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The structure of some classes of *K*-contact manifolds

MUKUT MANI TRIPATHI* and MOHIT KUMAR DWIVEDI

Department of Mathematics and Astronomy, Lucknow University, Lucknow 226 007, India
*Present Address: Department of Mathematics, Banaras Hindu University, Varanasi 221 005, India
E-mail: mmtripathi66@yahoo.com; dwivedimohit@yahoo.com

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Abstract. We study projective curvature tensor in K-contact and Sasakian manifolds. We prove that (1) if a K-contact manifold is quasi projectively flat then it is Einstein and (2) a K-contact manifold is ξ-projectively flat if and only if it is Einstein Sasakian. Necessary and sufficient conditions for a K-contact manifold to be quasi projectively flat and φ-projectively flat are obtained. We also prove that for a $(2n+1)$-dimensional Sasakian manifold the conditions of being quasi projectively flat, φ-projectively flat and locally isometric to the unit sphere $S^{2n+1}(1)$ are equivalent. Finally, we prove that a compact φ-projectively flat K-contact manifold with regular contact vector field is a principal S^1-bundle over an almost Kaehler space of constant holomorphic sectional curvature 4.

Keywords. K-contact manifold; regular K-contact manifold; Sasakian manifold; projective curvature tensor.

1. Introduction

Let M be an almost contact metric manifold equipped with an almost contact metric structure (φ, ξ, η, g). Since at each point $p \in M$ the tangent space T_pM can be decomposed into the direct sum $T_pM = \varphi(T_pM) \oplus \{\xi_p\}$, where $\{\xi_p\}$ is the 1-dimensional linear subspace of T_pM generated by ξ_p, the conformal curvature tensor $\mathcal{C}$ is a map

$$\mathcal{C}\colon T_pM \times T_pM \times T_pM \to \varphi(T_pM) \oplus \{\xi_p\}, \qquad p \in M.$$

One has the following well-known particular cases: (1) the projection of the image of $\mathcal{C}$ in $\varphi(T_pM)$ is zero; (2) the projection of the image of $\mathcal{C}$ in $\{\xi_p\}$ is zero; and (3) the projection of the image of $\mathcal{C}|_{\varphi(T_pM)\times\varphi(T_pM)\times\varphi(T_pM)}$ in $\varphi(T_pM)$ is zero. An almost contact metric manifold satisfying the case (1), (2) and (3) is said to be conformally symmetric [8], ξ-conformally flat [9] and φ-conformally flat [3] respectively. In [8], it is proved that a conformally symmetric K-contact manifold is locally isometric to the unit sphere. In [9], it is proved that a K-contact manifold is ξ-conformally flat if and only if it is an η-Einstein Sasakian manifold. In [3], some necessary conditions for a K-contact manifold to be φ-conformally flat are proved.

Apart from conformal curvature tensor, the projective curvature tensor is another important tensor from the differential geometric point of view. Let M be an m-dimensional Riemannian manifold. If there exists a one-to-one correspondence between each coordinate neighborhood of M and a domain in Euclidean space such that any geodesic of the

Riemannian manifold corresponds to a straight line in the Euclidean space, then M is said to be locally projectively flat. For $m \geq 3$, M is locally projectively flat if and only if the well-known *projective curvature tensor* $\mathcal{P}$ vanishes. Here, $\mathcal{P}$ is defined by

$$\mathcal{P}(X, Y)Z = R(X, Y)Z - \frac{1}{m-1}\{g(Y, Z)QX - g(X, Z)QY\} \tag{1.1}$$

for $X, Y, Z \in TM$, where R is the curvature tensor and Q is the Ricci operator. In fact, M is projectively flat (that is, $\mathcal{P} = 0$) if and only if it is of constant curvature (pp. 84–85 of [7]). Thus, the projective curvature tensor is a measure of the failure of a Riemannian manifold to be of constant curvature.

In this paper we study projective curvature tensor in K-contact and Sasakian manifolds. Section 2 contains some preliminaries. In §3, we consider three cases of projective curvature tensor, analogous to those of conformal curvature tensor, and give definitions of quasi projectively flat, ξ-projectively flat and φ-projectively flat almost contact metric manifolds. It is proved that if a K-contact manifold is quasi projectively flat then it is Einstein. We also prove that a K-contact manifold is ξ-projectively flat if and only if it is Einstein Sasakian. Necessary and sufficient conditions for a K-contact manifold to be quasi projectively flat and φ-projectively flat are obtained. Thus in §3, we prove that for a $(2n+1)$-dimensional Sasakian manifold the conditions of being quasi projectively flat, φ-projectively flat and locally isometric to the unit sphere $S^{2n+1}(1)$ are equivalent. In the last section, it is established that a φ-projectively flat compact regular K-contact manifold is a principal S^1-bundle over an almost Kaehler space of constant holomorphic sectional curvature 4.

2. Preliminaries

Let M be an *almost contact metric manifold* of dimension $(2n+1)$ equipped with an almost contact metric structure (φ, ξ, η, g) consisting of a $(1, 1)$ tensor field φ, a vector field ξ, a 1-form η and a Riemannian metric g. Then

$$\varphi^2 = -I + \eta \otimes \xi, \quad \eta(\xi) = 1, \quad \varphi\xi = 0, \quad \eta \circ \varphi = 0, \tag{2.1}$$

$$g(X, Y) = g(\varphi X, \varphi Y) + \eta(X)\eta(Y), \qquad X, Y \in TM. \tag{2.2}$$

From (2.1) and (2.2) we easily get

$$g(X, \varphi Y) = -g(\varphi X, Y), \qquad g(X, \xi) = \eta(X), \qquad X, Y \in TM. \tag{2.3}$$

An almost contact metric manifold is

(1) a contact metric manifold if $g(X, \varphi Y) = d\eta(X, Y)$ for all $X, Y \in TM$;
(2) a K-contact manifold if $\nabla\xi = -\varphi$, where ∇ is Levi-Civita connection; and
(3) a Sasakian manifold if $(\nabla_X\varphi)Y = g(X, Y)\xi - \eta(Y)X$ for all $X, Y \in TM$.

A K-contact manifold is a contact metric manifold, while converse is true if the Lie derivative of φ in the characteristic direction ξ vanishes. A Sasakian manifold is always a K-contact manifold. A 3-dimensional K-contact manifold is a Sasakian manifold. A contact metric manifold is Sasakian if and only if

$$R(X, Y)\xi = \eta(Y)X - \eta(X)Y, \qquad X, Y \in TM. \tag{2.4}$$

In a Sasakian manifold M equipped with the structure (φ, ξ, η, g), the following relations are well-known:

$$R(\xi, X)Y = g(X, Y)\xi - \eta(Y)X, \qquad X, Y \in TM, \tag{2.5}$$

$$S(X, \xi) = 2n\eta(X), \qquad X \in TM, \tag{2.6}$$

where S is the Ricci tensor.

In a $(2n + 1)$-dimensional almost contact metric manifold M, if $\{e_1, \dots, e_{2n}, \xi\}$ is a local orthonormal basis of vector fields in M, then $\{\varphi e_1, \dots, \varphi e_{2n}, \xi\}$ is also a local orthonormal basis. It is easy to verify that

$$\sum_{i=1}^{2n} g(e_i, e_i) = \sum_{i=1}^{2n} g(\varphi e_i, \varphi e_i) = 2n, \tag{2.7}$$

$$\sum_{i=1}^{2n} g(e_i, Z)S(Y, e_i) = \sum_{i=1}^{2n} g(\varphi e_i, Z)S(Y, \varphi e_i) = S(Y, Z) - S(Y, \xi)\eta(Z) \tag{2.8}$$

for all $Y, Z \in TM$. In particular, in view of $\eta \circ \varphi = 0$, we get

$$\sum_{i=1}^{2n} g(e_i, \varphi Z)S(Y, e_i) = \sum_{i=1}^{2n} g(\varphi e_i, \varphi Z)S(Y, \varphi e_i) = S(Y, \varphi Z) \tag{2.9}$$

for all $Y, Z \in TM$. If M is a K-contact manifold then it is known that

$$R(X, \xi)\xi = X - \eta(X)\xi, \qquad X \in TM \tag{2.10}$$

and

$$S(\xi, \xi) = 2n. \tag{2.11}$$

Moreover, M is Einstein if and only if

$$S = 2ng. \tag{2.12}$$

From (2.11) we get

$$\sum_{i=1}^{2n} S(e_i, e_i) = \sum_{i=1}^{2n} S(\varphi e_i, \varphi e_i) = r - 2n, \tag{2.13}$$

where r is the scalar curvature. In a K-contact manifold we also get

$$R(\xi, Y, Z, \xi) = g(\varphi Y, \varphi Z), \qquad Y, Z \in TM. \tag{2.14}$$

Consequently,

$$\sum_{i=1}^{2n} R(e_i, Y, Z, e_i) = \sum_{i=1}^{2n} R(\varphi e_i, Y, Z, \varphi e_i) = S(Y, Z) - g(\varphi Y, \varphi Z). \tag{2.15}$$

For more details we refer to [1].

3. Some structure theorems

In a $(2n+1)$-dimensional almost contact metric manifold $(M, \varphi, \xi, \eta, g)$ the projective curvature tensor $\mathcal{P}$ is given by

$$\mathcal{P}(X, Y)Z = R(X, Y)Z - \frac{1}{2n}\{g(Y, Z)QX - g(X, Z)QY\}, \quad X, Y, Z \in TM. \tag{3.1}$$

Analogous to the consideration of conformal curvature tensor, we give the following definition:

DEFINITION 3.1

An almost contact metric manifold M is said to be
quasi projectively flat if

$$g(\mathcal{P}(X, Y)Z, \varphi W) = 0, \qquad X, Y, Z, W \in TM, \tag{3.2}$$

ξ*-projectively flat* if

$$\mathcal{P}(X, Y)\xi = 0, \qquad X, Y \in TM \tag{3.3}$$

and φ*-projectively flat* if (see also Definition 5 of [6])

$$g(\mathcal{P}(\varphi X, \varphi Y)\varphi Z, \varphi W) = 0, \qquad X, Y, Z, W \in TM. \tag{3.4}$$

We begin with the following:

Lemma 3.2. *Let M be a $(2n+1)$-dimensional K-contact manifold. If M satisfies*

$$g(\mathcal{P}(\varphi X, Y)Z, \varphi W) = 0, \qquad X, Y, Z, W \in TM, \tag{3.5}$$

then M is Einstein.

Proof. Let M be a $(2n+1)$-dimensional K-contact manifold. From (3.1) we have

$$g(\mathcal{P}(\varphi X, Y)Z, \varphi W) = R(\varphi X, Y, Z, \varphi W) - \frac{1}{2n}\{g(Y, Z)S(\varphi X, \varphi W) - g(\varphi X, Z)S(Y, \varphi W)\} \tag{3.6}$$

for all $X, Y, Z, W \in TM$. If $\{e_1, \ldots, e_{2n}, \xi\}$ is a local orthonormal basis of vector fields in M, then from (3.6) we get

$$\sum_{i=1}^{2n} g(\mathcal{P}(\varphi e_i, Y)Z, \varphi e_i) = \sum_{i=1}^{2n} R(\varphi e_i, Y, Z, \varphi e_i) - \frac{1}{2n}\sum_{i=1}^{2n}\{g(Y, Z)S(\varphi e_i, \varphi e_i) - g(\varphi e_i, Z)S(Y, \varphi e_i)\}$$

for all $Y, Z \in TM$. Using (2.15), (2.2), (2.13) and (2.8) in the above equation, we get

$$2n \sum_{i=1}^{2n} g(\mathcal{P}(\varphi e_i, Y)Z, \varphi e_i) = (2n+1)S(Y,Z) + 2n\eta(Y)\eta(Z) - rg(Y,Z) - \eta(Z)S(Y,\xi) \tag{3.7}$$

for all $Y, Z \in TM$. If M satisfies (3.5) then from (3.7) we get

$$(2n+1)S(Y,Z) = rg(Y,Z) - 2n\eta(Y)\eta(Z) + \eta(Z)S(Y,\xi) \tag{3.8}$$

for all $Y, Z \in TM$. Putting $Y = \xi$ in (3.8) and using (2.11), (2.3) and $\eta(\xi) = 1$, we get

$$S(Z,\xi) = \frac{r}{2n+1}\eta(Z), \qquad Z \in TM \tag{3.9}$$

from which we get

$$r = 2n(2n+1). \tag{3.10}$$

Using (3.9) and (3.10) in (3.8) we get (2.12). ■

In view of Lemma 3.2 we have the following.

Theorem 3.3. *If a K-contact manifold is quasi projectively flat then it is Einstein.*

Next, we have the following.

Theorem 3.4. *A K-contact manifold M is quasi projectively flat if and only if*

$$R(X,Y,Z,\varphi W) = g(Y,Z)g(X,\varphi W) - g(X,Z)g(Y,\varphi W) \tag{3.11}$$

for all $X, Y, Z, W \in TM$.

Proof. If M is quasi projectively flat, using (2.12) in

$$g(\mathcal{P}(X,Y)Z,\varphi W) = R(X,Y,Z,\varphi W) - \frac{1}{2n}\{g(Y,Z)S(X,\varphi W) - g(X,Z)S(Y,\varphi W)\}$$

we obtain (3.11). The converse is straightforward. ■

Theorem 3.5. *A K-contact manifold is ξ-projectively flat if and only if it is Einstein Sasakian.*

Proof. Using (3.1) and $g(X,\xi) = \eta(X)$ we get

$$g(\mathcal{P}(X,Y)\xi, W) = R(X,Y,\xi,W) - \frac{1}{2n}\{\eta(Y)S(X,W) - \eta(X)S(Y,W)\} \tag{3.12}$$

for all $X, Y, W \in TM$. For a local orthonormal basis $\{e_1, \dots, e_{2n}, \xi\}$ of vector fields in M, from (3.12) we get

$$\sum_{i=1}^{2n} g(\mathcal{P}(e_i, Y)\xi, e_i) = \sum_{i=1}^{2n} R(e_i, Y, \xi, e_i) - \frac{1}{2n}\sum_{i=1}^{2n} \eta(Y)S(e_i, e_i)$$

for all $Y \in TM$. Using (2.15) and (2.13) in the above equation we get

$$\sum_{i=1}^{2n} g(\mathcal{P}(e_i, Y)\xi, e_i) = S(Y, \xi) - \frac{r-2n}{2n}\eta(Y), \qquad Y \in TM. \tag{3.13}$$

If M is ξ-projectively flat then from (3.13) we get

$$S(Y, \xi) = \frac{r-2n}{2n}\eta(Y), \qquad Y \in TM. \tag{3.14}$$

Putting $Y = \xi$ in (3.14) and using (2.11) and $\eta(\xi) = 1$ we get (3.10). In view of (3.10), eq. (3.14) becomes (2.6). Since M is ξ-projectively flat, putting $Y = \xi$ in (3.12) and using (2.10), (2.6) and $\eta(\xi) = 1$ we obtain (2.12), which shows that M is Einstein. Using (3.3) and (2.12) in (3.12), we obtain (2.4), which shows that M is Sasakian.

Conversely, if M is Einstein Sasakian, then in view of (2.4) and (2.12) from (3.1) we get (3.3). ■

Remark 3.6. In [4], it is proved that if a Sasakian manifold is projectively flat then it is an Einstein manifold. The assumption of K-contact is weaker than that of Sasakian and the assumptions of quasi projectively flat and ξ-projectively flat are weaker than that of projectively flat. Thus Theorems 3.3 and 3.5 are stronger results than the result of [4].

Next, we prove the following:

Theorem 3.7. *A K-contact manifold M is φ-projectively flat if and only if M satisfies*

$$R(\varphi X, \varphi Y, \varphi Z, \varphi W) = g(\varphi Y, \varphi Z)\, g(\varphi X, \varphi W) - g(\varphi X, \varphi Z)\, g(\varphi Y, \varphi W) \tag{3.15}$$

for all $X, Y, Z, W \in TM$.

Proof. Let M be a $(2n + 1)$-dimensional K-contact manifold. From (3.1) we have

$$g(\mathcal{P}(\varphi X, \varphi Y)\varphi Z, \varphi W) = R(\varphi X, \varphi Y, \varphi Z, \varphi W) - \frac{1}{2n}\{g(\varphi Y, \varphi Z)S(\varphi X, \varphi W) - g(\varphi X, \varphi Z)S(\varphi Y, \varphi W)\} \tag{3.16}$$

for all $X, Y, Z, W \in TM$. For an orthonormal basis of vector fields $\{e_1, \dots, e_{2n}, \xi\}$ in M, from (3.16) it follows that

$$\sum_{i=1}^{2n} g(\mathcal{P}(\varphi e_i, \varphi Y)\varphi Z, \varphi e_i) = \sum_{i=1}^{2n} R(\varphi e_i, \varphi Y, \varphi Z, \varphi e_i) - \frac{1}{2n}\sum_{i=1}^{2n}\{g(\varphi Y, \varphi Z)S(\varphi e_i, \varphi e_i) - g(\varphi e_i, \varphi Z)S(\varphi Y, \varphi e_i)\},$$

which in view of (2.15), (2.13) and (2.9) becomes

$$\sum_{i=1}^{2n} g(\mathcal{P}(\varphi e_i, \varphi Y)\varphi Z, \varphi e_i) = \left(1 + \frac{1}{2n}\right) S(\varphi Y, \varphi Z) - \frac{r}{2n} g(\varphi Y, \varphi Z) \tag{3.17}$$

for all $Y, Z \in TM$. If M is φ-projectively flat then from (3.17) we get

$$S(\varphi Y, \varphi Z) = \frac{r}{2n+1} g\left(\varphi Y, \varphi Z\right), \qquad Y, Z \in TM. \tag{3.18}$$

Using (2.13) and (2.7) in the above equation we get (3.10). In view of (3.10), eq. (3.18) becomes

$$S(\varphi Y, \varphi Z) = 2n g(\varphi Y, \varphi Z), \qquad Y, Z \in TM. \tag{3.19}$$

Since M is φ-projectively flat, in view of (3.19), eq. (3.16) yields (3.15). The converse is obvious. ∎

Now we prove the following:

Theorem 3.8. *Let M be a $(2n+1)$-dimensional Sasakian manifold. Then the following statements are equivalent*:

(a) *M is quasi projectively flat.*
(b) *M is φ-projectively flat.*
(c) *M is locally isometric to the unit sphere $S^{2n+1}(1)$.*

Proof. Let M be a Sasakian manifold of dimension $(2n+1)$. From (3.2) and (3.4) it is obvious that (a) implies (b). Now, assume that (b) is true. In a Sasakian manifold, in view of (2.5) and (2.4) we can verify

$$\begin{aligned} R(\varphi^2 X, \varphi^2 Y, \varphi^2 Z, \varphi^2 W) = {} & R(X, Y, Z, W) - g(Y, Z)\eta(X)\eta(W) + g(X, Z)\eta(Y)\eta(W) \\ & + g(Y, W)\eta(X)\eta(Z) - g(X, W)\eta(Y)\eta(Z) \end{aligned} \tag{3.20}$$

for all $X, Y, Z, W \in TM$. Replacing X, Y, Z, W by $\varphi X, \varphi Y, \varphi Z, \varphi W$ respectively in (3.15) and using (3.20) we get

$$R(X, Y)Z = g(Y, Z)X - g(X, Z)Y, \qquad X, Y, Z \in TM.$$

This proves the statement (c).

Thus, it is easy to check that (c) implies (a). This completes the proof. ∎

4. Compact regular *K*-contact manifolds

A $(2n+1)$-dimensional K-contact manifold M is said to be regular if for each point $p \in M$ there is a cubical coordinate neighborhood U of p such that the integral curves of ξ in U pass through U only once. Moreover, if M is compact also, the orbits of ξ are closed curves. Let the space of orbits of ξ be denoted by B. Then we have the natural projection $\pi\colon M \to B$ and B is a $2n$-dimensional differentiable manifold such that π is a differentiable map. In [2], it is proved that if M is a $(2n+1)$-dimensional compact regular contact manifold, then M is a principal S^1-bundle over B, where S^1 is a 1-dimensional compact Lie group which is isomorphic to the 1-parameter group of global transformations generated by ξ.

Now, we prove the following:

Theorem 4.1. *A φ-projectively flat compact regular K-contact manifold is a principal S^1-bundle over an almost Kaehler space of constant holomorphic sectional curvature* 4.

Proof. Let M be a compact regular K-contact manifold. Since in a K-contact manifold ξ is a Killing vector field, the metric g is invariant under the action of the group S^1. Hence a metric $\tilde{g}$ and a (1, 1) tensor field J on B can be defined by

$$\tilde{g}(X,Y) = g(X^*,Y^*), \tag{4.1}$$

$$JX = \pi_*\varphi X^* \tag{4.2}$$

for any vector fields $X, Y \in TB$, where * denotes the horizontal lift with respect to η. It is well-known that $(J, \tilde{g})$ is an almost Kaehler structure on B [5]. Let $\tilde{R}$ denote the Riemann curvature tensor on B. Then we have [3]

$$\begin{aligned}\tilde{R}(X,Y,Z,W) &= R(X^*,Y^*,Z^*,W^*) + 2g(X^*,\varphi Y^*)g(\varphi Z^*,W^*)\\ &\quad - g(Z^*,\varphi X^*)g(\varphi Y^*,W^*) + g(Z^*,\varphi Y^*)g(\varphi X^*,W^*)\end{aligned}$$

for all $X, Y, Z, W \in TB$. So from (4.2), we obtain [3]

$$\begin{aligned}\tilde{R}(JX,JY,JZ,JW) &= R(\varphi X^*,\varphi Y^*,\varphi Z^*,\varphi W^*) + 2g(X^*,\varphi Y^*)g(\varphi Z^*,W^*)\\ &\quad - g(Z^*,\varphi X^*)g(\varphi Y^*,W^*) + g(Z^*,\varphi Y^*)g(\varphi X^*,W^*).\end{aligned} \tag{4.3}$$

Moreover, if M is φ-projectively flat then from Theorem 3.7 and eq. (4.3) we have

$$\begin{aligned}\tilde{R}(JX,JY,JZ,JW) &= g(\varphi Y^*,\varphi Z^*)g(\varphi X^*,\varphi W^*) - g(\varphi X^*,\varphi Z^*)g(\varphi Y^*,\varphi W^*)\\ &\quad + 2g(X^*,\varphi Y^*)g(\varphi Z^*,W^*) - g(Z^*,\varphi X^*)g(\varphi Y^*,W^*)\\ &\quad + g(Z^*,\varphi Y^*)g(\varphi X^*,W^*).\end{aligned}$$

In the above equation, replacing X and W by JX and JW respectively, we get

$$\begin{aligned}\tilde{R}(X,JY,JZ,W) &= g(Y^*,Z^*)g(X^*,W^*) - g(\varphi X^*,Z^*)g(Y^*,\varphi W^*)\\ &\quad + 2g(X^*,Y^*)g(Z^*,W^*) + g(X^*,Z^*)g(Y^*,W^*)\\ &\quad + g(\varphi Y^*,Z^*)g(\varphi X^*,W^*),\end{aligned}$$

which for a unit vector field $X \in TB$ gives

$$\tilde{R}(X,JX,JX,X) = 4.$$

Thus the base manifold B is of constant holomorphic sectional curvature 4. ∎

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