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Semisymmetric cubic graphs of order $16p^2$

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Abstract. An undirected graph without isolated vertices is said to be semisymmetric if its full automorphism group acts transitively on its edge set but not on its vertex set. In this paper, we inquire the existence of connected semisymmetric cubic graphs of order $16p^2$. It is shown that for every odd prime p, there exists a semisymmetric cubic graph of order $16p^2$ and its structure is explicitly specified by giving the corresponding voltage rules generating the covering projections.

Keywords. Automorphism group; regular cover; vertex-transitive graph; edge-transitive graph; semisymmetric graph.

1. Introduction

Throughout this paper, graphs are assumed to be finite, simple, undirected and connected. For the group-theoretic concepts and notations not defined here, we refer to [8, 16].

Given a graph X, we let $V(X)$, $E(X)$, $A(X)$ and $\mathrm{Aut}(X)$ be the *vertex set*, the *edge set*, the *arc set* and the *full automorphism group* of X, respectively. For $u, v \in V(X)$, we denote by $\{u, v\}$ the edge incident to u and v in X.

If a subgroup G of $\mathrm{Aut}(X)$ acts transitively on $V(X)$ and $E(X)$, we say that X is *G-vertex-transitive* and *G-edge-transitive*, respectively. In the special case when $G = \mathrm{Aut}(X)$ we say that X is vertex-transitive and edge-transitive, respectively. It can be shown that a G-edge-transitive but not G-vertex-transitive graph X is necessarily bipartite, where the two parts of the bipartition are orbits of $G \leq \mathrm{Aut}(X)$. Moreover, if X is regular then these two parts have the same cardinality. A regular G-edge-transitive but not G-vertex-transitive graph will be referred to as a *G-semisymmetric graph*. In particular, if $G = \mathrm{Aut}(X)$ the graph is said to be semisymmetric.

An *s-arc* in a graph X is an ordered $(s+1)$-tuple $(v_0, v_1, \ldots, v_s)$ of vertices of X such that v_{i-1} is adjacent to v_i for $1 \leq i \leq s$, and $v_{i-1} \neq v_{i+1}$ for $1 \leq i < s$; in other words, a directed walk of length s which never includes a backtracking. A graph X is said to be *s-arc-transitive* if $\mathrm{Aut}(X)$ is transitive on the set of s-arcs in X. In particular, 0-arc-transitive means vertex-transitive, and 1-arc-transitive means *arc-transitive* or *symmetric*.

The study of semisymmetric graphs was initiated by Folkman [7]. Semisymmetric graphs of order $2pq$ and semisymmetric cubic graphs of orders $2p^3$, $6p^2$ and $2p^2q$ are classified in [4, 11, 10, 15], and also in [1] it is proved that every edge-transitive cubic graph of order $8p^2$ is vertex-transitive. In [3], an overview of known families of cubic semisymmetric graphs is given. Recently, Bretto and Gillibert [2] exhibit an efficient algorithm, based on G-graphs, for constructing cubic semisymmetric graphs.

Let X be a graph and K a finite group. By a^{-1} we mean the reverse arc to an arc a. A *voltage assignment* (or, *K-voltage assignment*) of X is a function $\xi\colon A(X) \to K$ with the property that $\xi(a^{-1}) = (\xi(a))^{-1}$ for each arc $a \in A(X)$. The values of ξ are called *voltages*, and K is the *voltage group*. The graph $X \times_{\xi} K$ derived from a voltage assignment $\xi\colon A(X) \to K$ has vertex set $V(X) \times K$ and edge set $E(X) \times K$, so that an edge (e, g) of $X \times K$ joins a vertex (u, g) to $(v, g\xi(a))$ for $a = (u, v) \in A(X)$ and $g \in K$, where $e = \{u, v\}$. Given a spanning tree T of the graph X, a voltage assignment ξ is said to be *T-reduced* if the voltages on the tree arcs are the identity.

For $p = 2$, there is no semisymmetric cubic graph of order 64 (see [3]). So, in this paper we can assume that p is an odd prime. By [3], it is shown that there exists a unique semisymmetric cubic graph of order $16p^2$ for $p = 3$ or 5. Also, by [2], for $p = 7$, there exist three such graphs. This paper deals with the existence of semisymmetric cubic graphs of order $16p^2$, where p is an odd prime. Our aim is to specify the structure of these graphs in terms of the derived graphs.

Theorem 1.1. *For every odd prime p, there exists a semisymmetric cubic graph X of order $16p^2$. If $p \neq 7$, X can be seen as a derived graph* $\mathrm{GP}(8,3) \times_{\xi} \mathbb{Z}_p^2$, *where* $\mathrm{GP}(8,3)$ *is generalized Petersen graph.*

The corresponding voltages assignments have two types as follows:

Type 1.

$\xi(x)$	$\xi(x_1)$	$\xi(x_2)$	$\xi(x_3)$	$\xi(x_4)$
$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} s \\ 1+(1+s)i \end{pmatrix}$	$\begin{pmatrix} 1 \\ -1-s-si \end{pmatrix}$	$\begin{pmatrix} si \\ 1+s-i \end{pmatrix}$	$\begin{pmatrix} i \\ -s+(1+s)i \end{pmatrix}$

$\xi(x_5)$	$\xi(x_6)$	$\xi(x_7)$	$\xi(x_8)$
$\begin{pmatrix} -s \\ -1-(1+s)i \end{pmatrix}$	$\begin{pmatrix} -1 \\ 1+s+si \end{pmatrix}$	$\begin{pmatrix} -si \\ -1-s+i \end{pmatrix}$	$\begin{pmatrix} -i \\ s-(1+s)i \end{pmatrix}$

and p is one of the following:

$$p \equiv 5 (\mathrm{mod}\ 8),\ i^2 = -1,\ s \in \mathbb{Z}_p,\ s \neq -1, i;$$

$$p \equiv 1 (\mathrm{mod}\ 8),\ i^2 = -1,\ s \in \mathbb{Z}_p,\ s \neq -1, i, v, -v, -1+v-v^2,$$
$$-1-v-v^2,\ \text{where } v^2 = -i;$$

$$p \equiv 1 (\mathrm{mod}\ 8),\ i^2 = -1,\ s \in \mathbb{Z}_p,\ s = -1+v-v^2,\ \text{where } v^2 = -i.$$

Type 2.

$\xi(x)$	$\xi(x_1)$	$\xi(x_2)$	$\xi(x_3)$	$\xi(x_4)$
$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} \alpha+1 \\ \beta \end{pmatrix}$	$\begin{pmatrix} -1 \\ 1 \end{pmatrix}$	$\begin{pmatrix} \beta \\ -(\alpha+1) \end{pmatrix}$	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\xi(x_5)$	$\xi(x_6)$	$\xi(x_7)$	$\xi(x_8)$
$\begin{pmatrix} -(\alpha+1) \\ -\beta \end{pmatrix}$	$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	$\begin{pmatrix} -\beta \\ \alpha+1 \end{pmatrix}$	$\begin{pmatrix} -1 \\ -1 \end{pmatrix}$

and also p is one of the following:

$$p \equiv 7(\text{mod } 8),\ \alpha^2 + \beta^2 = -1(\text{mod } p);$$

$$p \equiv 3(\text{mod } 8),\ \alpha^2 + \beta^2 = -1(\text{mod } p),\ (\alpha, \beta) \neq (\pm 1, \pm\sqrt{-2})$$

(Note: empty if $p = 3$);

$$p \equiv 3(\text{mod } 8),\ \alpha^2 + \beta^2 = -1(\text{mod } p),\ (\alpha, \beta) = (1, \sqrt{-2}).$$

2. Preliminaries

Let X be a graph and N be a subgroup of $\text{Aut}(X)$. The *quotient graph* X/N or X_N induced by N is the graph defined in the following way:

(i) The set of vertices Σ is the set of orbits under the action N on $V(X)$;
(ii) Let $A, B \in \Sigma$ be two vertices of X_N. $\{A, B\}$ is an edge of X_N if and only if there are $u \in A$ and $v \in B$ such that $\{u, v\} \in E(X)$.

Let $\tilde{X}$ and X be two graphs and $\wp\colon \tilde{X} \to X$ be a graph epimorphism. Then $\wp$ is a *covering projection* if $\wp$ is a local isomorphism, that is, if for each $v \in V(\tilde{X})$, the restriction of $\wp$ to the neighborhood of v is a bijection onto the neighborhood of $\wp(v) \in V(X)$ in X. A graph $\tilde{X}$ is called a *covering* of a graph X, if there is a covering projection $\wp\colon \tilde{X} \to X$.

Two covering projections $\wp\colon \tilde{X} \to X$ and $\wp'\colon \tilde{X}' \to X$ of a graph X are *isomorphic* if there exist an automorphism $\alpha \in \text{Aut}(X)$ and an isomorphism $\tilde{\alpha}\colon \tilde{X} \to \tilde{X}'$ such that $\alpha \circ \wp = \wp' \circ \tilde{\alpha}$. In particular, if $\alpha = id$ then $\wp$ and $\wp'$ are *equivalent*.

A covering projection $\wp\colon \tilde{X} \to X$ is called a *regular covering projection* if there exists a subgroup $\text{CT}(\wp) \leq \text{Aut}\tilde{X}$ acting semiregularly (that is, with trivial stabilizers) on $V(\tilde{X})$ such that its orbits coincide with the (vertex) *fibres* $\wp^{-1}(v)$, $v \in V(X)$. If $\text{CT}(\wp)$, also known as the group of covering transformations, is isomorphic to an abstract group N, then we speak of a regular N-covering projection and also $\tilde{X}$ is said to be a regular covering (or N-covering) of X. The regular N-covering projection $\wp$ is p-elementary abelian if N is an elementary abelian p-group. Let $\tilde{X}$ be a N-covering of X with a covering projection $\wp\colon \tilde{X} \to X$. If $\alpha \in \text{Aut}(X)$ and $\tilde{\alpha} \in \text{Aut}(\tilde{X})$ satisfy $\alpha \circ \wp = \wp \circ \tilde{\alpha}$, we call $\tilde{\alpha}$ a *lift* of α, and α the *projection* of $\tilde{\alpha}$. A regular covering projection $\wp$ is called *edge-transitive* (*vertex-transitive*) if some subgroup $G \leq \text{Aut}(\tilde{X})$ lifts along $\wp$, in which G is an edge-transitive (vertex-transitive) subgroup. Similarly, a regular covering projection $\wp$ is called *semisymmetric* if some subgroup $G \leq \text{Aut}(\tilde{X})$ lifts along $\wp$, in which G is an edge- but not vertex-transitive subgroup.

Note that every regular covering $\tilde{X}$ of a graph X can be derived from a T-reduced voltages assignment ξ with respect to an arbitrary fixed spanning tree T of X [9]. It is clear that if ξ is reduced, the derived graph $X \times_\xi K$ is connected if and only if the voltages on the cotree arcs generate the voltages group K.

The next proposition is a special case of Lemma 3.2 of [11].

PROPOSITION 2.1

Let X be a connected G-semisymmetric cubic graph with bipartition sets $U(X)$ and $W(X)$, where $G \leq \text{Aut}(X)$. Moreover, suppose that N is a normal subgroup of G. If N is intransitive on bipartition sets, then N acts semiregularly on both $U(X)$ and $W(X)$, and X is an N-covering of a G/N-semisymmetric cubic graph.

PROPOSITION 2.2 (Proposition 2.4 of [10])

The vertex stabilizers of a connected G-semisymmetric cubic graph X have order $2^r 3$, *where* $0 \le r \le 7$. *Moreover, if u and v are two adjacent vertices, then the edge stabilizer* $G_u \cap G_v$ *is a common Sylow* 2*-subgroup of* G_u *and* G_v.

PROPOSITION 2.3 [5]

Let X be a connected cubic symmetric graph of order $4p$ *or* $4p^2$ *for a prime p. Then X is isomorphic to the* 2*-regular hypercube* Q_3 *of order* 8, *the* 2*-regular generalized Petersen graphs* $P(8, 3)$ *or* $P(10, 7)$ *of order* 16 *or* 20 *respectively, the* 3*-regular Desargues graph of order* 20 *or the* 3*-regular Coxeter graph* C_{28} *of order* 28.

PROPOSITION 2.4 [14]

Every both edge-transitive and vertex-transitive cubic graph is symmetric.

PROPOSITION 2.5 (Burnside Theorem of [8])

Every group of order $p^a q^b$, *p and q primes, is solvable.*

PROPOSITION 2.6 (Corollary 4.5 of [12])

Let X be a connected cubic graph admitting an edge-transitive solvable subgroup of automorphism. Then X is a regular cover either of the 3*-dipole* Dip_3 *or of the complete graph* K_4. *Moreover, the corresponding covering projection decomposes into a sequence of elementary abelian covering projections.*

3. Proof of theorem 1.1

We identify the vertex set of the generalized Petersen graph GP(8, 3) with $V = \{1, 2, \ldots, 16\}$ and the edge set with the union of *outer edges* $E_1 = \{\{i, 1 + i (\mathrm{mod}\ 8)\} | i \in \{1, \ldots, 8\}\}$, the *inner edges* $E_2 = \{\{8 + i, 9 + ((i + 2)\mathrm{mod}\ 8)\} | i \in \{1, \ldots, 8\}\}$, and the *spokes* $E_3 = \{\{i, i + 8\} | i \in \{1, \ldots, 8\}\}$, see figure 1.

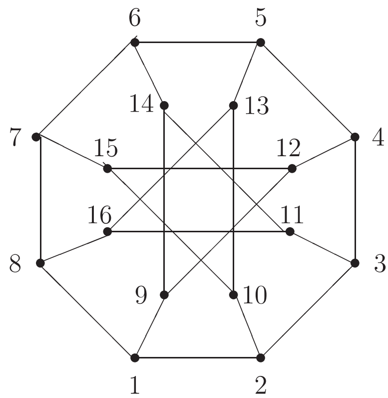

Figure 1. The generalized Petersen graph GP(8, 3).

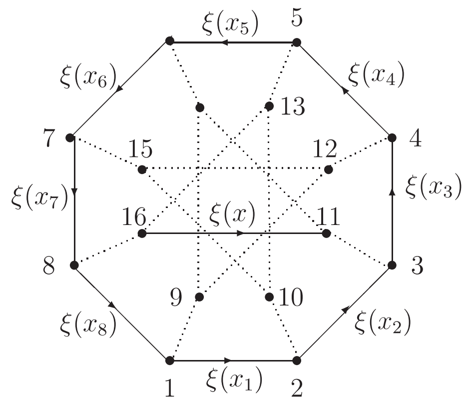

Figure 2. A spanning tree and voltage assignment on GP(8, 3).

The automorphism group of the generalized Petersen graph GP(8, 3) has order 96 and moreover, Aut(GP(8, 3)) $= \langle \rho, \omega, \sigma \rangle$, where

$$\rho = (1, 2, 3, 4, 5, 6, 7, 8)(9, 10, 11, 12, 13, 14, 15, 16),$$

$$\omega = (2, 8, 9)(3, 16, 14)(4, 13, 6)(7, 12, 10),$$

$$\sigma = (1, 14, 7, 12, 5, 10, 3, 16)(2, 11, 8, 9, 15, 4, 13).$$

Let T be a spanning tree of GP(8, 3) containing all spokes and inner edges except for the edge $\{11, 16\}$. Let x denote the arc $(16, 11)$ and let x_i denote arc $(i, 1 + i (\mathrm{mod}\ 8))$, $i \in \{1, \ldots, 8\}$. Let ξ be such a K-voltage assignment defined by $\xi = 0$ on T and $\xi(x), \xi(x_1), \ldots, \xi(x_8)$ on the cotree arcs $x, x_1, \ldots, x_8$, respectively, where 0 is the identity element of K and $\xi(x), \xi(x_i) \in \mathbb{Z}_p^2$ $(1 \leq i \leq 8)$, see figure 2.

Note that for a semisymmetric cubic graph X of order $16p^2$, p is a prime, we have:

(i) If $p = 2$, by [3] there is no semisymmetric cubic graph of order $16p^2$;
(ii) If $p = 3$ or 5, by [3] there exists a unique semisymmetric cubic graph of order $16p^2$;
(iii) If $p = 7$, by Table 2 of [2] there exist three semisymmetric cubic graphs of order $16p^2$.

Hence we can assume that $p \geq 11$ and so we have the following lemma.

Lemma 3.1. *Assume that there exists a connected semisymmetric cubic graph X of order $16p^2$, where $p(\geq 11)$ is a prime. Set A :=*Aut(X)*. Moreover suppose that $Q := O_p(A)$ be the maximal normal p-subgroup of A. Then A is solvable and also $|Q| = p^2$ and X is a Q-covering of the generalized Petersen graph* GP(8, 3).

Proof. Let X be a cubic graph satisfying the assumptions. Therefore X is a bipartite graph. Denote by $U(X)$ and $W(X)$, the bipartition sets of X, where $|U(X)| = |W(X)| = 8p^2$. By Proposition 2.2 and orbit-stabilizer theorem, we have $|A| = 2^{r+3}3p^2$ $(0 \leq r \leq 7)$. We also have A is solvable, for if it was not then its composition factors would have to be one of the non-abelian simple groups A_5 or $PSL(2, 7)$. So, 5 or 7 must be one of the prime factors in $|A|$, a contradiction to $p \geq 11$. Let $Q = O_p(A)$ be the maximal normal p-subgroup of A. We want to show that $|Q| = p^2$.

Suppose first that $|Q| = 1$. Let N be a minimal normal subgroup of A. Then N is solvable and also elementary abelian and so $8p^2 \nmid |N|$. Hence N is intransitive on bipartition sets $U(X)$ and $W(X)$ and by Proposition 2.1, N acts semiregularly on these sets. Therefore, $|N| = 2, 4$ or 8. In each case, with repeated use of Proposition 2.1, we get a contradiction. Since the details of the proofs are similar, we investigate only the case when $|N| = 2$.

Let $|N| = 2$. By Proposition 2.1, the quotient graph X_N of X induced by N is an A/N-semisymmetric cubic graph. Let $U(X_N)$ and $W(X_N)$ be the bipartition sets of X_N. It is obvious that $|U(X_N)| = |W(X_N)| = 4p^2$. Suppose that T/N be a minimal normal subgroup of A/N. Note that T/N is solvable and then elementary abelian. So, by Proposition 2.1, T/N acts semiregularly on bipartition sets $U(X_N)$ and $W(X_N)$. Therefore, by minimality $|T/N| = 2$ or 4.

Suppose first that $|T/N| = 2$. Note that by Proposition 2.1, the quotient graph X_T is an A/T-semisymmetric cubic graph. We denote the bipartition sets of X_T by $U(X_T)$ and $W(X_T)$. It is easy to see that $|U(X_T)| = |W(X_T)| = 2p^2$. Let M/T be a minimal normal subgroup of A/T. Since A/T is solvable, M/T is also solvable and then elementary abelian. It is obvious that $2p^2 \nmid |M/T|$ and so by Proposition 2.1, M/T acts semiregularly on bipartition sets $U(X_T)$ and $W(X_T)$. Then $|M/T| = 2$. Again, consider the quotient graph X_M and let L/M be a minimal normal subgroup of A/M. It is easy to check that $|L/M| = p$ or p^2. Now suppose that P is a Sylow p-subgroup of L. Then one can see that P is normal and hence is characteristic in L. Therefore, A has a normal subgroup of order p or p^2, a contradiction to $|Q| = 1$ and then $|T/N| \neq 2$. Therefore, $|T/N| = 4$. But, in this case similarly as above, we can get a contradiction. Thus $|Q| \neq 1$.

Finally, suppose that $|Q| = p$. Let X_Q be the quotient graph of X induced by Q, and N/Q be a minimal normal subgroup of A/Q. Hence N/Q is solvable, and by Proposition 2.1, $|N/Q| = 2, 4$ or 8. Similarly as the case $|Q| = 1$, in these cases we get a contradiction. Therefore, $|O_p(A)| = p^2$. Now by Proposition 2.1, X is a Q-covering of the quotient graph X_Q. Note that the vertex-set of X_Q is the set of Q-orbits in $V(X)$. So, the quotient graph X_Q is of order 16. Hence, X is a Q-covering of a cubic graph of order 16. But, by [3], every edge-transitive cubic graph of order 16 is vertex-transitive and so by Proposition 2.4, this graph is symmetric. Now by Proposition 2.3, it is unique and therefore the quotient graph X_Q is the generalized Petersen graph GP(8, 3). ■

Proof of Theorem 1.1. Let X be a semisymmetric cubic graph of order $16p^2$, where $p \geq 11$. By Lemma 3.1, A =Aut(X) is an edge-transitive solvable group. Then by Proposition 2.6, X is a regular covering either of the 3-dipole Dip_3 or the complete graph K_4 and the corresponding regular covering projection decomposes into a sequence of elementary abelian covering projections. By Burnside theorem, Aut(GP(8, 3)) is solvable. Now since X is a regular Q-covering of GP(8, 3), we can assume that $Q \cong \mathbb{Z}_p^2$. By Lemma 3.1, X is a Q-covering of the graph GP(8, 3) with the corresponding covering projection as $\wp$. Now by the normality of Q in A, we have the semisymmetric subgroup A/Q lifts along $\wp$ eventually to A. Therefore, $\wp$ is a semisymmetric covering projection.

Now we consider the general case. Let $\wp\colon X \to$ GP(8, 3) be a semisymmetric $\mathbb{Z}_p^2$-covering of GP(8, 3), where p is an odd prime. One can see that all of such covering projections are obtained in [13]. Note that for every $p \equiv 1$ or 5 (mod 8) there exist $(p-1)/4$ pairwise nonisomorphic semisymmetric covering projections. Also, for every $p \equiv 3$ or 7 (mod 8) there exist $(p+1)/4$ pairwise nonisomorphic semisymmetric covering projections. We remind that X can be derived from a T-reduced voltage assignment ξ with respect to the spanning tree T of GP(8, 3), for which T was introduced in the beginning of this section. Now from [13], the corresponding voltage assignments ξ have two types as follows:

Type 1.

$\xi(x)$	$\xi(x_1)$	$\xi(x_2)$	$\xi(x_3)$	$\xi(x_4)$
$\begin{pmatrix}0\\0\end{pmatrix}$	$\begin{pmatrix}s\\1+(1+s)i\end{pmatrix}$	$\begin{pmatrix}1\\-1-s-si\end{pmatrix}$	$\begin{pmatrix}si\\1+s-i\end{pmatrix}$	$\begin{pmatrix}i\\-s+(1+s)i\end{pmatrix}$

$\xi(x_5)$	$\xi(x_6)$	$\xi(x_7)$	$\xi(x_8)$
$\begin{pmatrix}-s\\-1-(1+s)i\end{pmatrix}$	$\begin{pmatrix}-1\\1+s+si\end{pmatrix}$	$\begin{pmatrix}-si\\-1-s+i\end{pmatrix}$	$\begin{pmatrix}-i\\s-(1+s)i\end{pmatrix}$

and p is one of the following:

$$p \equiv 5 (\mathrm{mod}\ 8),\ i^2 = -1,\ s \in \mathbb{Z}_p,\ s \neq -1, i;$$
$$p \equiv 1 (\mathrm{mod}\ 8),\ i^2 = -1,\ s \in \mathbb{Z}_p,\ s \neq -1, i, v, -v, -1+v-v^2,$$
$$-1-v-v^2,\ \text{where } v^2 = -i;$$
$$p \equiv 1 (\mathrm{mod}\ 8),\ i^2 = -1,\ s \in \mathbb{Z}_p,\ s = -1+v-v^2,\ \text{where } v^2 = -i.$$

Type 2.

$\xi(x)$	$\xi(x_1)$	$\xi(x_2)$	$\xi(x_3)$	$\xi(x_4)$
$\begin{pmatrix}0\\0\end{pmatrix}$	$\begin{pmatrix}\alpha+1\\\beta\end{pmatrix}$	$\begin{pmatrix}-1\\1\end{pmatrix}$	$\begin{pmatrix}\beta\\-(\alpha+1)\end{pmatrix}$	$\begin{pmatrix}1\\1\end{pmatrix}$

$\xi(x_5)$	$\xi(x_6)$	$\xi(x_7)$	$\xi(x_8)$
$\begin{pmatrix}-(\alpha+1)\\-\beta\end{pmatrix}$	$\begin{pmatrix}1\\-1\end{pmatrix}$	$\begin{pmatrix}-\beta\\\alpha+1\end{pmatrix}$	$\begin{pmatrix}-1\\-1\end{pmatrix}$

and also p is one of the following:

$$p \equiv 7 (\mathrm{mod}\ 8),\ \alpha^2+\beta^2 = -1 (\mathrm{mod}\ p);$$
$$p \equiv 3 (\mathrm{mod}\ 8),\ \alpha^2+\beta^2 = -1 (\mathrm{mod}\ p),\ (\alpha,\beta) \neq (\pm 1, \pm\sqrt{-2})$$
(Note: empty if $p = 3$);
$$p \equiv 3 (\mathrm{mod}\ 8),\ \alpha^2+\beta^2 = -1 (\mathrm{mod}\ p),\ (\alpha,\beta) = (1, \sqrt{-2}).$$

In Theorem 2.4 of [6], it is proved that all these covering graphs are semisymmetric. Since the corresponding derived graphs to nonisomorphic regular covering projections are nonisomorphic graphs, it is obvious that for odd prime p, there are $(p-1)/4$ or $(p+1)/4$ nonisomorphic semisymmetric cubic graphs.

Since for $p = 3$ and 5, there exists a unique semisymmetric cubic graph of order 144 and 400 respectively, we can identify these graphs as one of the above graphs. Also, for $p = 7$, there exist 3 semisymmetric cubic graph of order 784. Finally, for $p \geq 11$, X is certainly one of the above graphs. Therefore, the proof of our main theorem is complete. ∎

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