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Normalized unit groups and their conjugacy classes

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Abstract. Let $G = H \times A$ be a finite 2-group, where H is a non-abelian group of order 8 and A is an elementary abelian 2-group. We obtain a normal complement of G in the normalized unit group $V(FG)$ and in the unitary subgroup $V_*(FG)$ over the field F with 2 elements. Further, for a finite field F of characteristic 2, we derive class size of elements of $V(FG)$. Moreover, we provide class representatives of $V_*(FH)$.

Keywords. Group ring; unitary subgroup; unit group; conjugacy class.

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1. Introduction

Let FG be a group algebra of a finite p-group G over a field F of characteristic p. Let $\omega(FG)$ denote the set of elements of FG with augmentation 0. For a normal subgroup H of G, let $\Gamma(H)$ denote the ideal of FG generated by the elements $\{h - 1 \mid 1 \neq h \in H\}$. Clearly, $\Gamma(H) = \omega(FH) \cdot FG$. The set

$$V(FG) = \left\{ \sum_{g \in G} a_g g \in \mathcal{U}(FG) \;\middle|\; \sum_{g \in G} a_g = 1 \right\}$$

is called the normalized unit group of the group algebra FG. It is known that $V(FG) = 1 + \omega(FG)$. Finding the structure of this class of p-groups is a very interesting problem in modular group algebras.

The natural group homomorphism $G \to G/H$ defined by the rule $g \mapsto gH$ can be extended to F-algebra homomorphism from FG onto $F(G/H)$ with $\Gamma(H)$ as kernel. Since F is a field of characteristic p and G is a finite p-group, $\omega(FG)$ and hence $\Gamma(H)$ is a nilpotent ideal of FG. Therefore,

$$\frac{V(FG)}{1 + \Gamma(H)} \cong V(F(G/H)).$$

The anti-automorphism $g \mapsto g^{-1}$ of G can be extended linearly to an involution $*$ of FG. An element v of $V(FG)$ is said to be unitary if $v^* = v^{-1}$. The set of all such elements forms a subgroup of $V(FG)$ and is called the unitary subgroup $V_*(FG)$.

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In [6], Dennis posed the question: for which group G and ring R there exists a normal subgroup N of $\mathcal{U}(RG)$ such that $\mathcal{U}(RG) = N \rtimes G$? If N exists, then finding it is also a difficult problem. Several results regarding this problem in modular group algebras can be found in [8,9,12].

The conjugacy classes of a non-abelian group influence the study of its structure effectively. In [4], Coleman initiated the study of conjugacy classes of $V(FG)$. In this direction, firstly, Rao and Sandling [13] proved that p can never occur as the cardinality of any conjugacy class of $V(FG)$. Further, the results regarding conjugacy classes of $V(FG)$ can be found in [1,2].

Let v be an element of $V(FG)$. Then the centralizer of v in FG, $C_{FG}(v) = F \oplus C_{\omega(FG)}(v)$, where $C_{\omega(FG)}(v)$ is the centralizer of v in $\omega(FG)$. Thus the length of conjugacy class C_v of v in $V(FG)$ is

$$|C_v| = \frac{|V(FG)|}{|C_{V(FG)}(v)|} = \frac{|F|^{|G|-1}}{|C_{\omega(FG)}(v)|} = |F|^{|G|-(\dim_F(C_{FG}(v)))}.$$

Assume that $G = H \times A$ is a finite 2-group, where H is a non-abelian group of order 8 and A is a finite elementary abelian 2-group. In this paper, we find normal complement of G in the normalized unit group and unitary subgroup over the field with 2 elements. We also obtain length of conjugacy classes of units of FG over a finite field F of characteristic 2. Further, we provide cardinalities and representatives of conjugacy classes in $V(FH)$. Finally, we give a partition of $V_*(FH)$ into conjugacy classes.

Assume that $A = \langle x_1\rangle \times \langle x_2\rangle \times \cdots \times \langle x_k\rangle$ is an elementary abelian 2-group and F is a field of characteristic 2. Then

$$V(FG) = (\cdots((1+\Gamma\langle x_k\rangle) \rtimes (1+\Gamma\langle x_{k-1}\rangle)) \rtimes \cdots \rtimes (1+\Gamma\langle x_1\rangle)) \rtimes V(FH),$$

where $\Gamma\langle x_i\rangle$ is a 2-sided ideal of $F(H \times \langle x_1\rangle \times \langle x_2\rangle \times \cdots \times \langle x_i\rangle)$ and $(1+\Gamma\langle x_i\rangle)$ is an elementary abelian 2-group (see [10]). Note that for any $1 \le i \le k$, $(1+\Gamma\langle x_i\rangle) = \langle x_i\rangle \times W_i$. Thus $V(FG) = (A \times W) \rtimes V(FH)$, where $W = (\cdots((W_k \rtimes W_{k-1}) \rtimes W_{k-2}) \rtimes \cdots \rtimes W_1)$. Moreover, let A_i denote $V_*(F(H \times \langle x_1\rangle \times \langle x_2\rangle \times \cdots \times \langle x_i\rangle)) \cap (1+\Gamma\langle a_i\rangle)$ for any $1 \le i \le k$. Then $V_*(FG) = (\cdots((A_k \rtimes A_{k-1}) \rtimes A_{k-2}) \rtimes \cdots \rtimes A_1) \rtimes V_*(FH)$.

In section 2, we obtain structure of the normalized unit group $V(FG)$ and unitary subgroup $V_*(FG)$, when H is a non-abelain group of order 8 and F is the field with 2 elements. Let $\widehat{C_g}$ denote the sum of elements of conjugacy class of g in G. Then we have the following:

Theorem 1. *Let F be the field with 2 elements and $G = H \times A$, where H is a non-abelian group of order 8 and $A = \langle x_1\rangle \times \langle x_2\rangle \times \cdots \times \langle x_k\rangle$ is an elementary abelian 2-group. Let $\Gamma\langle x_i\rangle$ denote the 2-sided ideal of $F(H \times \langle x_1\rangle \times \langle x_2\rangle \times \cdots \times \langle x_i\rangle)$. If $(1+\Gamma\langle x_i\rangle) = \langle x_i\rangle \times W_i$ and $W = (\cdots((W_k \rtimes W_{k-1}) \rtimes W_{k-2}) \rtimes \cdots \rtimes W_1)$, then*

$$V(F(D_8 \times A)) = (W \rtimes (\langle 1+a+b\rangle \times \langle 1+\widehat{C_a}\rangle \times \langle 1+\widehat{C_b}\rangle)) \rtimes (D_8 \times A)$$

and

$$V(F(Q_8 \times A)) = (W \rtimes (\langle 1+a+b\rangle \times \langle a^2+\widehat{C_a}\rangle \times \langle a^2+\widehat{C_b}\rangle)) \rtimes (Q_8 \times A).$$

In section 3, we obtain the length of conjugacy class of elements of $V(FG)$, where $G = H \times C_2$ is a finite 2-group such that $C_2 = \langle y \rangle$ is the cyclic group of order 2. Clearly, $V(FG) = (1 + \Gamma\langle y \rangle) \rtimes V(FH)$. The main result of this section is as follows:

Theorem 2. *Let F be a finite field of characteristic 2 and $G = H \times C_2$ be a finite 2-group. Let $C_2 = \langle y \rangle$ and $v = (1+\alpha(y+1))w$ be a non-central element of $V(FG)$, where $\alpha \in FH$ and $w \in V(FH)$. If C'_w denotes the conjugacy class of w in $V(FH)$, then*

$$|C_v| = \begin{cases} |F|^{|H|-\dim_F(C_{FH}(\alpha w))}, & \text{if } w \in Z(FH) \\ |C'_w|^2, & \text{otherwise.} \end{cases}$$

In section 4, we obtain the length of conjugacy class of each unit of FH, where H is a non-abelian group of order 8, and the result is as follows:

Theorem 3. *Let F be the field with 2^n elements and H be a non-abelian group of order 8. Then the length of conjugacy class of each non-central element of $V(FH)$ is 2^{2n}.*

By using Theorem 2, we obtain the cardinality of conjugacy classes of elements of $V(F(H \times C_2))$ and then of $V(F(H \times C_2 \times C_2))$. Inductively, one can obtain the same for elements of $V(FG)$.

In section 5, we give representatives of conjugacy classes of $V(FH)$. Further, we provide a partition of the unitary subgroup $V_*(FH)$ into conjugacy classes.

2. Normal complement of G in $V(FG)$ over the field F with 2 elements

Let H be a non-abelian group of order 8. Then H can be either

$$Q_8 = \langle a, b \mid a^4 = 1, b^2 = a^2, b^{-1}ab = a^{-1} \rangle$$

or

$$D_8 = \langle a, b \mid a^4 = b^2 = 1, bab = a^{-1} \rangle.$$

The conjugacy classes of H are $\{1\}$, $\{a^2\}$, $\{a, a^3\}$, $\{b, a^2b\}$, $\{ab, a^3b\}$. Let F be the field with 2^n elements. It is known that class sums form a basis of the center of group algebra FH as an F-module. Clearly, $Z(V(FH))$ is the set of all central elements of FH with augmentation 1. Now any element of $Z(V(FH))$ can be written as $1 + s$, where s is an F-linear combination of the elements in $S = \{a^2 + 1, \widehat{C_a}, \widehat{C_b}, \widehat{C_{ab}}\}$. Since the product of any two elements of S is 0, $Z(V(FH))$ is an elementary abelian 2-group of order 2^{4n}.

Let $f(x)$ be a monic irreducible polynomial of degree n over F_2. Then $F \cong \frac{F_2[x]}{\langle f(x) \rangle}$. Assume that γ is the residue class of x mod $\langle f(x) \rangle$.

It is known that a non-abelian group H of order 8 has a normal complement in $V(FH)$, where F is the field with 2 elements (see [12]). In this section, we find a normal complement of D_8 and Q_8 in their normalized unit groups and unitary subgroups and finally obtain the structure of $V(F(D_8 \times A))$ and $V(F(Q_8 \times A))$. The following lemma plays a very crucial role in it.

Lemma 4. *Let F be the field with 2^n elements and H be a non-abelian group of order* 8. *Let V be the normalized unit group of the group algebra FH and $Z(V)$ be its center. Then*

$$\frac{V}{Z(V)} = \left(\prod_{i=0}^{n-1}\langle 1+\gamma^i(a+1)\rangle \times \prod_{i=0}^{n-1}\langle 1+\gamma^i(b+1)\rangle \times \prod_{i=0}^{n-1}\langle 1+\gamma^i(a+b)\rangle\right) Z(V)$$

is an elementary abelian 2-group of order 2^{3n}.

Proof. Observe that for any $v \in V$, $v^2 \in Z(V)$. It is known that nilpotency class of $V(FH)$ is 2 (see Theorem 2 of [11]). Thus $\frac{V}{Z(V)}$ is an elementary abelian 2-group.

Suppose $v = 1 + \sum_{i=1}^{3}\alpha_i(a^i+1) + \sum_{j=0}^{3}\beta_j(a^jb+1)$ is a non-central element of V. By using the identity $(gh+1) = (g+1)+(h+1)+(g+1)(h+1)$, one can write $v = 1+y+z$, where $y = \alpha(a+1)+\beta(b+1)+\delta(ab+1)$ for some $\alpha, \beta, \delta \in F$ and $z \in Z(FH)$. As product of any element of H and an element of S is in S, it implies that $v \equiv (1+y) \bmod Z(V)$. Further,

$$\begin{aligned} 1+y &= 1+\alpha(a+1)+\beta(b+1)+\delta(a+b)+\delta(a+1)(b+1) \\ &= (1+\alpha(a+1)+\beta(b+1)+\delta(a+b)) \bmod Z(V). \end{aligned}$$

Thus $v \equiv (1+\alpha(a+1))(1+\beta(b+1))(1+\delta(a+b)) \bmod Z(V)$. Consider the subgroups

$$A = \{(1+\alpha(a+1))Z(V) \mid \alpha \in F\} = \prod_{i=0}^{n-1}\langle 1+\gamma^i(a+1)\rangle Z(V),$$

$$B = \{(1+\alpha(b+1))Z(V) \mid \alpha \in F\} = \prod_{i=0}^{n-1}\langle 1+\gamma^i(b+1)\rangle Z(V)$$

and

$$C = \{(1+\alpha(a+b))Z(V) \mid \alpha \in F\} = \prod_{i=0}^{n-1}\langle 1+\gamma^i(a+b)\rangle Z(V)$$

of $V/Z(V)$. Therefore, any element of V mod $Z(V)$ can be written as a product of elements of A, B and C. Since $\left|\frac{V}{Z(V)}\right| = 2^{3n}$ and each of the subgroups A, B and C has order 2^n, we have

$$\frac{V}{Z(V)} = A \times B \times C.$$

□

Now we give a proof of Theorem 1.

Proof of Theorem 1. It follows from Lemma 4 that an element of $V(FH)$ can be written as $a^i b^j (1+a+b)^k z$, where $0 \le i, j, k \le 1$ and $z \in Z(V(FH))$. Write $Z(V(FD_8)) = \langle a^2 \rangle \times \langle 1+\widehat{C_a} \rangle \times \langle 1+\widehat{C_b} \rangle \times \langle a^2+\widehat{C_{ab}} \rangle$. Define a map

$$\theta : V(FD_8) \to D_8$$

by the rule $a \mapsto a, b \mapsto b, (1+a+b) \mapsto e, (1+\widehat{C_a}) \mapsto e, (1+\widehat{C_b}) \mapsto e$. Since $V' \subseteq Z(V)$ and $(1+a+b)^2 = (a^2+\widehat{C_{ab}})$, we get that θ is an epimorphism with $\ker(\theta) = (\langle 1+a+b \rangle \times \langle 1+\widehat{C_a} \rangle \times \langle 1+\widehat{C_b} \rangle)$. If $i : D_8 \to V(FD_8)$ denotes the inclusion map, then $\theta \circ i$ is an identity map on D_8 and hence the short exact sequence

$$\{1\} \to \ker(\theta) \to V(FD_8) \xrightarrow{\theta} D_8 \to \{1\}$$

splits. Thus $V(FD_8) = (\langle 1+a+b \rangle \times \langle 1+\widehat{C_a} \rangle \times \langle 1+\widehat{C_b} \rangle) \rtimes D_8$.

Now it follows from [9] that if $V(FH) = N \rtimes H$, then $V_*(FH) = (N \cap V_*(FH)) \rtimes H$. It is known that if F is the field with 2^n elements, then $|V_*(FD_8)| = 2^{6n}$ (see Theorem 2, [3]) and so $|N \cap V_*(FD_8)| = 2^3$. Since $(1+a+b)^2 = (a^2+\widehat{C_{ab}})$ is a unitary unit, $V_*(FD_8) = (\langle a^2+\widehat{C_{ab}} \rangle \times \langle 1+\widehat{C_a} \rangle \times \langle 1+\widehat{C_b} \rangle) \times D_8$.

To find the structure of $V(FQ_8)$, write $Z(V(FQ_8)) = \langle a^2 \rangle \times \langle a^2+\widehat{C_a} \rangle \times \langle a^2+\widehat{C_b} \rangle \times \langle 1+\widehat{C_{ab}} \rangle$. Define $\delta : V(FQ_8) \to Q_8$ by the rule

$$a \mapsto a, b \mapsto b, (1+a+b) \mapsto e, (a^2+\widehat{C_a}) \mapsto e, (a^2+\widehat{C_b}) \mapsto e.$$

One can obtain that $V(FQ_8) = (\langle 1+a+b \rangle \times \langle a^2+\widehat{C_a} \rangle \times \langle a^2+\widehat{C_b} \rangle) \rtimes Q_8$ and $V_*(FQ_8) = (\langle 1+\widehat{C_{ab}} \rangle \times \langle a^2+\widehat{C_a} \rangle \times \langle a^2+\widehat{C_b} \rangle) \times Q_8$.

If N is a normal complement of H in $V(FH)$, then $(W \rtimes N)$ is a normal complement of G in $V(FG)$ (see [9]). Hence the result follows. □

As a corollary, we obtain the structure of $V_*(FG)$:

COROLLARY 5

Let $A_i = V_i \times \langle a_i \rangle$ *for all* $1 \le i \le k$. *Then*

$$V_*(F(D_8 \times A)) = (U \rtimes (\langle a^2+\widehat{C_{ab}} \rangle \times \langle 1+\widehat{C_a} \rangle \times \langle 1+\widehat{C_b} \rangle)) \rtimes (D_8 \times A)$$

and

$$V_*(F(Q_8 \times A)) = (U \rtimes (\langle 1+\widehat{C_{ab}} \rangle \times \langle a^2+\widehat{C_a} \rangle \times \langle a^2+\widehat{C_b} \rangle)) \rtimes (Q_8 \times A),$$

where $U = (\cdots((V_k \rtimes V_{k-1}) \rtimes V_{k-2}) \rtimes \cdots \rtimes V_1)$.

Proof. Let M denote the normal complement of H in $V_*(FH)$. Then

$$V(FG) = (U \times A) \rtimes (M \times H) = (U \rtimes M) \rtimes (H \times A)$$

and the result follows. □

3. Class length of elements of $V(F(H \times C_2))$

Let $G = H \times C_2$ be a finite 2-group, where $C_2 = \langle y \rangle$ is the cyclic group of order 2. In this section, we provide the length of conjugacy class of elements of $V(FG)$.

Proof of Theorem 2. Let $x = x_0 + x_1 y$ be a non-central element of FG, where $x_0, x_1 \in FH$. Consider the homomorphism $\phi : G \to H$ defined by the rule $h \mapsto h$ and $y \mapsto e$, for any $h \in H$. It can be extended to algebra homomorphism ϕ' from FG onto FH with kernel $\Gamma\langle y \rangle$. If $x_0 + x_1 \in Z(FH)$, then $C_{FG}(x) = C_{FG}(x_1\hat{y})$, where $\hat{y} = 1 + y$. Note that for any $c = c_0 + c_1 y$ in $C_{FG}(x_1\hat{y})$, the element $\phi'(c) \in C_{FH}(x_1)$. Thus

$$\phi'_{|C_{FG}(x_1\hat{y})} : C_{FG}(x_1\hat{y}) \to C_{FH}(x_1)$$

is a homomorphism. Since $\Gamma\langle y \rangle \subseteq C_{FG}(x_1\hat{y})$ and ϕ' is onto, we have $\frac{C_{FG}(x_1\hat{y})}{\Gamma(\langle y \rangle)} \cong C_{FH}(x_1)$ and therefore, $|C_{FG}(x)| = |F|^{|H|+\dim_F(C_{FH}(x_1))}$. Next assume that $x_0 + x_1 \notin Z(FH)$. Further, note that if $c_0 + c_1 y \in C_{FH}(x)$, then $c_0 + c_1$ is an element of $C_{FH}(x_0 + x_1)$. Hence

$$\phi'_{|C_{FG}(x)} : C_{FG}(x) \to C_{FH}(x_0 + x_1)$$

is an F-homomorphism. It is also an epimorphism as for any element $z \in C_{FH}(x_0+x_1)$, $z\hat{y}$ is an element of $C_{FG}(x)$. Note that the elements of the set $\{c_0 + c_1 y \in C_{FG}(x) \mid c_0 = c_1\}$, which form the kernel of this epimorphism, have one-to-one correspondence with $C_{FH}(x_0 + x_1)$. Thus $|C_{FG}(x)| = |C_{FH}(x_0 + x_1)|^2$.

Therefore, if $v = (1 + \alpha(y + 1))w = (w + \alpha w) + (\alpha w)y$ is a non-central element of $V(FG)$, then

$$|C_v| = \begin{cases} |F|^{|H|-\dim_F(C_{FH}(\alpha w))}, & \text{if } w \in Z(FH) \\ |C'_w|^2, & \text{otherwise.} \end{cases}$$

□

4. Cardinality of conjugacy classes in $V(FH)$, where H is a group of order 8

The aim of this section is to compute the cardinality of conjugacy class of each non-central element of $V(FH)$, where H is a non-abelian group of order 8. Finally, we obtain class length of any element of $V(F(H \times A))$. So, first we give a proof of Theorem 3.

Proof of Theorem 3. Let $v = 1 + w$, where $w \in \omega(FH)$ be a non-central element of $V(FH)$. Let $C_V(v)$ denote the centralizer of v in $V(FH)$. Consider the subgroup $T = \prod_{i=0}^{n-1} \langle (1+\gamma^i w) \rangle$ of $V(FH)$. Since $\omega^4(FH) = 0$, the exponent of $V(FH)$ is 4. If $o(v)$ is 2, then T is an elementary abelian 2-group and so $T \cdot Z(V(FH)) = T \times Z(V(FH))$ is of order 2^{5n}. Now assume that order of v is 4. Let $T^2 = \{t^2 \mid t \in T\}$ and $T[2] = \{t \in T \mid t^2 = 1\}$. The map $\phi : T \to T^2$ defined by the rule $y \mapsto y^2$ is a group homomorphism with $\ker(\phi) = T[2]$. Hence $|T^2| = \frac{|T|}{|T[2]|} = \frac{4^n}{2^n} = 2^n$. Since $T \cap Z(V(FH)) = T^2$,

$$|T \cdot Z(V(FH))| = \frac{|T| \cdot |Z(V(FH))|}{|T \cap Z(V(FH))|} = 2^{5n}.$$

Since in both the cases $T \cdot Z(V(FH)) \subseteq C_V(v)$, we have $|C_V(v)| \geq 2^{5n}$ and so $|C_v| \leq 2^{2n}$. As it is known from [2] that for any non-central element $v \in V(FH)$, $|C_v|$ is divisible by 2^{2n}, the result follows. □

Note that any $x \in FH$ can be written as $\beta + w$, where $\beta \in F$ and $w \in \omega(FH)$ and so $x = \beta v$, where $v = (1 + \beta^{-1} w) \in V(FH)$. Hence for any non-central element $x \in FH$,

$$|C_{FH}(x)| = |C_{FH}(v)| = |F| \cdot |C_V(v)| = 2^{6n}.$$

COROLLARY 6

Let $G = H \times C_2$. If $v = (1 + \alpha(y+1))w$ is a non-central element of $V(FG)$, then

$$|C_v| = \begin{cases} |F|^2, & \text{if } w \in Z(V(FH)) \\ |F|^4, & \text{otherwise}. \end{cases}$$

Proof. Since $v = (w + \alpha w) + \alpha w y$ is a non-central element of $V(FG)$, if $w \in Z(V(FH))$, then $\alpha w \notin Z(FH)$. So from Theorem 2 and Theorem 3, we have that $|C_{FG}(v)| = |C_{FG}(\alpha w \hat{y})| = |F|^{|H| + \dim_F(C_{FH}(\alpha w))} = |F|^{14}$. But if $w \notin Z(V(FH))$, then $|C_{FG}(v)| = |C_{FH}(w)|^2 = |F|^{12}$. □

Now we give an idea to obtain the length of conjugacy class of any element of $V(FG)$, where $G = H \times \langle y_1 \rangle \times \langle y_2 \rangle$ such that $o(y_1) = o(y_2) = 2$. Let $H_1 = H \times \langle y_1 \rangle$. Hence $V(FG) = (1 + \Gamma\langle y_2 \rangle) \rtimes V(FH_1)$.

COROLLARY 7

Let $G = H \times \langle y_1 \rangle \times \langle y_2 \rangle$, where $o(y_i) = 2$ for all $1 \leq i \leq 2$. Let $H_1 = H \times \langle y_1 \rangle$. Consider the map $\phi : H_1 \to H$ defined by the rule $y_1 \mapsto e$. If $v = (1 + \alpha(y_2 + 1))w$, where $\alpha \in FH_1$ and $w \in V(FH_1)$ is a non-central element of $V(FG)$, then

$$|C_v| = \begin{cases} |F|^2, & \text{if } w \in Z(V(FH_1)) \text{ and } \phi(\alpha w) \in Z(FH) \\ |F|^4, & \text{if } w \in Z(V(FH_1)) \text{ and } \phi(\alpha w) \notin Z(FH) \\ |F|^4, & \text{if } w \notin Z(V(FH_1)) \text{ and } \phi(w) \in Z(V(FH)) \\ |F|^8, & \text{if } w \notin Z(V(FH_1)) \text{ and } \phi(w) \notin Z(V(FH)). \end{cases}$$

Proof. Let $v = (w + \alpha w) + \alpha w y_2$, where $w \in Z(V(FH_1))$ be a non-central element of $V(FG)$. Then $C_{FG}(v) = C_{FG}(\alpha w \widehat{y_2})$. Hence from Theorem 2, $|C_{FG}(v)| = |F|^{H_1 + \dim_F(C_{FH_1}(\alpha w))}$. Now either $\alpha w \in V(FH_1)$ or $1 + \alpha w \in V(FH_1)$. Thus from the last corollary, we have that if $\phi(\alpha w) \in Z(FH)$, then $|C_{FH_1}(\alpha w)| = |F|^{14}$ and if $\phi(\alpha w) \notin Z(FH)$, then $|C_{FH_1}(\alpha w)| = |F|^{12}$.

Further, if $w \notin Z(VH_1)$, then $|C_v| = |C'_w|^2$ implies that $|C_v| = |F|^4$, when $\phi(w) \in Z(V(FH))$ and $|C_v| = |F|^8$, when $\phi(w) \notin Z(V(FH))$. □

Similarly, one can obtain the length of the conjugacy class of any element of $V(FG)$, where $G = H \times A$.

5. Representatives of conjugacy classes in $V(FH)$ and $V_*(FH)$

In this section, we provide conjugacy class representatives of $V(FH)$. For that we need the following lemma:

Lemma 8. *Let F be the field with 2^n elements and H be a non-abelian group of order* 8. *Let V be the normalized unit group of the group algebra FH and $Z(V)$ be its center. Let $v \in V$ be a non-central element. Then the elements of the set*

$$A_v = \{(v, w) \mid w \in V\}$$

form a central subgroup of order 2^{2n}.

Proof. Since $V' \subseteq Z(V)$, the elements of the set A_v are central and have exponent 2. If $(v, w_1), (v, w_2) \in A_v$, then

$$(v, w_1)(v, w_2) = v^{-1}w_2^{-1}v(v^{-1}w_1^{-1}vw_1)w_2 = (v, w_1w_2) \in A_v.$$

For a non-central element $v \in V$, define a map $\psi_v : V(FG) \to A_v$ by the assignment $w \mapsto (v, w)$. Clearly, ψ_v is an epimorphism with $C_V(v)$ as kernel. By Theorem 3, $|C_V(v)| = 2^{5n}$. Thus $|A_v| = 2^{2n}$. □

Any conjugate of v in V is of the form $w^{-1}vw = v(v, w)$, where $(v, w) \in A_v$. Note that index of A_v in $Z(V)$ is 2^{2n}. The following theorem provides conjugacy class representatives corresponding to non-central elements of V.

Theorem 9. *Let $V = \bigcup_{\alpha,\beta,\delta\in F} v_{\alpha,\beta,\delta}Z(V)$, where*

$$v_{\alpha,\beta,\delta} = (1 + \alpha(a + 1))(1 + \beta(b + 1))(1 + \delta(a + b)).$$

Then the conjugacy class representatives corresponding to non-central elements of V are $t_1^{(\alpha,\beta,\delta)}v_{\alpha,\beta,\delta},\ t_2^{(\alpha,\beta,\delta)}v_{\alpha,\beta,\delta}, \ldots,\ t_{2^{2n}}^{(\alpha,\beta,\delta)}v_{\alpha,\beta,\delta}$, where $\{t_i^{(\alpha,\beta,\delta)} \mid 1 \le i \le 2^{2n}\}$ is a transversal of $A_{v_{\alpha,\beta,\delta}}$ in $Z(V)$.

Proof. Since order of $Z(V)$ is 2^{4n}, the number of non-central elements in V is $(2^{7n} - 2^{4n})$. Therefore, the number of conjugacy classes corresponding to non-central elements are $2^{2n}(2^{3n} - 1)$. Consider the element $v_{\alpha,\beta,\delta}$, where $\alpha, \beta, \delta \in F$ such that $(\alpha, \beta, \delta) \neq (0, 0, 0)$. Then

$$cl(v_{\alpha,\beta,\delta}) = \{v_{\alpha,\beta,\delta}z \mid z \in A_{v_{\alpha,\beta,\delta}}\}.$$

If $\{t_1^{(\alpha,\beta,\delta)} = 1,\ t_2^{(\alpha,\beta,\delta)}, \ldots,\ t_{2^{2n}}^{(\alpha,\beta,\delta)}\}$ is a transversal of $A_{v_{\alpha,\beta,\delta}}$ in $Z(V)$, then

$$\{v_{\alpha,\beta,\gamma} \cdot z \mid z \in Z(V)\} = \bigcup_{i=1}^{2^{2n}} t_i^{(\alpha,\beta,\delta)}(v_{\alpha,\beta,\delta}A_{v_{\alpha,\beta,\delta}}).$$

Hence the conjugacy class representatives corresponding to non-central elements of the coset $v_{\alpha,\beta,\delta}Z(V)$ are $v_{\alpha,\beta,\delta},\ t_2^{(\alpha,\beta,\delta)}v_{\alpha,\beta,\delta}, \ldots,\ t_{2^{2n}}^{(\alpha,\beta,\delta)}v_{\alpha,\beta,\delta}$. □

Let $V_*(FH)$ denote the unitary subgroup of V. Since any non-abelian group of order 8 is an extra-special 2-group, $V_*(FH)$ is a normal subgroup of V (see Lemma 1, [3]). Note that all the central elements of V are unitary. Therefore, $Z(V) = Z(V_*(FH))$. Now we obtain a partition of $V_*(FQ_8)$ and $V_*(FD_8)$ into conjugacy classes.

5.1 *Conjugacy classes in* $V_*(FQ_8)$

Let V_* denote the unitary subgroup of $V(FQ_8)$. Note that order of $V_*(FQ_8)$ is 2^{4n+2}. The following lemma plays an important role in obtaining a partition of V_*.

Lemma 10. *Let V_* be the unitary subgroup of $V(FQ_8)$. Then the elements of V_* are of the form $z \cdot g$, where $z \in Z(V(FQ_8))$ and $g \in Q_8$.*

Proof. The result follows from Theorem 2.3 of [5]. □

Theorem 11. *Let V_* be the unitary subgroup of $V(FQ_8)$. Then the $3 \cdot 2^{2n}$ conjugacy class representatives corresponding to non-central elements of V_* are given by*

$$\{(1+\rho\widehat{C_a})(1+\eta\widehat{C_b})g \mid \rho, \eta \in F;\ g = a, b\}$$

and

$$\{(1+\rho\widehat{C_{ab}})(1+\eta\widehat{C_b})ab \mid \rho, \eta \in F\}.$$

Proof. Assume that $v = a \cdot z$, where $z \in Z(V(FQ_8))$. Then for any $w = (1+\alpha(a+1))(1+\beta(b+1))(1+\delta(a+b))z' \in V(FQ_8)$, where $z' \in Z(V(FQ_8))$, we have that

$$w^{-1}aw = (1+\delta(a+b))^{-1}(1+\beta(b+1))^{-1}a(1+\beta(b+1))(1+\delta(a+b)).$$

As $(1+\beta(b+1))^{-1}a(1+\beta(b+1)) = a(1+(\beta+\beta^2)\widehat{C_b})(1+\beta^2(1+a^2))$ and $(1+\delta(a+b))^{-1}a(1+\delta(a+b)) = (1+\delta(a+b))^2(1+\delta(a+b))a(1+\delta(a+b)) = (1+\delta(a+b))^2a(1+\delta\widehat{C_b})$, we get that $cl(a) = \{w^{-1}vw \mid w \in V(FQ_8)\} = \{a(1+(\delta^2+\beta^2)(a^2+1))(1+(\delta+\beta^2+\beta)\widehat{C_b})(1+\delta^2\widehat{C_{ab}}) \mid \beta, \delta \in F\}$. If $x \in (1+\rho_1\widehat{C_a})(1+\eta_1\widehat{C_b})cl(a) \cap (1+\rho_2\widehat{C_a})(1+\eta_2\widehat{C_b})cl(a)$, then for some $w_1 = (1+\alpha_1(a+1))(1+\beta_1(b+1))(1+\delta_1(a+b))$ and $w_2 = (1+\alpha_2(a+1))(1+\beta_2(b+1))(1+\delta_2(a+b)) \in V(FQ_8)$,

$$\begin{aligned} x &= (1+\rho_1\widehat{C_a})(1+\eta_1\widehat{C_b})(w_1^{-1}aw_1) \\ &= (1+\rho_2\widehat{C_a})(1+\eta_2\widehat{C_b})(w_2^{-1}aw_2) \end{aligned}$$

As $w_i^{-1}aw_i \in cl(a)$, write $w_i^{-1}aw_i = a \cdot z_i$, where for any $1 \le i \le 2$, $z_i = (1+(\delta_i^2+\beta_i^2)(a^2+1))(1+(\delta_i+\beta_i^2+\beta_i)\widehat{C_b})(1+\delta_i^2\widehat{C_{ab}})$. So the above equation gives

$$(1+\rho_1\widehat{C_a})(1+\eta_1\widehat{C_b})az_1 = (1+\rho_2\widehat{C_a})(1+\eta_2\widehat{C_b})az_2.$$

Since $Z(V(FQ_8))$ is an elementary abelian 2-group, we have $(1+(\rho_1+\rho_2)\widehat{C_a})(1+(\eta_1+\eta_2)\widehat{C_b})z_1z_2 = 1$. Hence

$$\begin{aligned} 1 &+ (\rho_1+\rho_2)\widehat{C_a} + (\beta_1^2+\delta_1^2+\beta_2^2+\delta_2^2)(a^2+1) + (\delta_1^2+\delta_2^2)\widehat{C_{ab}} \\ &+ (\beta_1(1+\beta_1)+\delta_1+\beta_2(1+\beta_2)+\delta_2+\eta_1+\eta_2)\widehat{C_b} = 1 \end{aligned}$$

Comparing the coefficients of (a^2+1), $(ab+a^3b)$, $(b+a^2b)$ and $(a+a^3)$, we get $\rho_1 = \rho_2$ and $\eta_1 = \eta_2$. Therefore,

$$\left\{a.z \mid z \in Z(V)\right\} = \bigcup_{\substack{\rho,\eta\in F \\ \text{disjoint}}} (1+\rho\widehat{C_a})(1+\eta\widehat{C_b})cl(a).$$

Now assume that $v = b \cdot z$. Then $cl(b)$ is given by the following set $\{b(1+(\alpha^2+\delta^2)(a^2+1))(1+(\alpha+\alpha^2+\delta)\widehat{C_a})(1+\delta^2\widehat{C_{ab}}) \mid \alpha, \delta \in F\}$. Similarly one can show that

$$\left\{b \cdot z \mid z \in Z(V)\right\} = \bigcup_{\substack{\rho,\eta\in F \\ disjoint}} (1+\rho\widehat{C_a})(1+\eta\widehat{C_b})cl(b).$$

Assume that $\alpha \in F$ is fixed. Then for different β and δ, the conjugates of ab are distinct. Therefore, for fixed α, the distinct 2^{2n} elements $(1+\alpha(a+1))(1+\beta(b+1))(1+\delta(a+b))$, $\beta, \delta \in F$ determine the conjugacy class of ab. Hence $cl(ab)$ in V is given by the set $\{abz\}$, where z is an element of the form

$$(1+(\alpha^2+\beta^2)(a^2+1))(1+(\delta+\alpha+\alpha^2)(\widehat{C_a}))(1+(\delta+\beta+\beta^2)(\widehat{C_b}))$$

and then it can be proved that

$$\{ab \cdot z \mid z \in Z(V)\} = \bigcup_{\substack{\rho,\eta\in F \\ \text{disjoint}}} (1+\rho(\widehat{C_{ab}}))(1+\eta(\widehat{C_b}))cl(ab).$$

□

5.2 *Conjugacy classes of $V_*(FD_8)$*

In this section, we provide conjugacy class representatives for the unitary subgroup $V_*(FD_8)$ of $V(FD_8)$. Note that the order of $V_*(FD_8)$ is 2^{6n}. We need the following lemma to obtain the class representatives.

Lemma 12. *Let $V_*(FD_8)$ denote the unitary subgroup of $V(FD_8)$. Then any of its element can be written as $zu_\beta v_\alpha$, where $u_\beta = 1+\beta(b+1)$, $v_\alpha = 1+\alpha(ab+1)$ for some $\alpha, \beta \in F$, and $z \in Z(V(FD_8))$.*

Proof. It is known from [7] that $V_*(FD_8) = H \rtimes K$, where

$$H = \{1+\beta_0\widehat{C_a}+\beta_1(a^2+1)+\beta_2(b+1)+\beta_3\widehat{C_{ab}}+\beta_4(a^2b+1) \mid \beta_i \in F\}$$

and

$$K = \left\{1+\alpha\left(\sum_{i=1}^{3} a^i\right)+\alpha\left(\sum_{j=0}^{2} a^j b\right) \;\middle|\; \alpha \in F\right\}.$$

If $h = 1+\beta_0\widehat{C_a}+\beta_1(a^2+1)+\beta_2(b+1)+\beta_3\widehat{C_{ab}}+\beta_4(a^2b+1)$ is any element of H, then $h \equiv (1+(\beta_2+\beta_4)(b+1)) \bmod Z(V(FD_8))$. Assume that $H_1 = \prod_{i=0}^{n-1}\langle 1+\gamma^i(b+1)\rangle$. Since H is abelian, it follows that $H = Z(V(FD_8)) \times H_1$.

Further, suppose that $k = 1+\alpha\Big(\sum_{i=1}^{3} a^i + \sum_{j=0}^{2} a^j b)\Big)$ for some $\alpha \in F$, is an element of K. Then one can write k as $(1+\alpha(ab+1))z_1$, where $z_1 = (1+(\alpha+\alpha^2)(a^2+1)+\alpha(a+a^3+b+a^2b)+\alpha^2(ab+a^3b)) \in Z(V(FD_8))$. Now the result follows. □

Theorem 13. *Let $V_*(FD_8)$ be the unitary subgroup of $V(FD_8)$. Suppose that $u_\beta = 1+\beta(b+1)$ and $v_\alpha = 1+\alpha(ab+1)$ for some $\alpha, \beta \in F$. Then the $2^{2n}(2^{2n}-1)$ conjugacy class representatives corresponding to non-central elements of $V_*(FD_8)$ are*

$$\{(1+\lambda\widehat{C_b})(1+\mu\widehat{C_{ab}})v \mid \lambda, \mu \in F;\ v = u_\beta, v_\alpha \textit{ for } \alpha, \beta \in F^*\}$$

and

$$\{(1+\sigma\widehat{C_b})(1+\nu\widehat{C_a})(u_\beta v_\alpha) \mid \sigma, \nu \in F;\ \alpha, \beta \in F^*\}.$$

Proof. Note that $cl(u_\beta) = \{w^{-1}u_\beta w \mid w \in V(FD_8)\}$. Assume that $w = (1+\eta_1(a+1))(1+\eta_2(b+1))(1+\eta_3(a+b))z$, where $\eta_1, \eta_2, \eta_3 \in F$ and $z \in Z(V(FD_8))$. Then $cl(u_\beta) = \{u_\beta(1+(\beta(\eta_1+\eta_3+\eta_1^2)+\beta^2(\eta_1+\eta_3+\eta_1^2+\eta_3^2))\widehat{C_{ab}})\ (1+\beta^2(\eta_1^2+\eta_3^2)(a^2+1))(1+(\beta^2(\eta_1+\eta_1^2+\eta_3+\eta_3^2)+\beta\eta_3^2)\widehat{C_a})(1+\beta(1+\beta)(\eta_3^2+\eta_1^2)\widehat{C_b}) \mid \eta_i \in F\}$.

If $y \in (1+\lambda\widehat{C_b})(1+\mu\widehat{C_{ab}})\ cl(u_\beta) \ \cap\ (1+\lambda'\widehat{C_b})(1+\mu'\widehat{C_{ab}})\ cl(u_\beta)$, then $y = (1+\lambda\widehat{C_b})(1+\mu\widehat{C_{ab}})(w^{-1}u_\beta w) = (1+\lambda'\widehat{C_b})(1+\mu'\widehat{C_b})((w')^{-1}u_\beta w')$ for some $w' = (1+\eta_1'(a+1))(1+\eta_2'(b+1))(1+\eta_3'(a+b))$ in $V(FD_8)$. It implies that

$$\begin{aligned}1 &+ (\beta^2(\eta_1+\eta_3+\eta_1'+\eta_3'+\eta_1^2+\eta_3^2+(\eta_1')^2+(\eta_3')^2)+\beta(\eta_3^2+(\eta_3')^2))\widehat{C_a}\\ &+(\beta(1+\beta)(\eta_1^2+\eta_3^2+(\eta_1')^2+(\eta_3')^2)+\lambda+\lambda')\widehat{C_b}\\ &+(\beta^2(\eta_1+\eta_3+\eta_1'+\eta_3'+\eta_1^2+\eta_3^2+(\eta_1')^2+(\eta_3')^2)+\beta(\eta_1+\eta_3+\eta_1'+\eta_3'\\ &+\eta_1^2+(\eta_1')^2)+\mu+\mu')\widehat{C_{ab}}+(\beta^2(\eta_1^2+\eta_3^2+(\eta_1')^2+(\eta_3')^2))(a^2+1) = 1.\end{aligned}$$

Thus $\lambda = \lambda'$ and $\mu = \mu'$. Therefore,

$$\{u_\beta \cdot z \mid z \in Z(V)\} = \bigcup_{\substack{\lambda,\mu\in F\\ \text{disjoint}}} (1+\lambda\widehat{C_b})(1+\mu\widehat{C_{ab}})cl(u_\beta).$$

Now consider the element $v_\alpha = 1+\alpha(ab+1)$ for some $\alpha \in F$. Then $cl(v_\alpha)$ is $\{v_\alpha(1+((\eta_1+\eta_1^2)(\alpha+\alpha^2)+\alpha^2\eta_2(1+\eta_2)+\alpha\eta_3)\widehat{C_b})\ (1+(\alpha(1+\alpha)\eta_1^2+\alpha\eta_2^2(1+\alpha))\widehat{C_{ab}})\ (1+(\alpha^2(\eta_1^2+\eta_2^2))(a^2+1))\ (1+(\eta_1(1+\eta_1)\alpha^2+\alpha(1+\alpha)\eta_2(1+\eta_2)+\alpha\eta_3)\widehat{C_a}) \mid \eta_1, \eta_2, \eta_3 \in F\}$.

Similarly, one can prove that

$$\{v_\alpha \cdot z \mid z \in Z(V)\} = \bigcup_{\substack{\lambda,\mu\in F\\ \text{disjoint}}} (1+\lambda\widehat{C_b})(1+\mu\widehat{C_{ab}})cl(v_\alpha).$$

Now consider the element $u_\beta v_\alpha = (1+\beta(b+1))(1+\alpha(ab+1))$, where $\alpha, \beta \in F^*$. Since $w^{-1}(u_\beta v_\alpha)w = (w^{-1}u_\beta w)(w^{-1}v_\alpha w)$, the conjugacy class of $u_\beta v_\alpha$ can be obtained.

Further, if $y \in (1+\sigma\widehat{C_b})(1+\nu\widehat{C_a})cl(u_\beta v_\alpha) \cap (1+\sigma'\widehat{C_b})(1+\nu'\widehat{C_a})cl(u_\beta v_\alpha)$, then for some $w, w' \in V(FD_8)$,

$$y = (1+\sigma\widehat{C_b})(1+\nu\widehat{C_a})(w^{-1}u_\beta v_\alpha w) = (1+\sigma'\widehat{C_b})(1+\nu'\widehat{C_a})((w')^{-1}u_\beta v_\alpha w').$$

Comparing coefficients of (a^2+1), $(ab+a^3b)$, $(b+a^2b)$ and $(a+a^3)$ in the above equation respectively, we have

$$\beta^2(\eta_1^2+\eta_3^2+(\eta_1')^2+(\eta_3')^2)+\alpha^2(\eta_1^2+\eta_2^2+(\eta_1')^2+(\eta_2')^2)=0 \tag{1}$$

$$\begin{aligned}&\beta^2(\eta_1+\eta_3+(\eta_1')+(\eta_3')+\eta_1^2+\eta_3^2+(\eta_1')^2+(\eta_3')^2)+\alpha(1+\alpha)\\&\quad(\eta_1^2+\eta_2^2+(\eta_1')^2+(\eta_2')^2)+\beta(\eta_1^2+\eta_3+(\eta_1')^2+(\eta_3')+\eta_1+\eta_1')=0,\end{aligned} \tag{2}$$

$$\begin{aligned}&\beta(1+\beta)(\eta_1^2+\eta_3^2+(\eta_1')^2+(\eta_3')^2)+\alpha(1+\alpha)(\eta_1^2+\eta_1+(\eta_1')^2+(\eta_1'))\\&\quad+\alpha^2(\eta_2^2+\eta_2+(\eta_2')^2+(\eta_2'))+\alpha(\eta_3+\eta_3')+\sigma+\sigma'=0,\end{aligned} \tag{3}$$

$$\begin{aligned}&\beta^2(\eta_1^2+\eta_1+(\eta_1')^2+\eta_1'+\eta_3^2+\eta_3+(\eta_3')^2+\eta_3')+\beta(\eta_3^2+(\eta_3')^2)+\alpha(\eta_3\\&\quad+\eta_3')+\alpha(1+\alpha)(\eta_2^2+\eta_2+(\eta_2')^2+\eta_2')+\alpha^2(\eta_1^2+\eta_1+(\eta_1')^2+\eta_1')+\\&\quad\nu+\nu'=0.\end{aligned} \tag{4}$$

If $\alpha=\beta\in F^*$, then (1) implies that $\eta_2+\eta_2'=\eta_3+\eta_3'$ and putting this in (2), we get $\beta^2(\eta_1+\eta_1'+\eta_3+\eta_3')+\beta(\eta_1+\eta_1'+\eta_3+\eta_3'+\eta_2^2+(\eta_2')^2)=0$. Then it follows from (3) and (4) that $\sigma=\sigma'$ and $\nu=\nu'$.

Further, if $\alpha\neq\beta$, then substituting (1) in (2), we get the relation $\alpha(1+\beta)(\eta_2+\eta_2'+\eta_3+\eta_3')+\alpha(\eta_2+(\eta_2')^2)+\beta(\eta_3+(\eta_3')^2)=0$. Then it follows from (3) and (4) that $\sigma=\sigma'$ and $\nu=\nu'$. Therefore,

$$\{(u_\beta v_\alpha)\cdot z \mid z\in Z(V)\} = \bigcup_{\substack{\sigma,\nu\in F\\ \text{disjoint}}}(1+\sigma\widehat{C_b})(1+\nu\widehat{C_a})cl(u_\beta v_\alpha).$$

□

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