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C^*-algebra-valued partial metric space and fixed point theorems

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Abstract. In this paper, we introduce the notion of C^*-algebra-valued partial metric space which is more general than partial metric space. Some fixed point results using C-class functions on such spaces are obtained. Moreover, some illustrated examples are also provided.

Keywords. C^*-algebra-valued partial metric space; C_*-class function; fixed point.

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1. Introduction

Matthews [10] introduced the notion of a partial metric space and proved the contraction principle of Banach in this new framework. Ma *et al.* [9] introduced the new concept of C^*-algebra-valued metric space, which is a more generalized concept than metric space. The main idea consists in using the set of all positive elements of a unital C^*-algebra instead of the set of real numbers. As an application, the existence and uniqueness results for an integral type operator are given and solved. Many researchers worked on the concept of C-class function covering a class of contractive conditions for reference, and many researchers obtained fixed point results in partial metric space and C^*-algebra-valued metric spaces (see [1,2,4,6–8,11,12]).

In the present paper, we introduce the concept of C^*-algebra-valued partial metric space and obtain some interesting fixed point results using C-class function in this setting.

2. C^*-algebra-valued partial metric spaces

Let $\mathbb{A}$ be a unital C^*-algebra with unit I. We introduce the concept of C^*-algebra-valued partial metric space, which is more general than partial metric space.

If $x \in \mathbb{A}$ is of the form yy^* for some $y \in \mathbb{A}$, then x is called a positive element in $\mathbb{A}$ (see [5]). Let $\mathbb{A}_+$ be the positive cone of positive elements in $\mathbb{A}$. Define a partial ordering $\preceq$ on the elements of $\mathbb{A}$, that is,

$$b \succeq a \Leftrightarrow b - a \in \mathbb{A}_+ \Leftrightarrow b - a \succeq \theta,$$

where θ is the zero element in $\mathbb{A}$.

DEFINITION 2.1

Let X be a nonempty set. A function $p : X \times X \to \mathbb{A}$ is called a C^*-algebra-valued partial metric space on X if the following conditions are satisfied:

(1) $\theta \preceq p(x, y)$ for all $x, y \in X$ and $p(x, x) = p(y, y) = p(x, y)$ if and only if $x = y$;
(2) $p(x, x) \preceq p(x, y)$ for all $x, y \in X$;
(3) $p(x, y) = p(y, x)$ for all $x, y \in X$;
(4) $p(x, y) \preceq p(x, z) + p(z, y) - p(z, z)$ for all $x, y \in X$.

Then the pair $(X, \mathbb{A}, p)$ is called a C^*-algebra-valued partial metric space.

Remark 2.1. If we take $\mathbb{A} = \mathbb{R}$, then the new notion of C^*-algebra-valued metric space becomes equivalent to the definition of the real partial metric space.

Example 2.1. Let $X = [0, 1]$ and $x \in \mathbb{A}$ be a nonzero element. Define $p(s, t) = \max\{1 + s, 1 + t\}xx^*$. Then we can easily show that $p : X \times X \to \mathbb{A}$ is a C^*-algebra-valued partial metric. But $p : X \times X \to \mathbb{A}$ is not a C^*-algebra-valued metric, since $p(s, s) = (1 + s)xx^* \neq \theta$.

It is clear that each C^*-algebra-valued partial metric p on X generates a T_0 topology τ_p on X. Let $\theta \prec \epsilon \in \mathbb{A}$. Then the set $\{B_p(x, \epsilon) : x \in X, \epsilon \succ \theta\}$, where $B_p(x, \epsilon) = \{y \in X, p(x, y) \prec p(x, x) + \epsilon\}$ for all $x \in X$ and $\epsilon \succ \theta$ forms the base τ_m.

DEFINITION 2.2

Let $(X, \mathbb{A}, p)$ be a C^*-algebra-valued partial space, $x \in X$ and $\{x_n\}$ be a sequence in X.

(1) $\{x_n\}$ converges to x with respect to $\mathbb{A}$, whenever for every $\epsilon > 0$, there is a natural number N such that $\|p(x_n, x) - p(x, x)\| \leq \epsilon$ for all $n \geq N$. We denote it by

$$\lim_{n\to\infty} \big(p(x_n, x) - p(x, x)\big) = \theta.$$

(2) $\{x_n\}$ is a partial Cauchy sequence with respect to $\mathbb{A}$, whenever for every $\epsilon > 0$, there is a natural number N such that

$$\left(p(x_n, x_m) - \frac{1}{2}p(x_n, x_n) - \frac{1}{2}p(x_m, x_m)\right)$$
$$\left(p(x_n, x_m) - \frac{1}{2}p(x_n, x_n) - \frac{1}{2}p(x_m, x_m)\right)^* \preceq \epsilon^2$$

for all $m, n \geq N$.

(3) $(X, \mathbb{A}, p)$ is said to be complete with respect to $\mathbb{A}$ if every partial Cauchy sequence with respect to $\mathbb{A}$ converges to a point x in X such that

$$\lim_{n\to\infty} \left(p(x_n, x) - \frac{1}{2}p(x_n, x_n) - \frac{1}{2}p(x, x)\right) = \theta.$$

From a given C^*-algebra-valued partial metric, we can obtain a C^*-algebra-valued metric as follows.

Remark 2.2. Let p be a C^*-algebra-valued partial metric. Put

$$p^s(x, y) = 2p(x, y) - p(x, x) - p(y, y).$$

Then p^s is a C^*-algebra-valued metric.

Solution. We have $p^s(x, y) \succeq \theta$, since $p(x, x) \preceq p(x, y)$ and $p(y, y) \preceq p(y, x) = p(x, y)$. If $p^s(x, y) = \theta$, then $2p(x, y) - p(x, x) - p(y, y) = \theta$, i.e.,

$$p(x, y) = \frac{p(x, x) + p(y, y)}{2}. \tag{1}$$

From (1) and the fact that $p(x, x) \preceq p(x, y)$, $p(y, y) \preceq p(x, y)$, we get

$$p(x, x) \preceq p(y, y) \tag{2}$$

and

$$p(y, y) \preceq p(x, x). \tag{3}$$

Combining (2) and (3), we get

$$p(x, x) = p(y, y). \tag{4}$$

Also, from (1), we have

$$p(x, y) = p(x, x) \quad \text{or} \quad p(x, y) = p(y, y). \tag{5}$$

Thus combining (4) and (5), we get

$$p(x, y) = p(x, x) = p(y, y).$$

Therefore, $x = y$.

If $x = y$, then $p^s(x, x) = 2p(x, x) - p(x, x) - p(x, x) = \theta$. Thus

$$p^s(x, y) = 2p(x, y) - p(x, x) - p(y, y) = 2p(y, x) - p(x, x) - p(y, y) = p^s(y, x).$$

For the triangular inequality,

$$\begin{aligned} p^s(x, z) &= 2p(x, z) - p(x, x) - p(z, z) \\ &\preceq 2\big(p(x, y) + p(y, z) - p(y, y)\big) - p(x, x) - p(z, z) \\ &= \big(2p(x, y) - p(x, x) - p(y, y)\big) + \big(2p(y, z) - p(y, y) - p(z, z)\big) \\ &= p^s(x, y) + p^s(y, z). \end{aligned}$$

Therefore, $p^s(x, z) \preceq p^s(x, y) + p^s(y, z)$. Hence $p^s(x, y)$ is a C^*-algebra-valued metric.

Lemma 2.2. *Let $(X, \mathbb{A}, p)$ be a C^*-algebra-valued partial metric space.*

(1) *$\{x_n\}$ is a partial Cauchy sequence in $(X, \mathbb{A}, p)$ if and only if it is Cauchy in the C^*-algebra-valued metric space $(X, \mathbb{A}, p^s)$;*
(2) *A C^*-algebra-valued partial metric space $(X, \mathbb{A}, p)$ is complete if and only if C^*-algebra-valued metric $(X, \mathbb{A}, p^s)$ is complete. Furthermore,*

$$\lim_{n\to\infty} p^s(x_n, x) = \theta \Leftrightarrow \lim_{n\to\infty} (2p(x_n, x) - p(x_n, x_n) - p(x, x) = \theta;$$

or

$$\lim_{n\to\infty} p^s(x_n, x) = \theta$$
$$\Leftrightarrow \big(\lim_{n\to\infty} (p(x_n, x) - p(x_n, x_n)) = \theta \text{ and } \lim_{n\to\infty} (p(x_n, x) - p(x, x)) = \theta \big).$$

Using Definitions 2.1 and 2.2 and Remark 2.2, we have the following.

Lemma 2.3. *Assume that $x_n \to x$ and $y_n \to y$ as $n \to \infty$ in a C^*-algebra-valued partial metric space $(X, \mathbb{A}, p)$. Then*

$$\lim_{n\to\infty} \big(p(x_n, y_n) - p(x_n, x_n)\big) = p(x, y) - p(x, x)$$

and

$$\lim_{n\to\infty} \big(p(x_n, y_n) - p(y_n, y_n)\big) = p(x, y) - p(y, y).$$

3. Main results

DEFINITION 3.1 (C_*-class function)

Suppose $\mathbb{A}$ is a unital C^*-algebra. Then a continuous function $F : \mathbb{A}_+ \times \mathbb{A}_+ \to \mathbb{A}$ is called a C_*-class function if for any $A, B \in \mathbb{A}_+$, the following conditions hold:

(1) $F(A, B) \preceq A$;
(2) $F(A, B) = A$ implies that either $A = \theta$ or $B = \theta$.

Note that $\mathbb{C}_+ = \{x = y\bar{y} : y \in \mathbb{C}\} = (0, \infty)$.

An extra condition on F such that $F(\theta, \theta) = \theta$ could be imposed in some cases if required. Here C_* will denote the class of all C_*-class functions.

Remark 3.1. If we take $\mathbb{A} = \mathbb{C}$ in Definition 3.1, then it will denote the set of complex C-class functions introduced by Ansari *et al.* [3].

Let Ψ be the set of all continuous functions $\psi : \mathbb{A}_+ \to \mathbb{A}_+$ satisfying the following conditions:

(1) ψ is continuous and nondecreasing;
(2) $\psi(T) = \theta$ if and only if $T = \theta$.

Theorem 3.1. *Let $(X, \mathbb{A}, p)$ be a C^*-algebra-valued partial metric space and $T : X \to X$ be a self mapping satisfying*

$$\psi(p(Tx, Ty)) \preceq F_*(\psi(p(x, y)), \phi(p(x, y))) \quad \text{for all } x, y \in X, \tag{6}$$

where $\psi, \phi \in \Psi$ and $F_* \in C_*$. Then T has a unique fixed point.

Proof. Fix $x_0 \in X$. Define $x_n \in T^n x_0$ for every $n = 1, 2, 3, \ldots$. Then we shall prove that

$$p(x_n, x_{n+1}) \to \theta \quad \text{as } n \to \infty.$$

We have

$$\begin{aligned} \psi(p(x_n, x_{n+1})) &= \psi(p(Tx_{n-1}, Tx_n) \\ &\preceq F_*(\psi(p(x_{n-1}, x_n)), \phi(p(x_{n-1}, x_n))) \\ &\preceq \psi(p(x_{n-1}, x_n)). \end{aligned} \tag{7}$$

Therefore we get

$$\psi(p(x_n, x_{n+1})) \preceq \psi(p(x_{n-1}, x_n)).$$

Hence ψ is nondecreasing and so the sequence $\{p(x_{n-1}, x_n)\}$ is monotonically decreasing in $\mathbb{A}_+$. So there exists $\theta \preceq t \in \mathbb{A}_+$ such that

$$p(x_n, x_{n+1}) \to t \quad \text{as } n \to \infty.$$

Letting $n \to \infty$ in (7), by the definition of F_* and the continuity of ψ, ϕ, we have

$$\psi(t) \preceq F_*(\psi(t), \psi(t)) \preceq \psi(t).$$

Thus $F_*(\psi(t), \phi(t)) = \psi(t)$ and so $\psi(t) = \theta$ or $\phi(t) = \theta$. Hence $t = \theta$. That is,

$$p(x_n, x_{n+1}) \to \theta \quad \text{as } n \to \infty. \tag{8}$$

Now, we want to show that $\{x_n\}$ is a partial Cauchy sequence in $(X, \mathbb{A}, p)$. By Lemma 2.2, it is sufficient to prove that $\{x_n\}$ is a Cauchy sequence in $(X, \mathbb{A}, p^s)$. We have proved $\lim_{n\to\infty} p(x_n, x_{n+1}) = \theta$. Keeping in mind that $\theta \preceq p(x_n, x_n) \preceq p(x_n, x_{n+1})$, we get

$$\lim_{n\to\infty} p(x_n, x_n) = \theta. \tag{9}$$

Also, $\theta \preceq p(x_{n+1}, x_{n+1}) \preceq p(x_n, x_{n+1})$. This implies

$$\lim_{n\to\infty} p(x_{n+1}, x_{n+1}) = \theta. \tag{10}$$

Assume that $\{x_n\}$ is not Cauchy in $(X, \mathbb{A}, p^s)$. Then there exist $\epsilon > 0$ and subsequences $\{x_{m(k)}\}$ and $\{x_{n(k)}\}$ with $n(k) > m(k) > k$ such that

$$\|p^s(x_{m(k)}, x_{n(k)})\| > \epsilon.$$

Now, corresponding to $m(k)$, we can choose $n(k)$ such that it is the smallest integer with $n(k) > m(k)$ and satisfying the above inequality. Hence

$$\|p^s(x_{m(k)}, x_{n(k)-1})\| \leq \epsilon.$$

So we have

$$\begin{aligned}\epsilon < \|p^s(x_{m(k)}, x_{n(k)})\| &\leq \|p^s(x_{m(k)}, x_{n(k)-1})\| + \|p^s(x_{n(k)-1}, x_{n(k)})\| \\ &\quad -\|p^s(x_{n(k)-1}, x_{n(k)-1})\| \\ &\leq \|p^s(x_{m(k)}, x_{n(k)-1})\| + \|p^s(x_{n(k)-1}, x_{n(k)})\| \\ &\leq \epsilon + \|p^s(x_{n(k)-1}, x_{n(k)})\|. \end{aligned} \tag{11}$$

We know that

$$p^s(x_{n(k)-1}, x_{n(k)}) = 2p(x_{n(k)-1}, x_{n(k)}) - p(x_{n(k)-1}, x_{n(k)-1}) - p(x_{n(k)}, x_{n(k)}). \tag{12}$$

Using (8), (9), (10) and (12), we have

$$\lim_{k\to\infty} \|p^s(x_{n(k)-1}, x_{n(k)})\| = \theta. \tag{13}$$

Using (11) and (13), we have

$$\epsilon < \lim_{k\to\infty} \|p^s(x_{m(k)}, x_{n(k)})\| < \epsilon + \theta.$$

This implies

$$\lim_{k\to\infty} \|p^s(x_{m(k)}, x_{n(k)})\| = \epsilon. \tag{14}$$

Again,

$$\begin{aligned}\|p^s(x_{n(k)}, x_{m(k)})\| &\leq \|p^s(x_{n(k)}, x_{n(k)-1})\| + \|p^s(x_{n(k)-1}, x_{m(k)}\| \\ &\quad -|p^s(x_{n(k)-1}, x_{n(k)-1})\| \\ &\leq \|p^s(x_{n(k)}, x_{n(k)-1})\| + \|p^s(x_{n(k)-1}, x_{m(k)})\| \\ &\leq \|p^s(x_{n(k)}, x_{n(k)-1})\| + \|p^s(x_{n(k)-1}, x_{m(k)-1})\| \\ &\quad +\|p^s(x_{m(k)-1}, x_{m(k)})\| - \|p^s(x_{m(k)-1}, x_{m(k)-1}) \\ &\leq \|p^s(x_{n(k)}, x_{n(k)-1})\| + \|p^s(x_{n(k)-1}, x_{m(k)-1})\| \\ &\quad + \|p^s(x_{m(k)-1}, x_{m(k)})\|. \end{aligned} \tag{15}$$

Also,

$$\begin{aligned}\|p^s(x_{n(k)-1}, x_{m(k)-1})\| &\leq \|p^s(x_{n(k)-1}, x_{n(k)})\| + \|p^s(x_{n(k)}, x_{m(k)-1})\| \\ &\quad - \|p^s(x_{n(k)}, x_{m(k)})\| \\ &\leq \|p^s(x_{n(k)-1}, x_{n(k)})\| + \|p^s(x_{n(k)}, x_{m(k)-1})\| \\ &\leq \|p^s(x_{n(k)-1}, x_{n(k)})\| + \|p^s(x_{n(k)}, x_{m(k)})\| \end{aligned}$$

$$
\begin{aligned}
&+\|p^s(x_{m(k)}, x_{m(k)-1})\| - \|p^s(x_{m(k)}, x_{m(k)})\| \quad (16)\\
&\leq \|p^s(x_{n(k)-1}, x_{n(k)})\| + \|p^s(x_{n(k)}, x_{m(k)})\|\\
&+ \|p^s(x_{m(k)}, x_{m(k)-1})\|.
\end{aligned}
$$

Letting $k \to \infty$ in (15) and (16) and using (13) and (14), we get

$$\lim_{k\to\infty} \|p^s(x_{n(k)-1}, x_{m(k)-1})\| = \epsilon.$$

Thus

$$
\begin{aligned}
&\lim_{k\to\infty} \|p(x_{n(k)-1}, x_{m(k)-1})\|\\
&= \frac{1}{2}\lim_{k\to\infty} \|2p(x_{n(k)-1}, x_{m(k)-1}) - p(x_{n(k)-1}, x_{n(k)-1}) - p(x_{m(k)-1}, x_{m(k)-1})\|\\
&= \frac{1}{2}\lim_{k\to\infty} \|p^s(x_{n(k)-1}, x_{m(k)-1})\|\\
&= \frac{\epsilon}{2}.
\end{aligned}
$$

Since $p(x_{n(k)-1}, x_{m(k)-1}), p(x_{n(k)}, x_{m(k)}) \in \mathbb{A}_+$ and

$$\lim_{k\to\infty} \|p(x_{n(k)-1}, x_{m(k)-1})\| = \lim_{k\to\infty} \|p(x_{n(k)}, x_{m(k)})\| = \frac{\epsilon}{2},$$

there exists $a \in \mathbb{A}_+$ with $\|a\| = \frac{\epsilon}{2}$ such that

$$\lim_{k\to\infty} p(x_{n(k)-1}, x_{m(k)-1}) = \lim_{k\to\infty} p(x_{n(k)}, x_{m(k)}) = a.$$

Now by (6), we have

$$
\begin{aligned}
\psi(a) = \lim_{k\to\infty} \psi(p(x_{n(k)}, x_{m(k)})) \preceq \lim\nolimits_{k\to\infty} F_*(&\psi(p(x_{n(k)-1}, x_{m(k)-1})),\\
&\phi(p(x_{n(k)-1}, x_{m(k)-1}))).
\end{aligned}
$$

Therefore,

$$\psi(a) \preceq F_*(\psi(a), \phi(a)) \preceq \psi(a).$$

Thus $\psi(a) = \theta$ or $\phi(a) = \theta$ and so $\theta = \theta$, which is a contradiction. Thus $\{x_n\}$ is a Cauchy sequence in the complete C^*-algebra-valued metric space $(X, \mathbb{A}, p^s)$ and so $\{x_n\}$ is partially Cauchy in the complete C^*-algebra-valued partial metric space $(X, \mathbb{A}, p)$. Hence there exists some $\nu \in X$ such that

$$\lim_{n\to\infty} (p(x_n, \nu) - p(x_n, x_n)) = \theta.$$

Using (9), we get

$$\lim_{n\to\infty} p(x_n, \nu) = \theta \quad \text{and thus } p(\nu, \nu) = \theta.$$

Now, we shall show that ν is a fixed point of T. Using (6), we get

$$\theta \preceq \psi(p(T\nu, T\nu)) \preceq F_*\big(\psi(p(\nu,\nu), p(\nu,\nu))\big) \preceq F_*(\psi(\theta), \phi(\theta)) = \theta.$$

Thus $\psi(p(T\nu, T\nu)) = \theta$, which implies $p(T\nu, T\nu)) = \theta$. On the other hand,

$$\psi(p(x_n, T\nu)) \preceq F_*(\psi(p(x_{n-1},\nu), \psi(p(x_{n-1},\nu))).$$

Letting $n \to \infty$ and using the concept of continuity of the functions ϕ, ψ and F_*, we have $p(\nu, T\nu) = \theta$. Hence we have

$$p(\nu,\nu) = p(T\nu, T\nu) = p(\nu, T\nu) = \theta.$$

By Definition 2.1, we have $T\nu = \nu$.

For the uniqueness, let $\mu, \xi \in X$ be two fixed points of T. Using (6), we get

$$\psi(p(\mu,\mu)) = \psi(p(T\mu, T\mu)) \preceq F_*(\psi(p(\mu,\mu), \phi(p(\mu,\mu)) \preceq \psi(p(\mu,\mu).$$

Therefore, $\psi(p(\mu,\mu)) = \theta$ or $\phi(p(\mu,\mu)) = \theta$. Thus $p(\mu,\mu) = \theta$. Similarly, one can prove that $p(\xi,\xi) = \theta$. Using (6), we have

$$\psi(p(\mu,\xi)) = \psi(p(T\mu, T\xi)) \preceq F_*(\psi(p(\mu,\xi), \psi(p(\mu,\xi)))) \preceq \psi(p(\mu,\xi)).$$

Hence $\psi(p(\mu,\xi)) = \theta$ or $\phi(p(\mu,\xi) = \theta$ and so $p(\mu,\xi) = \theta$. Thus we get

$$p(\mu,\mu) = p(\xi,\xi) = p(\mu,\xi) = \theta.$$

Therefore using Definition 2.1, we get $\mu = \xi$. This implies the uniqueness. □

For $F_* = A - B$, we have the following result.

COROLLARY 3.2

Let $(X, \mathbb{A}, p)$ be a C^*-algebra-valued partial metric space and $T : X \to X$ be a self mapping satisfying

$$\psi(p(Tx, Ty)) \preceq \psi(p(x,y)) - \phi(p(x,y)) \quad \textit{for all } x, y \in X,$$

where $\psi, \phi \in \Psi$ and $F_* \in C_*$. Then T has a unique fixed point.

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