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Hartman–Mycielski functor of non-metrizable compacta

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Abstract. We investigate certain topological properties of the normal functor H, introduced by the first author, which is a certain functorial compactification of the Hartman–Mycielski construction HM. We prove that H is always open and we also find the condition when HX is an absolute retract, homeomorphic to the Tychonov cube.

Keywords. Hartman–Mycielski construction; absolute retract; Tykhonov cube; normal functor.

1. Introduction

The general theory of functors acting on the category $\mathcal{C}omp$ of compact Hausdorff spaces (i.e. compacta) and continuous mappings was started by Shchepin [S2]. He described some elementary properties of these functors and defined the notion of the normal functor which has become very fruitful. The classes of all normal and weakly normal functors include many classical constructions: the hyperspace *exp*, the space of probability measures P, the superextension λ, the space of hyperspaces of inclusion G, and many other functors (cf. [FZ] and [TZ]).

Let X be any space and d any admissible metric on X bounded by 1. By $\mathrm{HM}(X)$ we shall denote the space of all maps from $[0, 1)$ to the space X such that $f|[t_i, t_{i+1}) \equiv \mathrm{const}$, for some $0 = t_0 \leq \cdots \leq t_n = 1$, with respect to the following metric:

$$d_{\mathrm{HM}}(f, g) = \int_0^1 d(f(t), g(t))\mathrm{d}t, \quad f, g \in \mathrm{HM}(X).$$

The construction of $\mathrm{HM}(X)$ is known as the Hartman–Mycielski construction (cf. [HM]). For every $Z \in \mathcal{C}omp$ consider

$$\mathrm{HM}_n(Z) = \{f \in \mathrm{HM}(Z)| \text{ there exist } 0 = t_1 < \cdots < t_{n+1} = 1 \text{ with } f|[t_i, t_{i+1}) \equiv z_i \in Z, i = 1, \ldots, n\}.$$

Let $\mathcal{U}$ be the unique uniformity of Z. For every $U \in \mathcal{U}$ and $\varepsilon > 0$, let

$$\langle \alpha, U, \varepsilon\rangle = \{\beta \in \mathrm{HM}_n(Z)| m\{t \in [0, 1)|(\alpha(t), \beta(t')) \notin U\} < \varepsilon\}.$$

The sets $\langle \alpha, U, \varepsilon \rangle$ form a base of a compact Hausdorff topology in $\mathrm{HM}_n Z$. Given a map $f\colon X \to Y$ in $\mathcal{C}omp$, define a map $\mathrm{HM}_n X \to \mathrm{HM}_n Y$ by the formula $\mathrm{HM}_n F(\alpha) = f \circ \alpha$. Then HM_n is a normal functor in $\mathcal{C}omp$ (cf. chapter 2.5.2 of [TZ]).

For $X \in \mathcal{C}omp$ we consider the space $\mathrm{HM}X$ with the topology described above. In general, $\mathrm{HM}X$ is not compact. Zarichnyi asked if there exists a normal functor in $\mathcal{C}omp$ which contains all functors HM_n as subfunctors (cf. [TZ]). Such a functor H was constructed in [Ra]. It was shown in [RR] that HX is homeomorphic to the Hilbert cube for each non-degenerate metrizable compactum X.

We investigate some topological properties of the space HX for non-metrizable compacta X. The main results of this paper are as follows:

Theorem 1.1. *Hf is open if and only if f is an open map.*

Theorem 1.2. *HX is an absolute retract if and only if X is an openly generated compactum of weight $\leq \omega_1$.*

Theorem 1.3. *HX is homeomorphic to the Tychonov cube if and only if X is an openly generated χ-homogeneous compactum of weight ω_1.*

2. Construction of *H* and its connection with the functor of probability measures *P*

Let $X \in \mathcal{C}omp$. By CX we denote the Banach space of all continuous functions $\varphi\colon X \to \mathbb{R}$ with the usual sup-norm: $\|\varphi\| = \sup\{|\varphi(x)| \mid x \in X\}$. We denote the segment $[0, 1]$ by I. For $X \in \mathcal{C}omp$ let us define the uniformity of $\mathrm{HM}X$. For each $\varphi \in C(X)$ and $a, b \in [0, 1]$ with $a < b$ we define the function $\varphi_{(a,b)}\colon \mathrm{HM}X \to \mathbb{R}$ by the following formula

$$\varphi_{(a,b)} = \frac{1}{(b-a)} \int_a^b \varphi \circ \alpha(t)\mathrm{d}t.$$

Define

$$S_{\mathrm{HM}}(X) = \{\varphi_{(a,b)} \mid \varphi \in C(X) \quad \text{and} \quad (a, b) \subset [0, 1)\}.$$

For $\varphi_1, \dots, \varphi_n \in S_{\mathrm{HM}}(X)$ we define a pseudometric $\rho_{\varphi_1,\dots,\varphi_n}$ on $\mathrm{HM}X$ by the following formula:

$$\rho_{\varphi_1,\dots,\varphi_n}(f, g) = \max\{|\varphi_i(f) - \varphi_i(g)| \mid i \in \{1, \dots, n\}\},$$

where $f, g \in \mathrm{HM}X$. The family of pseudometrics

$$\mathcal{P} = \{\rho_{\varphi_1,\dots,\varphi_n} \mid n \in \mathbb{N}, \text{ where } \varphi_1, \dots, \varphi_n \in S_{\mathrm{HM}}(X)\},$$

defines a totally bounded uniformity $\mathcal{U}_{\mathrm{HM}X}$ of $\mathrm{HM}X$ (cf. [Ra]).

For each compactum X we consider the uniform space $(HX, \mathcal{U}_{HX})$ which is the completion of $(\mathrm{HM}X, \mathcal{U}_{\mathrm{HM}X})$ and the topological space HX with the topology induced by the uniformity $\mathcal{U}_{HX}$. Since $\mathcal{U}_{\mathrm{HM}X}$ is totally bounded, the space HX is compact.

Let $f\colon X \to Y$ be a continuous map. Define the map $\mathrm{HM}f\colon \mathrm{HM}X \to \mathrm{HM}Y$ by the formula $\mathrm{HM}f(\alpha) = f \circ \alpha$, for all $\alpha \in \mathrm{HM}X$. It was shown in [Ra] that the map $\mathrm{HM}f\colon (\mathrm{HM}X, \mathcal{U}_{\mathrm{HM}X}) \to (\mathrm{HM}Y, \mathcal{U}_{\mathrm{HM}Y})$ is uniformly continuous. Hence there exists the continuous map $Hf\colon HX \to HY$ such that $Hf|\mathrm{HM}X = \mathrm{HM}f$. It is easy to see that $H\colon \mathcal{C}omp \to \mathcal{C}omp$ is a covariant functor and HM_n is a subfunctor of H for each $n \in \mathbb{N}$.

Let us remark that the family of functions $S_{\mathrm{HM}}(X)$ embed $\mathrm{HM}X$ in the product of closed intervals $\prod_{\varphi_{(a,b)}\in S_{\mathrm{HM}}(X)} I_{\varphi_{(a,b)}}$ where $I_{\varphi_{(a,b)}} = [\min_{x\in X} |\varphi(x)|, \max_{x\in X} |\varphi(x)|]$. Therefore the space HX is the closure of the image of $\mathrm{HM}X$. We denote by $p_{\varphi_{(a,b)}}\colon HX \to I_{\varphi_{(a,b)}}$ the restriction of the natural projection. Let us remark that the function Hf can be defined by the condition $p_{\varphi_{(a,b)}} \circ Hf = p_{(\varphi\circ f)_{(a,b)}}$, for each $\varphi_{(a,b)} \in S_{\mathrm{HM}}(Y)$.

It was shown in [RR] that HX is a convex subset of $\prod_{\varphi_{(a,b)}\in S_{\mathrm{HM}}(X)} I_{\varphi_{(a,b)}}$. Define the map $e_1\colon \mathrm{HM}X \times \mathrm{HM}X \times I \to \mathrm{HM}X$ by the condition that $e_1(\alpha_1, \alpha_2, t)(l)$ is equal to $\alpha_1(l)$ if $l < t$ and $\alpha_2(l)$ in the opposite case, for $\alpha_1, \alpha_2 \in \mathrm{HM}X$, $t \in I$ and $l \in [0, 1)$. We consider $\mathrm{HM}X$ with the uniformity $\mathcal{U}_{\mathrm{HM}X}$ and I with the natural metric. The map $e_1\colon \mathrm{HM}X \times \mathrm{HM}X \times I \to \mathrm{HM}X$ is uniformly continuous (cf. [RR]). Hence there exists the extension of e_1 to the continuous map $e\colon HX \times HX \times I \to HX$. It is easy to check that $e(\alpha, \alpha, t) = \alpha$, for each $\alpha \in HX$.

We recall that PX is the space of all nonnegative functionals $\mu\colon C(X) \to \mathbb{R}$ with norm 1, and considered in the weak* topology for a compactum X (cf. [TZ] or [FZ] for more details). Recall that the base of the weak* topology in PX consists of the sets of the form $O(\mu_0, f_1, \ldots, f_n, \varepsilon) = \{\mu \in PX \mid |\mu(f_i) - \mu_0(f_i)| < \varepsilon$ for every $1 \le i \le n\}$. Hence we can consider PX as a subspace of the product of closed intervals $\prod_{\varphi\in C(X)} I_\varphi$ where $I_\varphi = [\min_{x\in X} |\varphi(x)|, \max_{x\in X} |\varphi(x)|]$. We denote the restriction of the natural projection by $\pi_\varphi\colon PX \to I_\varphi$.

For each $(a, b) \subset (0, 1)$ we can define a map $rX_{(a,b)}\colon HX \to PX$ by the formula $\pi_\varphi \circ rX_{(a,b)} = p_{\varphi_{(a,b)}}$. It is easy to check that $rX_{(a,b)}$ is a well defined continuous affine map.

We also define a map $iX\colon PX \to HX$ by the formula $p_{\varphi_{(a,b)}} \circ iX = \pi_\varphi$. We have that $rX_{(a,b)} \circ iX = \mathrm{id}_{PX}$, hence $rX_{(a,b)}$ is a retraction for each $(a, b) \subset (0, 1)$. We denote the map $rX_{(0,1)}$ simply by rX. Let us remark that $r_{(a,b)}\colon H \to P$ is a natural transformation. (This means that for each map $f\colon X \to Y$ we have $Pf \circ rX_{(a,b)} = rY_{(a,b)} \circ Hf$.) The same property is valid for $i\colon P \to H$.

3. Openess of the functor *H*

A subset $A \subset HX$ is called *e-convex* if $e(\alpha, \beta, t) \in A$, for each $\alpha, \beta \in A$ and $t \in I$. If additionally, A is convex, then we say that A is *H-convex*. Throughout this section we shall assume that $f\colon X \to Y$ is a continuous surjective map between compacta. The proofs of the next three lemmas are easily verifiable on $\mathrm{HM}X$, which is a dense subset of HX.

Lemma 3.1. *For each $\mu, \nu \in HX$ and $t \in [0, 1]$ we have $e(Hf(\mu), Hf(\nu), t) = Hf(e(\mu, \nu, t))$.*

Lemma 3.2. *Consider any $\nu \in HX$ and $a, b, c \in \mathbb{R}$ such that $0 \le a < c < b \le 1$. Then we have $p_{\varphi_{(a,b)}}(\nu) = \frac{c-a}{b-a} p_{\varphi_{(a,c)}}(\nu) + \frac{b-c}{b-a} p_{\varphi_{(c,b)}}(\nu)$, for each $\nu \in HX$.*

Lemma 3.3. *Let $t \in (0, 1)$ and $(a, b) \subset (0, 1)$. For each $\mu, \nu \in HX$ and $\varphi \in C(X)$ we have $p_{\varphi_{(a,b)}}(e(\mu, \nu, t)) = p_{\varphi_{(a,b)}}(\mu)$ if $b \le t$ and $p_{\varphi_{(a,b)}}(e(\mu, \nu, t)) = p_{\varphi_{(a,b)}}(\nu)$ if $t \le a$.*

Lemma 3.4. *Let A be a closed H-convex subset of HX and $\nu \notin A$. Then there exist $\varphi \in C(X)$ and $(a, b) \subset (0, 1)$ such that $p_{\varphi_{(a,b)}}(\nu) < p_{\varphi_{(a,b)}}(\mu)$, for each $\mu \in A$.*

Proof. Suppose to the contrary. We can for each $\mu \in A$ choose $\psi_\mu \in S_{\mathrm{HM}}(X)$ such that $p_{\psi_\mu}(\nu) < p_{\psi_\mu}(\mu)$. Since A is compact, there exist $\mu_1, \dots, \mu_n \in A$ such that for each $\mu \in A$ there exists $i \in \{1, \dots, n\}$ such that $p_{\psi_{\mu_i}}(\nu) < p_{\psi_{\mu_i}}(\mu)$. By Lemma 3.2 we can choose a family of intervals $\{(a_i, b_i)\}_{i=1}^k$ such that $b_i \le a_{i+1}$ and for each $i \in \{1, \dots, k\}$ a family of function $\varphi^1_{(a_i,b_i)}, \dots, \varphi^{n_i}_{(a_i,b_i)} \in S_{\mathrm{HM}}(X)$ such that for each $\mu \in A$ there exist $i \in \{1, \dots, k\}$ and $l \in \{1, \dots, n_i\}$ such that $p_{\varphi^l_{(a_i,b_i)}}(\nu) < p_{\varphi^l_{(a_i,b_i)}}(\mu)$.

Consider the set $K = \{\mu \in A | p_{\varphi^l_{(a_i,b_i)}}(\mu) \le p_{\varphi^l_{(a_i,b_i)}}(\nu)$ for each $i \in \{2, \dots, k\}$ and $l \in \{1, \dots, n_i\}\}$. Then K is a compact convex subset of A and for each $\mu \in A$ there exists $l \in \{1, \dots, n_1\}$ such that $p_{\varphi^l_{(a_1,b_1)}}(\nu) < p_{\varphi^l_{(a_1,b_1)}}(\mu)$. Then $rX_{(a_1,b_1)}(K)$ is a convex compact subset of PX which does not contain $rX_{(a_1,b_1)}(\nu)$. Then there exists $\psi^1 \in C(X)$ such that $\pi_{\psi^1}(rX_{(a_1,b_1)}(\nu)) < \pi_{\psi^1}(\eta)$ for each $\eta \in rX_{(a_1,b_1)}(K)$. Hence, for each $\mu \in K$ we have $p_{\psi^1_{(a_1,b_1)}}(\nu) < p_{\psi^1_{(a_1,b_1)}}(\mu)$.

Proceeding in this way, we obtain $\psi^1, \dots, \psi^k \in C(X)$ such that for each $\mu \in A$ there exists $i \in \{1, \dots, k\}$ such that $p_{\psi^i_{(a_i,b_i)}}(\nu) < p_{\psi^i_{(a_i,b_i)}}(\mu)$. By our hypotheses we can choose $\mu_i \in A$ for each $i \in \{1, \dots, k\}$ such that $p_{\psi^i_{(a_i,b_i)}}(\mu_i) \le p_{\psi^i_{(a_i,b_i)}}(\nu)$. Put $\xi_1 = \mu_1$ and $\xi_{i+1} = e(\xi_i, \mu_{i+1}, b_i)$ for $i \in \{1, \dots, k-1\}$. Since A is e-convex, $\xi_k \in A$. By Lemma 3.3 we have $p_{\psi^i_{(a_i,b_i)}}(\xi_k) \le p_{\psi^i_{(a_i,b_i)}}(\nu)$ for each $i \in \{1, \dots, k\}$. Thus we obtain a contradiction and the lemma is proved.

The proof of the next lemma follows from Lemma 3.1 and the fact that Hf is an affine map.

Lemma 3.5. *$(Hf)^{-1}(\nu)$ is H-convex for each $\nu \in HY$.*

Let $f\colon X \to Y$ be a map and $\varphi \in C(X)$. By φ_* we denote the function $\varphi_*\colon Y \to \mathbb{R}$ defined by the formula $\varphi_*(y) = \inf(\varphi(f^{-1}(y)), y \in Y$. It is well known (cf. [DE]) that if f is open then the function φ_* is continuous.

Proof of Theorem 1.1. Let $f\colon X \to Y$ be a map such that the map $Hf\colon HX \to HY$ is open. Let us show that then the map Pf is also open. Consider any open set $U \subset PX$ and $\mu \in U$. Then $(rX)^{-1}(U)$ is an open set in HX and $iX(\mu) \in (rX)^{-1}(U)$ Since Hf is an open map, $Hf((rX)^{-1}(U))$ is open in HY and $Hf(iX(\mu)) \in Hf((rX)^{-1}(U))$. Since r is a natural transformation, we have $Hf((rX)^{-1}(U)) \subset (rY)^{-1}(Pf(U))$. We have $iY(Pf(\mu)) = Hf(iX(\mu))$ or $Pf(\mu) \in (iY)^{-1}(Hf((rX)^{-1}(U))) \subset (iY)^{-1}((rY)^{-1}(Pf(U))) = Pf(U)$. Since $(iY)^{-1}(Hf((rX)^{-1}(U)))$ is open, the map Pf is open. Hence f is open as well by [DE].

Now let a map $f\colon X \to Y$ be open. Suppose that Hf is not open. Then there exists $\mu_0 \in HX$, a net $\{\nu_\alpha, \alpha \in \mathcal{A}\} \subset O(Y)$ converging to $\nu_0 = Hf(\mu_0)$ and a neighborhood W of μ_0 such that $(Hf)^{-1}(\nu_\alpha) \cap W = \emptyset$ for each $\alpha \in \mathcal{A}$. Since $\mathrm{HM}(Y)$ is a dense subset of HY, we can suppose that all $\nu_\alpha \in \mathrm{HM}(Y)$. Since HX is a compactum, we can assume that the net $A_\alpha = (Hf)^{-1}(\nu_\alpha)$ converges in $\exp(HX)$ to some closed subset $A \subset HX$. It is easy to check that $A \subset (Hf)^{-1}(\nu_0)$ and $\mu_0 \notin A$. By Lemma 3.5 all sets A_α are H-convex. It is easy to see that A is H-convex as well. Since $\mu_0 \notin A$, there exists by Lemma 3.4 $\varphi \in C(X)$ and $(a, b) \subset (0, 1)$ such that $p_{\varphi_{(a,b)}}(\mu_0) < p_{\varphi_{(a,b)}}(\mu)$ for each $\mu \in A$. Consider any $\alpha \in \mathcal{A}$. Let $\{y_1, \dots, y_s\} = \nu_\alpha([0, 1))$. Choose for each y_i the point x_i such that $f(x_i) = y_i$ and $\varphi(x_i) = \varphi_*(y_i)$. Define a map $j\colon \{y_1, \dots, y_s\} \to \{x_1, \dots, x_s\}$

by the formula $j(y_i) = x_i$ and put $\mu_\alpha(t) = j \circ \nu_\alpha(t)$ for $t \in [0, 1)$. Let μ be a limit point of the net μ_α. Then $\mu \in A$. Since $\varphi_{(a,b)}(\mu_\alpha) = \varphi_{*(a,b)}(\nu_\alpha)$, we have $p_{\varphi_{(a,b)}}(\mu) = p_{\varphi_{*(a,b)}}(\nu_0) = p_{(\varphi_* \circ f)_{(a,b)}}(\mu_0) \leq p_{\varphi_{(a,b)}}(\mu_0)$. We have obtained a contradiction and the theorem is thus proved.

4. Proofs

We will need some notations and facts from the theory of non-metrizable compacta (cf. [S2] for more details). Let τ be an infinite cardinal number. A partially ordered set $\mathcal{A}$ is called *τ-complete*, if every subset of cardinality $\leq \tau$ has a least upper bound in $\mathcal{A}$. An inverse system consisting of compacta and surjective bonding maps over a τ-complete indexing set is called τ-complete. A continuous τ-complete system consisting of compacta of weight $\leq \tau$ is called a *τ-system*.

As usually, by ω we denote the countable cardinal number. A compactum X is called *openly generated* if X can be represented as the limit of an ω-system with open bonding maps.

Proof of Theorem 1.2. It was shown in [RR] that HX is an absolute retract for each metrizable compactum X. Therefore we can consider only non-metrizable case. Let X be an openly generated compactum of weight $\leq \omega_1$. By Theorem 1.1 the compactum HX is also openly generated. Since the weight of X (and HX – cf. [Ra]) is $\leq \omega_1$, HX is $AE(0)$. Since HX is a convex compactum, HX is an AR (cf. [Fe]).

Suppose now that $HX \in AR$. Since $rX\colon HX \to PX$ is a retraction, PX is also an AR. Then X is an openly generated compactum of weight $\leq \omega_1$[Fe]. Theorem is thus proved.

By $w(X)$ we denote the weight of the compactum X, by $\chi(x, X)$ the character at the point x and by $\chi(X)$ the character of the space X. The space X is called *χ-homogeneous* if for each $x, y \in X$ we have $\chi(x, X) = \chi(y, X)$. We will use the following characterization of the Tychonov cube I^τ. An AR-compactum X of weight τ is homeomorphic to I^τ for an uncountable cardinal number τ if and only if X is χ-homogeneous (cf. [S1]).

Let $x \in X$. Define $\delta(x) \in HX$ by the condition $p_{\varphi_{(a,b)}}(\delta(x)) = \varphi(x)$, for each $\varphi_{(a,b)} \in S_{\mathrm{HM}}(X)$.

Lemma 4.1. *Let $f\colon X \to Y$ be an open map. Then Hf has a degenerate fiber if and only if f has a degenerate fiber.*

Proof. Let $f\colon X \to Y$ be an open map such that there exists $y \in Y$ with $f^{-1}(y) = \{x\}$, $x \in X$. Consider any $\mu \in HX$ with $Hf(\mu) = \delta(y)$. Let us show that $\mu = \delta(x)$. Consider any $\varphi_{(a,b)} \in S_{\mathrm{HM}}(X)$. Suppose that $p_{\varphi_{(a,b)}}(\mu) \neq \varphi(x)$. We can assume that $p_{\varphi_{(a,b)}}(\mu) < \varphi(x)$. By Lemma 1 of [Ra] there exists a function $\psi \in C(Y)$ such that $\psi(y) = \varphi(x)$ and $\psi \circ f \leq \varphi$. Then we have $p_{(\psi \circ f)_{(a,b)}}(\mu) \leq p_{\varphi_{(a,b)}}(\mu) < \varphi(x)$ and $p_{\psi_{(a,b)}}(\delta(y)) = p_{\psi_{(a,b)}} O(f)(\mu) = p_{(\psi \circ f)_{(a,b)}}(\mu) < \varphi(x) = \psi(y)$. Hence we obtain the contradiction. Thus, Hf has a degenerate fiber.

Suppose now that f has no degenerate fibers. Consider any $\mu \in HY$. Take any $y \in \operatorname{supp} \mu \subset Y$. Since f is an open map and $f^{-1}(y)$ is not a singleton, we can choose two closed subsets $A_1, A_2 \subset X$ such that $f(A_1) = f(A_2) = Y$ and $(A_1 \cap f^{-1}(y)) \cap (A_2 \cap f^{-1}(y)) = \emptyset$. Since the functor H preserves surjective maps (cf. [Ra]), there exist $\mu_1 \in H(A_1)$ and $\mu_2 \in H(A_2)$ such that $Hf(\mu_1) = Hf(\mu_2) = \mu$. Since $y \in \operatorname{supp} \mu$, there exist $y_1 \in \operatorname{supp} \mu_1 \subset A_1$ and $y_2 \in \operatorname{supp} \mu_2 \subset A_2$ such that $f(y_1) = f(y_2) = y$. Hence $\mu_1 \neq \mu_2$ and the lemma is thus proved.

Lemma 4.2. *An openly generated compactum X of weight ω_1 is χ-homogeneous if and only if HX is χ-homogeneous.*

Proof. Let HX be χ-homogeneous. Since the functor H preserves the weight (cf. [Ra]), HX is an absolute retract such that $\chi(\mu, HX) = \omega_1$ for each $\mu \in HX$. Take now any $x \in X$ and suppose that there exists a countable base of open neighborhoods $\{U_i | i \in \mathbb{N}\}$. Consider a family of functions $\{\varphi_i \in C(X) | i \in \mathbb{N}\}$ such that $\varphi_i(x) = 1$, $\varphi_i | X \backslash U_\subset = 0$. Then the family of function $\{\varphi_{i(a,b)} | i \in \mathbb{N}; a, b \in \mathbb{Q}\}$ defines a countable base of neighborhoods of $\delta(x)$ in HX. We obtain a contradiction. Hence X is χ-homogeneous.

Now let X be a χ-homogeneous openly generated compactum of weight ω_1. Then $\chi(X) = \omega_1$ by Lemma 4 of [Ra]. Suppose that there exists a point $\nu \in HX$ such that $\chi(\nu, HX) < \omega_1$. Represent X as the limit space of an ω-system $\{X_\alpha, p_\alpha, \mathcal{A}\}$ with open limit projections p_α. There exists $\alpha \in \mathcal{A}$ such that $(Hp_\alpha)^{-1}(Hp_\alpha(\nu)) = \{\nu\}$. By Lemma 4.2 there exists a point $z \in X_\alpha$ such that $p_\alpha^{-1}(z) = \{x\}$, $x \in X$. Hence $\chi(x, X) < \omega_1$ and we obtain a contradiction. The lemma is thus proved.

Proof of Theorem 1.3. The proof of the theorem follows from Theorem 1.2 and Lemma 4.2.

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