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Stokes flow with slip and Kuwabara boundary conditions

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Abstract. The forces experienced by randomly and homogeneously distributed parallel circular cylinder or spheres in uniform viscous flow are investigated with slip boundary condition under Stokes approximation using particle-in-cell model technique and the result compared with the no-slip case. The corresponding problem of streaming flow past spheroidal particles departing but little in shape from a sphere is also investigated. The explicit expression for the stream function is obtained to the first order in the small parameter characterizing the deformation. As a particular case of this we considered an oblate spheroid and evaluate the drag on it.

Keywords. Particle-in-cell models; Gegenbauer function; slip condtion; deformed sphere; vorticity.

1. Introduction

The study of slow flow problems, in which the inertial effects are negligible in comparison with the viscous effects was initiated by Stokes [6] who investigated the motion of pendulums. In engineering problems it is usual to have a swarm of particles instead of a single particle. But, it is sufficient to obtain a relatively simple analytical expression that takes into account the effect of the neighboring particles by developing particle-in-cell models.

Kuwabara [3] proposed a cell model in which both particle and the outer envelope are spherical. This formulation has the significant advantage that it leads to an axially symmetric flow that has a simple analytical solution in closed form, and thus can be used for drag calculations. Kuwabara assumes that the inner surface is stationary and that fluid passes through a cell enveloping it. The following boundary conditions are imposed: (i) no-slip and impenetrability at the inner surface, (ii) zero vorticity and uniform velocity conditions at the outer envelope.

In the present work the solution to the Stokes flow problem involving cylinders/spheres with Kuwabara boundary conditions is obtained, with slip boundary condition on the inner solid surface, instead of the no-slip condition on it. Also this model is extended to the case of a slightly deformed sphere; and as a particular case of it an oblate spheroid is considered and the drag force evaluated on it. The dependence of drag force on the slip parameter for spheres/cylinders are discussed analytically and graphically. The difference in our result for drag force from that obtained by Palaniappan [4] is also discussed.

2. The case of parallel circular cylinders

In the mathematical model, we assume that all the circular cylinders have the same radius a and are randomly and homogeneously distributed parallel to each other.

Let us suppose that the uniform velocity U perpendicular to each stationary solid circular cylinder is directed from left to right. We take the model to consist of a hypothetical circular cylinder of radius b termed cell surface, enclosing and coaxial with the given solid circular cylinder. On the inner solid circular cylinder slip boundary condition [2] apply, while on the hypothetical outer circular cylinder Kuwabara boundary conditions [3] are assumed.

Under Stokes approximation the fundamental equation for the two-dimensional steady motion lead to the biharmonic equation

$$\nabla^4 \psi = 0, \tag{2.1}$$

where ψ is the stream function in the cylindrical coordinates (r, θ) with velocity components (v_r, v_θ) and vorticity ω expressible as

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}, \quad v_\theta = -\frac{\partial \psi}{\partial r}, \tag{2.2}$$

$$\omega = \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} - \frac{1}{r}\frac{\partial v_r}{\partial \theta} = -\nabla^2 \psi. \tag{2.3}$$

Now, the boundary conditions of the model may be expressed as:

On the solid surface

(i) Normal component of velocity vanishes. This implies

$$\psi = 0, \text{ for } r = a. \tag{2.4}$$

(ii) Tangential stress tensor

$$\tau_{r\theta} = \mu \left[\frac{1}{r}\frac{\partial v_r}{\partial \theta} + r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) \right]$$

is proportional to the tangential velocity v_θ at the surface. This leads to

$$\frac{\partial \psi}{\partial r} = \lambda r \frac{\partial}{\partial r}\left[r^{-1}\frac{\partial \psi}{\partial r} \right], \quad \text{for } r = a \tag{2.5}$$

where λ is the 'slip coefficient' having the dimension of length.

On the cell surface, Kuwabara conditions

(iii) The continuity of normal component of velocity

$$\frac{\partial \psi}{\partial \theta} = Ub\cos\theta, \quad \text{for } r = b. \tag{2.6}$$

(iv) Vanishing of vorticity

$$\nabla^2\psi = 0, \quad \text{for } r = b. \tag{2.7}$$

A suitable solution of (2.1) as in ([2], p. 394) is

$$\psi = \left[A\frac{1}{r} + Br + Cr\ln\left(\frac{r}{a}\right) + Dr^3\right]\sin\theta, \tag{2.8}$$

where A, B, C and D are arbitrary constants to be determined from the above boundary conditions (2.4)–(2.7). Thus, we have

$$A = \frac{1}{2}\frac{Ua^2}{K}\left[\lambda l^{-2} + a\left(1 - \frac{1}{2}l^{-2}\right)\right], \tag{2.9}$$

$$B = -\frac{1}{2}\frac{Ua}{K}[1 - l^{-2}], \tag{2.10}$$

$$C = \frac{U}{K}[2\lambda + a], \tag{2.11}$$

$$D = -\frac{1}{4}\frac{U}{a^2l^2K}[2\lambda + a], \tag{2.12}$$

where

$$K = \left[2\lambda\left(P + \frac{1}{2} + \frac{1}{4}l^{-4}\right) + a\left(P + l^{-2} - \frac{1}{4}l^{-4}\right)\right], \tag{2.13}$$

$$P = \ln\, l - \frac{3}{4}, \tag{2.14}$$

and $l = \frac{b}{a}$ is a dimensionless parameter. Here, note that if we consider the limiting case $\lambda = 0$ which corresponds to the no-slip condition, these values of constants agree with the corresponding Kuwabara's [3] when $a = 1$. Further, the drag force X experienced by the solid cylinder in a cell can be calculated by integrating stress components over the surface and comes out as

$$X = 4\pi\mu C, \tag{2.15}$$

where C is given by (2.11).

Now, the drag coefficient C_D may be expressed as

$$C_D = \frac{X}{\frac{1}{2}\rho U^2 \cdot 2a} = \frac{8\pi}{RP}\left[1 + \frac{1}{2}\frac{\lambda_1}{P} + \frac{c}{P}(1 - \lambda_1) - \frac{1}{4P}\cdot c^2(1 - 2\lambda_1)\right]^{-1}, \tag{2.16}$$

where $R = 2Ua/\nu$ is the Reynolds number, $\lambda_1 = 2\lambda/(a + 2\lambda)$ and

$$P = -\frac{1}{2}\,\ln\, c - \frac{3}{4}, \tag{2.17}$$

c being the volume concentration defined as

$$c = \frac{\pi a^2}{\pi b^2} = l^{-2}.$$

It may be seen that $\lambda_1 = 0\,(\lambda = 0)$ corresponds to no-slip at a rigid surface and the value $\lambda_1 = 1\,(\lambda \to \infty)$ to pure slip at the surface.

3. The case of spheres

In the mathematical model, we again assume that all spheres have the same radius a and distributed at random and homogeneously. We take a sphere of radius b concentric with the solid sphere as the cell surface. Under Stokes approximation the fundamental equation for the axisymmetric steady flow in terms of Stokes stream function ψ may be expressed [2], in spherical polar coordinates (r, θ, φ) as

$$E^4\psi = E^2(E^2\psi) = 0 \tag{3.1}$$

where

$$E^2 = \frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right). \tag{3.2}$$

Further, the non-vanishing velocity components v_r, v_θ and vorticity ω are given as

$$v_r = -\frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial\theta}, \quad v_\theta = \frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r}, \tag{3.3}$$

$$\omega = \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} - \frac{1}{r}\frac{\partial v_r}{\partial\theta}. \tag{3.4}$$

The boundary conditions:

On the solid surface

(i) Vanishing of normal component of velocity

$$\psi = 0, \quad \text{for } r = a. \tag{3.5}$$

(ii) Slip condition

$$\frac{\partial\psi}{\partial r} = \lambda r^2\frac{\partial}{\partial r}\left(r^{-2}\frac{\partial\psi}{\partial r}\right), \quad \text{for } r = a. \tag{3.6}$$

On the cell surface

(iii) The continuity of normal velocity

$$\frac{\partial\psi}{\partial\theta} = -Ub^2\sin\theta\cos\theta, \quad \text{for } r = b. \tag{3.7}$$

(iv) Vanishing of vorticity

$$E^2\psi = 0, \quad \text{for } r = b. \tag{3.8}$$

The general solution of (3.1), as in [2] is

$$\psi(r,\theta) = \sum_{n=2}^{\infty} \left(A_n r^n + B_n r^{-n+1} + C_n r^{n+2} + D_n r^{-n+3}\right) \mathrm{I}_n(\zeta). \tag{3.9}$$

Here, $\zeta = \cos\theta$ and $\mathrm{I}_n(\zeta)$ is the Gegenbauer function of order n and degree $(-1/2)$ usually denoted [5] as $C_n^{-1/2}(\zeta)$, related to Legendre function $P_n(\zeta)$ as

$$I_n(\zeta) = \frac{P_{n-2}(\zeta) - P_n(\zeta)}{(2n-1)}, \quad n \geq 2. \tag{3.10}$$

Using the boundary conditions (3.5)–(3.8) and the identity

$$I_2 I_n' = \frac{(n-1)(n-2)I_{n-1} - n(n-1)I_{n+1}}{2(2n-1)}, \tag{3.11}$$

where I_n' is the derivative of I_n with respect to ζ, in the expression (3.9) we get the following values of the non-vanishing coefficients

$$A_2 = -\frac{5Ul^3}{\gamma_2}[6\lambda l^3 + a(2l^3+1)], \tag{3.12}$$

$$B_2 = -\frac{Ul^3 a^3}{\gamma_2}[6\lambda + a(5l^3 - 2)], \tag{3.13}$$

$$C_2 = \frac{3U}{a^2\gamma_2}[2\lambda + a]\, l^3, \tag{3.14}$$

$$D_2 = \frac{15aU}{\gamma_2}[2\lambda + a]\, l^6, \tag{3.15}$$

where

$$\gamma_2 = 2[3\lambda(5l^6 - 6l^5 + 1) + a(5l^6 - 9l^5 + 5l^3 - 1)], \tag{3.16}$$

and $l = b/a$ is a dimensionless parameter. Here, note that if we consider the limiting case $\lambda = 0$ which corresponds to the no-slip condition, the values of constants agree with the corresponding Kuwabara's [3]. (Also, see for correct values Dassios *et al* [1].)

Now, the drag X experienced by the sphere in a cell may be calculated using the formula (p. 115, [2])

$$X = \mu\pi \int_0^{\pi} \tilde{\omega}^3 \frac{\partial}{\partial r}\left[\frac{E^2\psi}{\tilde{\omega}^2}\right] r\, \mathrm{d}\theta. \tag{3.17}$$

Since, $\tilde{\omega} = r\sin\theta$ and $E^2\Psi = (5C_2 r^2 - \frac{D_2}{r})\sin^2\theta$, inserting these in (3.17) and integrating, we get

$$X = 4\pi\mu D_2, \tag{3.18}$$

where D_2 is given by (3.15). Further, the drag coefficient C_D is

$$C_D = \frac{X}{\frac{1}{2}\,\rho U^2 \cdot \pi a^2} = \frac{8(3-\lambda_2)L(c^{1/3})}{R}, \tag{3.19}$$

where R is the Reynolds number, $\lambda_2 = 3\lambda/(a+3\lambda)$ and

$$L(c^{1/3}) = \left[1 - \frac{3}{5}\,c^{1/3}(3-\lambda_2) + c(1-\lambda_2) - \frac{1}{5}c^2(1-2\lambda_2)\right]^{-1}, \tag{3.20}$$

c being the volume concentration defined as

$$c = \frac{\frac{4}{3}\pi a^3}{\frac{4}{3}\pi b^3} = l^{-3}.$$

Thus, rigid body (no-slip) corresponds to the value $\lambda_2 = 0$ and bubble (perfect-slip) corresponds to $\lambda_2 = 1$ ($\lambda \to \infty$).

4. Application to the problem of sedimentation

The terminal settling velocity of homogeneously distributed spheres may be found by equating the gravitational force to the viscous force on one of them.

The gravitational force is

$$G = \frac{4}{3}\pi a^3 g(\sigma - \rho), \tag{4.1}$$

where σ is the density of the particle, ρ the density of fluid and a the radius of the particle. Thus equating X, where

$$X = 2\pi\mu U a(3-\lambda_2)L(c^{1/3}), \tag{4.2}$$

and G as given by (4.1), we get the terminal velocity $V = U$ as

$$\frac{V}{V_0} = \frac{1}{L(c^{1/3})} \tag{4.3}$$

where

$$V_0 = \frac{2(\sigma-\rho)ga^2}{3(3-\lambda_2)\mu}, \tag{4.4}$$

is the terminal velocity of a single particle in a viscous fluid.

5. The case of deformed sphere

In this case, we extend the preceding analysis to deformed sphere. It is assumed that the surface of the deformed sphere is described by

$$r = a(1 + \varepsilon_m I_m(\zeta)), \tag{5.1}$$

in which ε_m is so small that squares and higher powers of it may be neglected; correspondingly outer cell will be taken as

$$r = b(1 + \varepsilon_m I_m(\zeta)). \tag{5.2}$$

Let us now assume that the stream function for the deformed sphere is expressed as

$$\psi(r,\theta) = \psi_0(r,\theta) + \varepsilon_m \psi_1(r,\theta) \tag{5.3}$$

where

$$\psi_0(r,\theta) = (A_2 r^2 + B_2 r^{-1} + C_2 r^4 + D_2 r) I_2(\zeta), \tag{5.4}$$

$$\psi_1(r,\theta) = \sum_{n=2}^{\infty} (A'_n r^n + B'_n r^{-n+1} + C'_n r^{n+2} + D'_n r^{-n+3}) I_n(\zeta). \tag{5.5}$$

The constants A_2, B_2, C_2 and D_2 are the values for a perfect sphere as obtained in (3.12)–(3.15). Now, the boundary conditions lead to

On the solid surface

(i) Vanishing of the normal component of velocity providing,

$$\frac{\partial \psi_1}{\partial \theta} = [(2A_2 a^2 - B_2 a^{-1} + 4C_2 a^4 + D_2 a)(I_2 I'_m + I_1 I_m)]\sin\theta, \quad \text{for } r = a. \tag{5.6}$$

(ii) The tangential component of stress tensor τ_{nt} is proportional to tangential velocity v_t at the surface. This yields

$$\begin{aligned}\frac{\partial \psi_1}{\partial r} - \lambda r^2 \frac{\partial}{\partial r}\left(r^{-2}\frac{\partial \psi_1}{\partial r}\right) &= [\lambda\{2(-6B_2 a^{-3} + 4C_2 a^2 + D_2 a^{-1}) \\ &\quad - m(m-1)(2A_2 - B_2 a^{-3} + 4C_2 a^2 + D_2 a^{-1})\} \\ &\quad - 2a(A_2 + B_2 a^{-3} + 6C_2 a^2)] I_2 I_m, \quad \text{for } r = a.\end{aligned} \tag{5.7}$$

On the cell surface

(iii) The continuity of normal velocity

$$\begin{aligned}\frac{\partial \psi_1}{\partial \theta} &= [(2A_2 b^2 - B_2 b^{-1} + 4C_2 b^4 + D_2 b + 2Ub^2) \\ &\quad \times (I_2 I'_m + I_1 I_m)]\sin\theta, \quad \text{for } r = b.\end{aligned} \tag{5.8}$$

(iv) Vanishing of vorticity

$$E^2 \psi_1 = -(5bC_2 + 2D_2 b^{-2}) I_2 I_m, \quad \text{for } r = b. \tag{5.9}$$

The above system (5.6)–(5.9) can be solved with the aid of the following identities:

$$I_1 I_m = -\frac{[(m+1)I_{m+1} + (m-2)I_{m-1}]}{2m-1}, \tag{5.10}$$

$$I_2 I_m = -\frac{(m-2)(m-3)}{2(2m-1)(2m-3)} I_{m-2} + \frac{m(m-1)}{(2m+1)(2m-3)} I_m - \frac{(m+1)(m+2)}{2(2m-1)(2m+1)} I_{m+2},\ (m \geq 2) \tag{5.11}$$

together with that given in (3.11).

Thus, using the boundary conditions (5.6)–(5.9), the non-vanishing constants A'_n, B'_n, C'_n and D'_n corresponding to $n = m-2, m, m+2$, may be determined but they are too lengthy to be reported here. Instead, we give below these constants for the particular case of an oblate spheroid only.

6. The case of an oblate spheroid

As a particular case of the deformed sphere, here we consider an oblate spheroid

$$\frac{x^2+y^2}{d^2} + \frac{z^2}{d^2(1-\varepsilon)^2} = 1 \tag{6.1}$$

whose equatorial radius is d in which ε is so small that squares and higher powers of it may be neglected. Then, its polar equation is written in the form

$$r = a(1 + 2\varepsilon I_2(\zeta)) \tag{6.2}$$

where $a = d(1-\varepsilon)$. Upon comparison with (5.1), we are led to the values $m = 2$, $\varepsilon_m = 2\varepsilon$. Since A'_0, B'_0, C'_0 and D'_0 all become zero and hence, availing ourselves (6.2) we find from (5.3) that the stream function around the oblate spheroid is

$$\begin{aligned}\psi(r,\theta) =& (A_2 r^2 + B_2 r^{-1} + C_2 r^4 + D_2 r) I_2(\zeta) \\ &+ 2\varepsilon(A'_2 r^2 + B'_2 r^{-1} + C'_2 r^4 + D'_2 r) I_2(\zeta) \\ &+ 2\varepsilon(A'_4 r^4 + B'_4 r^{-3} + C'_4 r^6 + D'_4 r^{-1}) I_4(\zeta),\end{aligned} \tag{6.3}$$

where

$$A'_2 = \frac{\left[\begin{array}{l}\{s_2 - (4\lambda + a)k_2 - 2(3\lambda + a)a^2 p_2\}(6b^5 - 5a^2b^3 - a^5) \\ +5a^2\{(q_2 - k_2) + p_2(b^2 - a^2)\}(a^4 + 2ab^3 + 6\lambda b^3)\end{array}\right]}{\gamma'_2} \tag{6.4}$$

$$B'_2 = k_2 - A'_2 a^3 - C'_2 a^5 - D'_2 a^5 \tag{6.5}$$

$$C'_2 = \frac{\left[\begin{array}{l}3a^4\{(q_2 - k_2) + p_2(b^2 - a^2)\}(10\lambda ab^3 + 5a^2b^3 - 6b^5 - 10\lambda a^4) \\ -(b^3 - a^3)(6b^5 - 5a^2b^3 - a^5)\{s_2 - (4\lambda + a)k_2 - 2(3\lambda + a)a^2 p_2\}\end{array}\right]}{(6b^5 - 5a^2b^3 - a^5)\gamma'_2}, \tag{6.6}$$

$$D_2' = \frac{\left[\begin{array}{l} 15a^4b^3(10\lambda ab^3 - 6b^5 + 5a^2b^3 - 10\lambda a^4\{(q_2 - k_2) + p_2(b^2 - a^2)\} \\ -5b^3(b^3 - a^3)(6b^5 - 5a^2b^3 - a^5)\{s_2 - (4\lambda + a)k_2\} + 2a^2p_2(6b^5 - 5a^2b^3 - a^5) \\ \{3\lambda(6ab^4 - 5a^3b^3 - a^6) + a(9ab^5 - 10a^3b^3 + a^6)\} \end{array}\right]}{(6b^4 - 5a^2b^3 - a^5)\gamma_2'}, \tag{6.7}$$

$$\gamma_2' = 2a^2[3\lambda(5b^6 - 6ab^5 + a^6) + a(5b^6 - 9ab^5 + 5a^3b^3 - a^6)], \tag{6.8}$$

Also,

$$A_4' = \frac{\left[\begin{array}{l} \{15(6\lambda + a)k_4 - 5s_4 + 2a^2(7\lambda + a)p_4\}(14b^9 - 9a^2b^7 - 5a^9) \\ -3a^2\{5(q_4 - k_4) + p_4(b^2 - a^2)\}(42\lambda b^7 + 15a^8 + 6ab^7) \end{array}\right]}{\gamma_4'}, \tag{6.9}$$

$$B_4' = \frac{1}{5}[5k_4 + a^2p_4 - 5A_4'a^7 - a^2C_4'(5a^7 + 9b^7)], \tag{6.10}$$

$$C_4' = \frac{[5(q_4 - k_4) + p_4(b^2 - a^2) - 5(b^7 - a^7)A_4']}{(14b^9 - 9a^2b^7 - 5a^9)}, \tag{6.11}$$

$$D_4' = \frac{1}{a^2}[k_4 - A_4'a^7 - B_4' - C_4'a^9], \tag{6.12}$$

$$\gamma_4' = 5a^2\left[\begin{array}{l} 7a^2(2\lambda + a)(14b^9 - 9a^2b^7 - 5a^9) - 3(b^7 - a^7) \\ (42\lambda b^7 + 15a^8 + 6ab^7) \end{array}\right], \tag{6.13}$$

with

$$a^2k_2 = -k_4 = -\frac{2a^2}{5}[2A_2a^3 - B_2 + 4C_2a^5 + D_2a^2], \tag{6.14}$$

$$a^2s_2 = -s_4 = \frac{4a^2}{5}[\lambda(2A_2a^3 + 5B_2) + a(A_2a^3 + B_2 + 6C_2a^5)], \tag{6.15}$$

$$b^2q_2 = -q_4 = -\frac{2b^2}{5}[2A_2b^3 - B_2 + 4C_2b^5 + D_2a^2 + 2Ub^3], \tag{6.16}$$

$$b^2p_2 = -p_4 = -\frac{2b^2}{5}[5C_2b^3 + 2D_2]. \tag{6.17}$$

Drag on the spheroid

We now calculate the drag F_z on oblate spheroid in a cell using (3.17), inserting ψ from (6.3) and integrating over the surface of the spheroid, we get

$$F_z = 4\pi\mu(D_2 + 2\varepsilon D_2'), \tag{6.18}$$

where D_2 is given by (3.15) and D_2' by (6.7).

Further, we deduce the drag force for unbounded fluid, i.e. ($b \to \infty$), under slip condition. This is expressed in dimensionless parameter $\lambda_2 = 3\lambda/(a + 3\lambda)$ and since $a = d(1 - \varepsilon)$, we get

$$F_z = 2\pi\mu U d\left[(3 - \lambda_2) - \frac{1}{15}(9 - 15\lambda_2 + 4\lambda_2^2)\varepsilon\right]. \tag{6.19}$$

When $\lambda_2 = 0$ (no-slip), we recover the Stokes resistance [2]

$$F_z = 6\pi\mu U d\left(1 - \frac{1}{5}\varepsilon\right) \tag{6.20}$$

for a solid oblate spheroid and when $\lambda_2 = 1$ (perfect-slip), (6.19) reduces the drag froce on the gaseous oblate spheroid as

$$F_z = 4\pi\mu U d\left(1 + \frac{1}{15}\varepsilon\right). \tag{6.21}$$

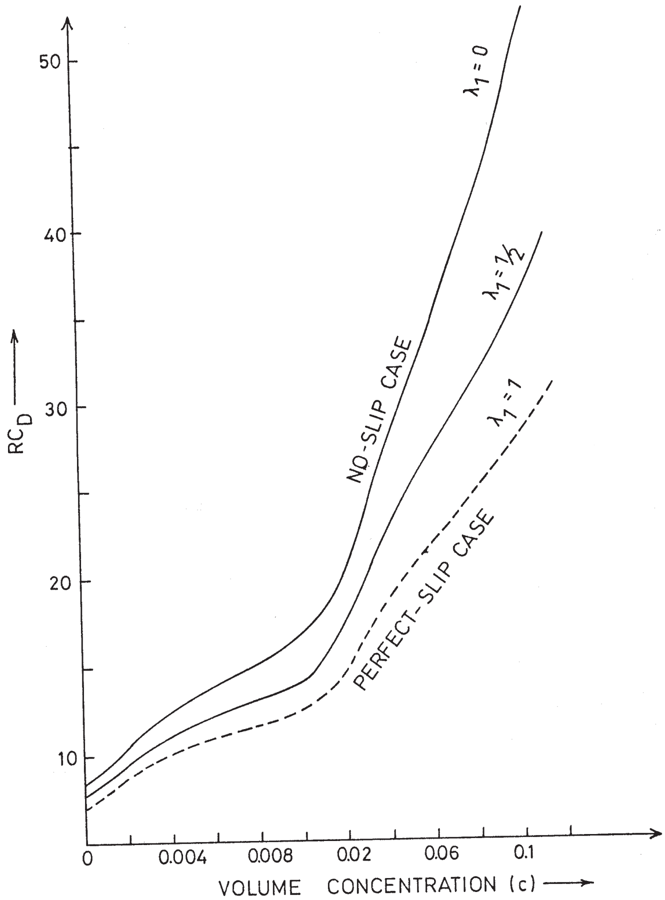

Figure 1. Variation of RC_D with respect to volume concentration (c) for circular cylinders.

If we put $\varepsilon = 0$ in (6.20) and (6.21), then these results correspond to that obtained in [2] for a sphere.

It may be remembered that Palaniappan [4] obtained instead of (6.19) the expression

$$F_z = 2\pi\mu U d\left[(3-\lambda_2) - \frac{1}{5}(3 - 5\lambda_2 + 4\lambda_2^2)\varepsilon\right]. \tag{6.22}$$

The reason for the differences is not difficult to comprehend. Palaniappan [4] has used the impenetrability condition $v_r = 0$ and taken the tangential stress as $\tau_{r\theta}$ in the slip boundary condition, while the correct conditions require $v_n = 0$, and replacement of $\tau_{r\theta}$ with τ_{nt} (n and t being the normal and tangential directions). In the case of no-slip as considered by Happel and Brenner [2], both are equivalent but when slipping is present the difference between v_r, v_n and $\tau_{r\theta}$, τ_{nt} must be taken into account.

It may be noticed that while the expression (6.20) for the no-slip case agrees with the corresponding result obtained by Palaniappan [4]. The expression (6.21) does not agree with the corresponding result where $4\pi\mu U d(1 - \frac{1}{5}\varepsilon)$ obtained by him.

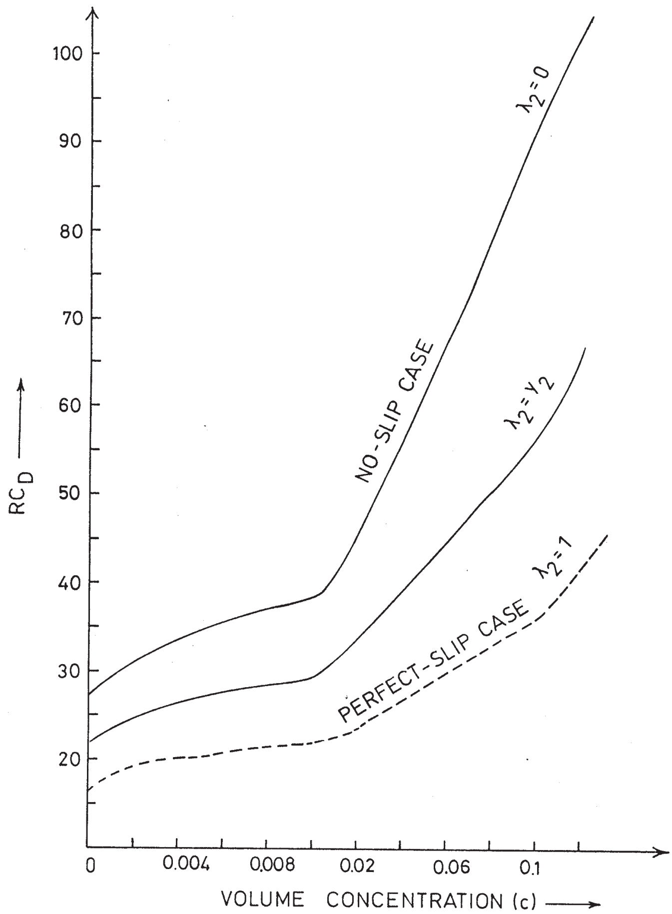

Figure 2. Variation of RC_{D} with respect to volume concentration (c) for spheres.

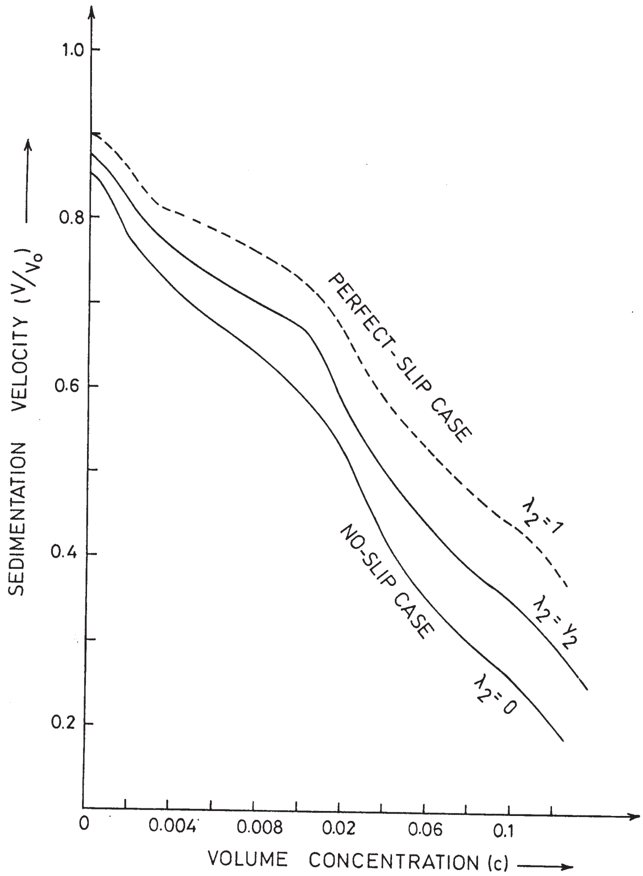

Figure 3. Variation of sedimentation velocity with respect to volume concentration (c).

7. Conclusion

In case of circular cylinders the relation between RC_D and volume concentration (c) has been numerically calculated which shows (figure 1) that as slip parameter λ_1 increases, RC_D decreases. For the case of spheres, the same result is depicted in figure 2, whereas in figure 3 the variation of (V/V_0) with volume concentration c shows that as slip parameter λ_2 increases, terminal settling velocity (V/V_0) also increases.

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