc., Ser. B* **24** (1962a) 46–72
[68] Rao C R, First and second order asymptotic efficiencies of estimators (with discussion), *Ann. Fac. Sci. Univ. Clemont. Ferrand* **8** (1962b) 33–40
[69] Rao C R, A note on a generalized inverse of a matrix with applications to problems in mathematical statistics, *J. R. Stat. Soc.* **B24** (1962c) 152–158
[70] Rao C R, Essays in Econometrics and Planning (1964a) (Oxford, UK: Pergamon Press)
[71] Rao C R, On discrete distributions arising out of methods of ascertainment: What population does a sample represent? in: Classical and Contagious Discrete Distributions (1965b) (Calcutta: Calcutta Statistical Publishing Society)
[72] Rao C R, Characterization of distribution of random variables in linear structural equations, *Sankhya* **28** (1966) 251–260
[73] Rao C R, Least squares theory with estimated dispersion matrix and its application to measurement of signals, Proceedings of the Fifth Berkeley Symposium on Mathematical Statistics and Probability (1967) vol. 1, pp. 355–372
[74] Rao C R, Estimation of variance and covariance components in linear models, *J. Amer. Stat. Assoc.* **67(337)** (1972a) 112–115
[75] Rao C R, Data, analysis and statistical thinking in economic and social development, Essays in Honour of Dr. C. D. Deshmukh (1972b) pp. 383–392
[76] Rao C R, Functional equations and characterization of probability distributions, in: Proceedings of the International Congress of Mathematics (Vancouver) (1974a) vol. 2, pp. 163–168
[77] Rao C R , Projectors, generalized inverses, and the blues, *J. R. Stat. Soc.* **36** (1974b) 442–448
[78] Rao C R, The inefficiency of least squares: extensions of the Kantorovich inequality, *Linear Algebra Appl.* **70** (1995) 249–255
[79] Rao C R, Bagging and boosting – applications to classification and regression methodologies, Keynote Address at Ohio State, Cleveland Clinic Foundation, and Case Western University Biostatistics Joint Symposium, May 18 (2006)
[80] Rao C R, Information and structure: Some explorations in econometric theory, Measurement, and Policy, in: Evolving Themes in Quantitative Economics, edited by V N Pandit and R Shanmugam (2009) (Academic Foundation)
[81] Rao C and P Slater, Multivariate analysis applied to differences between neurotic groups, *British J. Psychol.: Statistics Section* **2** (1949) 17–29
[82] Rothenberg T J, Identification in parametric models, *Econometrica* **39(3)** (1970) 577–591
[83] Silvey S, The Lagrangian multiplier test, *Ann. Math. Stat.* **30(2)** (1959) 389–407
[84] Sims C, Macroeconomics and reality, *Econometrica* **48(1)** (1980) 1–48
[85] Taguchi G, Quality engineering (Taguchi methods) for the development of electronic circuit technology, *IEEE Trans. on Reliability Soc.* **44(2)** (1995) 225–229
[86] Theil H, Economic Policy and Forecasts (1958) (Amsterdam: North-Holland Publishing Company)
[87] Tintner G, Multiple regressions for systems of equations, *Econometrica* **14(1)** (1946) 5–36
[88] Vinod H D, Rao's weighted distribution in econometrics: An application to unemployment statistics and Okun's law, *J. Quantitative Economics* **7(2)** (1991) 247–254
[89] Vinod H, Maximum entropy ensembles for time series inference in economics, *J. Asian Economics* **17(6)** (2006) 955–978
[90] Vinod H, Software-illustrated explanations of econometrics contributions by C R Rao for his upcoming 100-th birthday, Economics Department, Fordham University, Discussion Paper. to appear in *J. Quantitative Economics* (2019)
[91] Vinod H D and Rao C R, Handbook of Statistics: Conceptual Econometrics Using *R* (2019) (North-Holland–New York: Elsevier Science Publishers) vol. 41
[92] Vinod H D and Rao C R, Handbook of Statistics: Financial, Macro and Micro Econometrics Using *R* (2020) (North-Holland–New York: Elsevier Science Publishers) vol. 42
[93] Zellner A, An efficient method of estimating seemingly unrelated regressions and tests for aggregation bias, *J. Amer. Stat. Assoc.* **57(298)** (1962) 348–368