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Repdigits in Euler functions of associated pell numbers

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Abstract. In this paper, we discuss some properties of Euler totient function of associated Pell numbers which are repdigits in base 10.

Keywords. Pell numbers; associated Pell numbers; Euler totient function; repdigits.

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1. Introduction

The Euler totient function $\phi(n)$ of a positive integer n is the number of positive integers less than or equal to n and relatively prime to n. If n has the canonical decomposition $n = p_1^{a_1} \cdots p_r^{a_r}$, where $p_1, p_2, \ldots, p_r$ are distinct primes and $a_1, a_2, \ldots, a_r$ are positive integers, then it is well-known that

$$\phi(n) = p_1^{a_1-1}(p_1 - 1) \cdots p_r^{a_r-1}(p_r - 1).$$

The Pell sequence $\{P_n\}$ is defined by means of a binary recurrence relation $P_{n+1} = 2P_n + P_{n-1}$ with initial terms $P_0 = 0,\ P_1 = 1$. The associated Pell sequence $\{Q_n\}$ satisfies an identical recurrence relation with initial terms $Q_0 = 1,\ Q_1 = 1$. The sequence $\{R_n\}$, where $R_n = 2Q_n$, is known as the Pell–Lucas sequence. The Binet formulas for Pell and associated Pell numbers are

$$P_n = \frac{\alpha^n - \beta^n}{\alpha - \beta} \quad \text{and} \quad Q_n = \frac{\alpha^n + \beta^n}{2},$$

where $\alpha = 1 + \sqrt{2}$ and $\beta = 1 - \sqrt{2}$.

The Lucas and associated Lucas sequences are defined as $u_0 = 0,\ u_1 = 1,\ u_{n+1} = Au_n + Bu_{n-1}$ and $u_0 = 2,\ u_1 = A,\ v_{n+1} = Av_n + Bv_{n-1}$ respectively, where A and B are positive integers. Many well-known binary recurrence sequences are special cases of these sequences.

In the last two decades, Diophantine equations involving Euler functions of binary recurrence sequences have been extensively studied. Luca [6] explored all Fibonacci numbers whose Euler function is a power of 2. Luca and Stănică [8] proved that the Euler function of the n-th Pell number P_n or the n-th associated Pell number Q_n is a power of 2 only

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if $n \leq 8$. Damir *et al.* [3] extended their search for powers of 2 in Euler functions of the terms of the Lucas sequence corresponding to $B = \pm 1$ and proved that there are finitely many such n.

A repdigit is a positive integer with only one distinct digit in its decimal expansion. In particular, numbers of the form $d(10^m - 1)/9$ for some $m \geq 1$ and $1 \leq d \leq 9$ are called repdigits in base 10. In the year 2006, Luca [7] explored the repdigits associated with the Euler functions of Fibonacci numbers. In 2015, Faye and Luca [4] proved that there is no Pell or Pell–Lucas number larger than 10 with only one distinct digit. Subsequently, Bravo *et al.* [2] in 2016 investigated the presence of repdigits in the Euler functions of Lucas numbers. Sahukar and Panda [12] proved that Euler function of no Pell number is a repdigit having at least two digits.

In this paper, we study the presence of repdigits in the sequence $\{\phi(Q_n)\}$. In particular, we prove the following theorem on the solvability of the equation:

$$\phi(Q_n) = d(10^m - 1)/9 \tag{1.1}$$

for $d \in \{1, 2, \ldots, 9\}$.

Theorem 1.1. *Let* (1.1) *hold for some* $n > 16$ *and for some* $d \in \{1, 2, \ldots, 9\}$. *Then*

(a) $d \in \{4, 8\}$.
(b) n *is odd.*
(c) n *is an odd prime and* $n^2 | 10^m - 1$ *if* $d = 4$.

Throughout this paper, we use p with or without subscripts as a prime number, $\left(\frac{a}{b}\right)$ as Jacobi symbol of a and b and (a, b) as the greatest common divisor of a and b.

2. Preliminaries

In this section, we present some results about Pell and associated Pell numbers. The results of this section are necessary to prove the theorems stated in Section 1.

Lemma 2.1 [5]. *If* m *and* n *are natural numbers, then*

(1) $(P_n, Q_n) = 1$,
(2) $Q_n^2 - 2P_n^2 = (-1)^n$,
(3) $Q_m | Q_n$ *if and only if* $m | n$ *and* $\frac{n}{m}$ *is odd,*
(4) $v_2(P_n) = v_2(n)$ *and* $v_2(Q_n) = 0$, *where* $v_2(n)$ *is the exponent of* 2 *in the canonical decomposition of* n,
(5) $3 | Q_n$ *if and only if* $n \equiv 2 \pmod 4$,
(6) $5 \nmid Q_n$ *for any* n,
(7) $Q_{3\cdot 2^t} = Q_{2^t}(4Q_{2^t}^2 - 3)$.

Lemma 2.2 ([1], *Theorem A*). *If* n, y, m *are positive integers with* $m \geq 2$, *then the equation* $Q_n = y^m$ *has the only solution* $(n, y) = (1, 1)$.

Lemma 2.3 ([9], *Theorem* 3). *The solutions of the Diophantine equation* $Q_m Q_n = x^2$ *with* $0 \leq m < n$ *are* $n = 3m$, $3 \nmid m$, m *is odd.*

Table 1. Periods of Q_n.

k	$Q_n \pmod k$	Period
4	1, 1, 3, 3	4
5	1, 1, 3, 2, 2, 1, 4, 4, 2, 3, 3, 4	12
8	1, 1, 3, 7	4
20	1, 1, 3, 7, 17, 1, 19, 19, 17, 13, 3, 19	12

Lemma 2.4 ([1], *Lemma* 2.1). *A prime p is a primitive prime divisor of Lucas number u_n if p divides u_n but does not divide $(\alpha - \beta)^2 u_2 \cdots u_{n-1}$ and p is always congruent to ± 1 modulo n.*

Lemma 2.5 ([1], *Lemma* 2.1). *A primitive prime divisor of n-th associated Pell number Q_n exists only if $n \geq 2$.*

Lemma 2.6 ([13], *Pell–Lucas numbers*). *If the associated Pell number Q_n is a prime, then n is either a prime or a power of 2.*

3. Main results

In this section, we prove Theorem 1.1 stated at the end of Section 1. To prove this theorem, we need the least residues and periods of associated Pell sequence $\{Q_n\}_{n\geq 0}$ modulo 4, 5, 8 and 20. We list them in Table 1.

If $n \leq 16$, it is easy to see that 1, 3 and 7 are the only numbers in the associated Pell sequence $\{Q_n\}_{n\geq 0}$ such that $\phi(Q_n)$ is a repdigit. If $\phi(Q_n)$ is a repdigit for some $n > 16$, then by Lemma 2.6,

$$\phi(Q_n) \geq \frac{2}{\sqrt{3}}\sqrt{Q_n} \geq \frac{2}{\sqrt{3}}\sqrt{Q_{17}} > 450390 > 10^5,$$

and thus $m > 5$. Since Q_n is odd for all n, the canonical decomposition of Q_n is

$$Q_n = p_1^{a_1} p_2^{a_2} \cdots p_r^{a_r}, \ r \geq 0, \tag{3.1}$$

where $p_1, p_2, \ldots, p_r$ are distinct odd primes and $a_i \geq 1$ for all i and

$$\phi(Q_n) = p_1^{a_1-1} p_2^{a_2-1} \cdots p_r^{a_r-1}(p_1 - 1)(p_2 - 1)\cdots(p_r - 1). \tag{3.2}$$

If $\phi(Q_n) = d(10^m - 1)/9$ for some n, then

$$v_2(\phi(Q_n)) = \sum_{i=1}^{r} v_2(p_i - 1) = v_2(d), \quad \text{where } v_2(d) \in \{0, 1, 2, 3\}. \tag{3.3}$$

With these ideas, we now proceed with the proof of Theorem 1.1.

3.1 *Proof of Theorem* 1.1(*a*)

In view of (3.3), if $v_2(d) = 0$, then $d \in \{1, 3, 5, 7, 9\}$ and consequently, $\phi(Q_n)$ is odd, which is possible only when $n = 0, 1$. If $v_2(d) = 1$, then $d = 2, 6$ and because of (3.2), $r = 1$ and $p_1 \equiv 3 \pmod 4$. In particular, $Q_n = p_1^{a_1}$ and an application of Lemma 2.2 gives $a_1 = 1$ and hence

$$\phi(Q_n) = Q_n - 1 = d \cdot \frac{10^m - 1}{9}.$$

If $d = 2$, then $Q_n = 3 \pmod{20}$ and from Table 1, $n \equiv 2, 10 \pmod{12}$. Because of Lemma 2.6, n is either a prime or a power of 2. If $n \equiv 2 \pmod{12}$, we can write n as $n = 2(6k + 1)$ for some $k \geq 0$, which is a power of 2 only if $k = 0$ and consequently $n = 2$, which contradicts our assumption that $n > 16$. If $n \equiv 10 \pmod{12}$, then we can write n as $n = 2(6l + 5)$ for some $l \geq 0$, which is not a power of 2 for any l. If $d = 6$, then $Q_n = 7 \pmod{20}$ and from Table 1, it follows that $n \equiv 3 \pmod{12}$. Thus $n = 3(4k+1)$, which is a prime only if $k = 0$. Hence, the only possibility left is $n = 3$, a contradiction to $n > 16$.

3.2 *Proof of Theorem* 1.1(*b*)

From Theorem 1.1(a), it is clear that if (1.1) holds for $n > 16$, then $d \in \{4, 8\}$. To prove part (b) of Theorem 1.1, we need the following lemma.

Lemma 3.1. *If $n > 16$ is even and* (1.1) *holds for $d \in \{4, 8\}$, then all prime factors in the canonical decomposition of Q_n are congruent to* 3 *modulo* 4.

Proof. Since n is even, we can write n as $n = 2^t \cdot n_1$, where $t, n_1 \in \mathbb{N}$ and n_1 is odd. Assume on the contrary that there exist a prime $p \equiv 1 \pmod 4$ in the canonical decomposition of Q_n. In view of (3.3), the number of prime factors of Q_n is at most 2 and therefore, $r \leq 2$. If $d = 4$, then $r = 1$ and $Q_n = p_1^{a_1}$. From Lemma 2.2, it follows that $Q_n = p_1$ and hence

$$\phi(Q_n) = Q_n - 1 = 4 \cdot \frac{10^m - 1}{9} \equiv 4 \pmod{10}.$$

Thus

$$Q_n \equiv 5 \pmod{10},$$

which implies that $5 | Q_n$ and this contradicts Lemma 2.1. Reducing the identity of Lemma 2.1(2) modulo p_1, we get $-2P_n^2 \equiv 1 \pmod{p_1}$, which implies that $\left(\frac{-1}{p_1}\right) = 1$ and consequently $\left(\frac{2}{p_1}\right) = 1$. Therefore, $p_1 \equiv 1 \pmod 8$ and hence, $d = 8$, $r = 1$ and $Q_n = p_1^{a_1}$. Now, by Lemma 2.2, $Q_n = p_1$. Since $Q_{2^t} | Q_n$, there exist primitive prime factors of Q_{2^t} and Q_n that divide Q_n. Hence $n = 2^t$, otherwise Q_n would have more than one prime factor – a contradiction to $r = 1$. But

$$\phi(Q_n) = \phi(Q_{2^t}) = Q_{2^t} - 1 = 8 \cdot \frac{10^m - 1}{9} \equiv 3 \pmod 5$$

implies that $Q_{2^t} \equiv 4 \pmod 5$, which is not possible since $Q_{2^t} \equiv 2, 3 \pmod 5$ (see Table 1). Consequently, there does not exist any prime factor p such that $p \equiv 1 \pmod 4$. □

We are now in a position to prove Theorem 1.1(b). Let Q_n have the canonical decomposition $Q_n = p_1^{a_1} \cdots p_r^{a_r}$, where $p_1, p_2, \ldots, p_r$ are distinct odd primes and $a_i \geq 1$ for all i. Assume on the contrary that n is even, say $n = 2^t \cdot n_1$, where $t \geq 1$ and n_1 is odd. In view of Lemma 3.1, if p_i is a prime factor of Q_n, then $p_i \equiv 3 (\bmod\ 4)$ and consequently, $r = 2$ only if $d = 4$ and $r = 3$ only if $d = 8$. Reducing the identity of Lemma 2.1(2) modulo p_i, we get $\left(\frac{2}{p_i}\right) = -1$. Since p_i is a primitive prime factor of $Q_{2^t \cdot n_1}$, use of Lemma 2.4 results in $p_i \equiv -1 (\bmod\ 2^t \cdot n_1)$. Therefore, $p_i \equiv -1 (\bmod\ 2^t)$ for $i = 1, 2, \ldots, r$. If $d = 4$, then

$$d \cdot \frac{10^m - 1}{9} = \phi(Q_n) = \phi(p_1^{a_1} p_2^{a_2})$$
$$= p_1^{a_1 - 1} p_2^{a_2 - 1}(p_1 - 1)(p_1 - 1) \equiv \pm 4 (\bmod\ 2^t).$$

Similarly, if $d = 8$ and $r \in \{2, 3\}$, then $d \cdot \frac{10^m-1}{9} \equiv \pm 8 (\bmod\ 2^t)$, which implies that $10^m \equiv 10$ or $- 8 (\bmod\ 2^{\max\{0, t-r\}})$. If $10^m \equiv 10 (\bmod\ 2^{\max\{0, t-r\}})$, then $t \leq 3$ and if $10^m \equiv -8 (\bmod\ 2^{\max\{0, t-r\}})$, then $t \leq 5$ and therefore $t \leq 5$. Since $Q_{2^t} | Q_n$, it follows that

$$Q_n = Q_{2^t} p_1^{a_1} \tag{3.4}$$

or

$$Q_n = Q_{2^t} p_2^{a_2} p_3^{a_3}. \tag{3.5}$$

Now we distinguish two cases:

Case 1. $3 \nmid n_1$: From Table 1, it is clear that modulo 5, the period of $\{Q_{2n}\}$ is 6 and $Q_{2n} \equiv 3$ for $n \equiv 1, 5 \pmod 6$. Furthermore, the period of $Q_{2^t n}$ modulo 5 is 3 for $t \geq 2$ and $Q_{2^t n} \equiv 2 \pmod 5$ if $n \equiv 1 \pmod 3$. Hence, we can conclude that $Q_{2^t \cdot n_1} \equiv Q_{2^t} (\bmod\ 5)$ for $t \geq 1$ since n_1 is odd and is not a multiple of 3. Consequently, (3.4) reduces to $Q_{2^t} \equiv Q_{2^t} p_1^{a_1} (\bmod\ 5)$ and hence, $p_1^{a_1} \equiv 1 (\bmod\ 5)$. If a_1 is odd, then $p_1 \equiv 1 (\bmod\ 5)$ and $5 | p_1 - 1 | \phi(Q_n) = d \cdot \frac{10^m-1}{9}$, which is not possible since $d \in \{4, 8\}$. If a_1 is even, then $\frac{Q_n}{Q_{2^t}} = p_1^{a_1}$, which is not possible in view of Lemma 2.3.

On the other hand, if (3.5) holds, then for $t \in \{2, 3, 4\}$, Q_{2^t} is a prime and $Q_{2^t} \equiv 1 (\bmod\ 16)$ and hence $16 | \phi(Q_{2^t}) | \phi(Q_n) = 8 \cdot \frac{10^m-1}{9}$. This implies that $2 | \frac{10^m-1}{9}$, which is not possible. If $t = 5$, then $Q_{2^5} = 257 \cdot 1409 \cdot 2448769$. Since all the prime factors of Q_{2^5} are congruent to 1 modulo 16, it follows that $16^3 | \phi(Q_{2^t}) | 8 \cdot \frac{10^m-1}{9}$, which is again not possible.

If $t = 1$, then $Q_{2^t \cdot n_1} \equiv 3 (\bmod\ 4)$. Reducing (3.4) modulo 4, we get $3 \equiv 3^{a_2 + a_3 + 1} (\bmod\ 4)$ and therefore, $a_2 + a_3$ must be even. If a_2 is even, then a_3 is also even and then $\frac{Q_n}{3}$ is a perfect square, which is not possible in view of Lemma 2.3. Hence, a_2 and a_3 are odd. Since n_1 is odd and $(n_1, 3) = 1$, it follows that $2n_1 \equiv 2 (\bmod\ 4)$ and $2n_1 \equiv 2, 4 (\bmod\ 6)$

implying $2n_1 \equiv 2, 10 \pmod{12}$. Since the period of $\{Q_n\}_{n\geq 0}$ modulo 8 is 12, it follows that $Q_{2n_1} \equiv 3 \pmod 8$ if $2n_1 \equiv 2, 10 \pmod{12}$.

Reducing (3.5) modulo 8, we get $p_2^{a_2} p_3^{a_3} \equiv 1 (\bmod\ 8)$. Since a_2 and a_3 are odd, $p_2 p_3 \equiv 1 \pmod 8$, which together with $p_2 \equiv p_3 \equiv 3 \pmod 4$ implies that $p_2 \equiv p_3 \equiv 3, 7 \pmod 8$ and then $\frac{p_2-1}{2} \equiv \frac{p_3-1}{2} \equiv \pm 1 \pmod 4$. Thus, $\frac{p_2-1}{2} \cdot \frac{p_3-1}{2} \equiv 1 \pmod 4$. Therefore,

$$\begin{aligned}\phi(Q_n) &= \phi(3p_2^{a_2} p_3^{a_3}) = 2p_2^{a_2-1} p_3^{a_3-1}(p_2-1)(p_3-1)\\ &= 8 \cdot \frac{p_2-1}{2} \cdot \frac{p_3-1}{2} \cdot p_2^{a_2-1} p_3^{a_3-1} = 8 \cdot \frac{10^m-1}{9},\end{aligned}$$

which implies that

$$\frac{p_2-1}{2} \cdot \frac{p_3-1}{2} \cdot p_2^{a_2-1} p_3^{a_3-1} = \frac{10^m-1}{9}. \tag{3.6}$$

Since $\frac{p_2-1}{2} \cdot \frac{p_3-1}{2} \cdot p_2^{a_2-1} p_3^{a_3-1} \equiv 1 \pmod 4$ and $\frac{10^m-1}{9} \equiv 3 \pmod 4$, it follows from (3.6) that $1 \equiv 3 \pmod 4$, which is a contradiction.

Case 2. $3|n_1$: If n is even and $3|n_1$, then n is of the form $n = 2^t \cdot 3^s \cdot n_2$, where $t, s \geq 1$ and n_2 is odd. If $n_2 > 1$, then by Lemma 2.5, Q_n is a multiple of the primitive factors of Q_{2^t}, $Q_{3\cdot 2^t}$, $Q_{2^t\cdot n_2}$ and $Q_{2^t\cdot 3n_2}$, which implies that $r \geq 4$. But this is not true since $r \leq 3$. Hence $n_2 = 1$ and $n = 2^t \cdot 3^s$. If $s \geq 2$, then Q_n is divisible by its own primitive prime factor as well as primitive prime factors of Q_{2^t}, $Q_{3\cdot 2^t}$ and $Q_{3^2\cdot 2^t}$, which is not true since $r \leq 3$. Hence, n is either $n = 3^2 \cdot 2^t$ or $n = 3 \cdot 2^t$.

If $n = 3\cdot 2^t$, then $Q_{3\cdot 2^t} = Q_{2^t}(4Q_{2^t}^2 - 3)$. If $t \geq 2$, then $(Q_{2^t}, 4Q_{2^t}^2 - 3) = (Q_{2^t}, 3) = 1$ and if $t = 1$, then $(Q_{2^t}, 4Q_{2^t}^2 - 3) = 3$. Now, assume that $t \geq 2$. Then Q_{2^t} and $4Q_{2^t}^2 - 3$ are relatively prime and hence, we can write

$$Q_{2^t} = p_3^{a_3} \tag{3.7}$$

and

$$4Q_{2^t}^2 - 3 = p_4^{a_4}, \tag{3.8}$$

where both $p_3, p_4 \equiv 3 \pmod 4$ are primes, and in view of Lemma 2.2, $a_3 = 1$ and $Q_{2^t} = p_3$. Reducing (3.8) modulo 4, we get $3^{a_4} \equiv 1 \pmod 4$, which implies that a_4 is even and then $4Q_{2^t}^2 - 3 = (p_4^{a_4/2})^2$, which is possible only when $Q_{2^t} = 1$, or, equivalently, $t = 0$, which is a contradiction to $t \geq 2$. Hence, we are left with $t = 1$ leading to $n = 6$. But this contradicts our assumption that $n > 16$.

If $n = 2^t \cdot 3^2$, then $Q_{2^t 3^2} = Q_{2^t}(4Q_{2^t}^2 - 3)(4Q_{2^t 3}^2 - 3)$. If $t \geq 2$, then the factors on the right-hand side of the above equation are relatively prime and can be written as

$$Q_{2^t} = p_5^{a_5}, \tag{3.9}$$

$$4Q_{2^t}^2 - 3 = p_6^{a_6} \tag{3.10}$$

and

$$4Q_{2^t3}^2 - 3 = p_7^{a_7}, \tag{3.11}$$

where $p_5, p_6, p_7 \equiv 3 \pmod 4$ are all primes. Since by Lemma 2.2, Q_n is not a perfect power for $n > 1$, it follows that $a_5 = 1$. Hence (3.10) implies that a_6 is even. Therefore, $4Q_{2^t}^2 - 3$ is a perfect square, which is not possible for any $t \geq 2$. Hence, we are left with case $t = 1$, which corresponds to $n = 18$. One can easily check that $\phi(Q_{18})$ is not a repdigit. Hence n is odd.

3.3 *Proof of Theorem* 1.1(*c*)

If $\phi(Q_n) = 4 \cdot \frac{10^m-1}{9}$, then it follows from Theorem 1.1(b) that n is odd. If p is a prime factor of Q_n, then by Lemma 3.1, $p \equiv 3 \pmod 4$, which implies that $r = 2$ and $p \equiv 3, 7 \pmod 8$. Reducing $Q_n^2 - 2P_n^2 = -1$ modulo p, we get $2P_n^2 \equiv 1 \pmod p$, which implies that $\left(\frac{2}{p}\right) = 1$. Hence, $p \equiv 7 \pmod 8$ and $\frac{p-1}{2} \equiv 3 \pmod 4$. If Q_n has the canonical decomposition $Q_n = p_1^{a_1} p_2^{a_2}$, then

$$\phi(Q_n) = 4p_1^{a_1-1}p_2^{a_2-1} \cdot \frac{p_1 - 1}{2} \cdot \frac{p_2 - 1}{2} = 4 \cdot \frac{10^m - 1}{9}$$

implies that $3^{a_1+a_2-2} \equiv 3 \pmod 4$, which holds only when $a_1 + a_2 - 2$ is odd, so that a_1 and a_2 are of opposite parity. If $n = pq$, then Q_n is divisible by the primitive prime factors of Q_p, Q_q and Q_n, which is not true since $r = 2$. Hence $n = p$ or p^2.

If $n = p^2$, then $Q_{p^2} = Q_p \cdot \frac{Q_{p^2}}{Q_p}$, where both the factors are relatively prime and therefore, we take $Q_p = p_1^{a_1}$ and $\frac{Q_{p^2}}{Q_p} = p_2^{a_2}$. Since one of a_1 and a_2 is even, either Q_p or $\frac{Q_{p^2}}{Q_p}$ is a square, which is not possible in view of Lemma 2.2 and 2.3. If $n = p$, all prime factors of Q_p are primitive prime factors of Q_p and $p_1, p_2 \equiv 1 \pmod p$, which implies that $p^2 | (p_1 - 1)(p_2 - 1) | \phi(Q_p) | 4 \cdot \frac{10^m-1}{9}$ and hence $p^2 | 10^m - 1$.

Remark. As a consequence of Theorem 1.1(b), the Euler function of no Lucas balancing number C_n is a repdigit, since Lucas balancing numbers are nothing but even indexed associated Pell numbers (see [10,11]). It would be interesting to find some bound for n such that $\phi(Q_n)$ is a repdigit and explore more properties corresponding to odd primes n in Theorem 1.1(c). We leave these as open problems for the readers.

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