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Boundary controllability of integrodifferential systems in Banach spaces

K BALACHANDRAN and E R ANANDHI

Department of Mathematics, Bharathiar University, Coimbatore 641 046, India

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Abstract. Sufficient conditions for boundary controllability of integrodifferential systems in Banach spaces are established. The results are obtained by using the strongly continuous semigroup theory and the Banach contraction principle. Examples are provided to illustrate the theory.

Keywords. Boundary controllability; integrodifferential system; semigroup theory; fixed point theorem.

1. Introduction

Controllability of nonlinear systems represented by ordinary differential equations in Banach spaces has been extensively studied by several authors. Balachandran *et al* [1] studied the controllability of nonlinear integrodifferential systems whereas in [2] they have investigated the local null controllability of nonlinear functional differential systems in Banach spaces by using the Schauder fixed point theorem. Controllability of nonlinear functional integrodifferential systems in Banach spaces has been studied by Park and Han [10].

Several abstract settings have been developed to describe the distributed control systems on a domain Ω in which the control is acted through the boundary Γ. But in these approaches one can encounter the difficulty for the existence of sufficiently regular solution to state space system, the control must be taken in a space of sufficiently smooth functions. Balakrishnan [3] showed that the solution of a parabolic boundary control equation with L^2 controls can be expressed as a mild solution to an operator equation. Fattorini [6] discussed the general theory of boundary control systems. Barbu and Precupanu [4] studied a class of convex control problems governed by linear evolution systems covering the principal boundary control systems of parabolic type. In [5] Barbu investigated a class of boundary-distributed linear control systems in Banach spaces. Lasiecka [8] established the regularity of optimal boundary controls for parabolic equations with quadratic cost criterion. Recently Han and Park [7] derived a set of sufficient conditions for the boundary controllability of a semilinear system with a nonlocal condition. The purpose of this paper is to study the boundary controllability of nonlinear integrodifferential systems in Banach spaces by using the Banach fixed point theorem.

2. Preliminaries

Let E and U be a pair of real Banach spaces with norms $||\cdot||$ and $|\cdot|$, respectively. Let σ be a linear closed and densely defined operator with $D(\sigma)\subseteq E$ and let τ be a linear operator with $D(\tau)\subseteq E$ and $R(\tau)\subseteq X$, a Banach space.

Consider the boundary control integrodifferential system of the form

$$\dot{x}(t) = \sigma x(t) + f(t, x(t), \int_0^t g(t, s, x(s))\mathrm{d}s), \quad t \in J = [0, b],$$
$$\tau x(t) = B_1 u(t),$$
$$x(0) = x_0, \tag{1}$$

where $B_1 : U \to X$ is a linear continuous operator, the control function $u \in L^1(J, U)$, a Banach space of admissible control functions. The nonlinear operators $f : J \times E \times E \to E$ and $g : \Delta \times E \to E$ are given and $\Delta = \{(t, s); 0 \le s \le t \le b\}$. Let $A : E \to E$ be the linear operator defined by

$$D(A) = \{x \in D(\sigma); \tau x = 0\}, \quad Ax = \sigma x, \quad \text{for} \quad x \in D(A).$$

Let $B_r = \{y \in E : \|y\| \le r\}$, for some $r > 0$. We shall make the following hypotheses:

(i) $D(\sigma) \subset D(\tau)$ and the restriction of τ to $D(\sigma)$ is continuous relative to graph norm of $D(A)$.
(ii) The operator A is the infinitesimal generator of a C_0 semigroup $T(t)$ and there exists a constant $M > 0$ such that $\|T(t)\| \le M$.
(iii) There exists a linear continuous operator $B : U \to E$ such that $\sigma B \in L(U, E)$, $\tau(Bu) = B_1 u$, for all $u \in U$. Also $Bu(t)$ is continuously differentiable and $\|Bu\| \le C\|B_1 u\|$ for all $u \in U$, where C is a constant.
(iv) For all $t \in (0, b]$ and $u \in U$, $T(t)Bu \in D(A)$. Moreover, there exists a positive function $\nu \in L^1(0, b)$ such that $\|AT(t)B\| \le \nu(t)$, a.e. $t \in (0, b)$.
(v) $f : J \times E^2 \to E$ is continuous and there exist constants M_1 and M_2 such that for all $x_1, x_2 \in B_r$ and $y_1, y_2 \in E$ we have

$$\|f(t, x_1, y_1) - f(t, x_2, y_2)\| \le M_1[\|x_1 - x_2\| + \|y_1 - y_2\|]$$

and

$$M_2 = \max_{t \in J} \|f(t, 0, 0)\|.$$

(vi) $g : \Delta \times E \to E$ is continuous and there exist constants $L_1, L_2 > 0$ such that for all $x_1, x_2 \in B_r$

$$\|g(t, s, x_1) - g(t, s, x_2)\| \le L_1\|x_1 - x_2\|$$

and

$$L_2 = \max_{(t,s) \in \Delta} \|g(t, s, 0)\|.$$

Let $x(t)$ be the solution of (1). Then we can define a function $z(t) = x(t) - Bu(t)$ and from our assumption we see that $z(t) \in D(A)$. Hence (1) can be written in terms of A and B as

$$\dot{x}(t) = Az(t) + \sigma Bu(t) + f\left(t, x(t), \int_0^t g(t, s, x(s))\mathrm{d}s\right), \quad t \in J,$$
$$x(t) = z(t) + Bu(t),$$
$$x(0) = x_0. \tag{2}$$

If u is continuously differentiable on $[0, b]$, then z can be defined as a mild solution to the Cauchy problem

$$\dot{z}(t) = Az(t) + \sigma Bu(t) - B\dot{u}(t) + f\left(t, x(t), \quad \int_0^t g(t, s, x(s))\mathrm{d}s\right),$$
$$z(0) = x_0 - Bu(0)$$

and the solution of (1) is given by

$$x(t) = T(t)[x_0 - Bu(0)] + Bu(t)$$
$$+ \int_0^t T(t-s)\left[\sigma Bu(s) - B\dot{u}(s) + f\left(s, x(s), \int_0^s g(s, \tau, x(\tau)\mathrm{d}\tau)\right)\right]\mathrm{d}s. \quad (3)$$

Since the differentiability of the control u represents an unrealistic and severe requirement, it is necessary to extend the concept of the solution for the general inputs $u \in L^1$ (J, U). Integrating (3) by parts, we get

$$x(t) = T(t)x_0 + \int_0^t [T(t-s)\sigma - AT(t-s)]Bu(s)\mathrm{d}s$$
$$+ \int_0^t T(t-s)f\left(s, x(s), \int_0^s g(s, \tau, x(\tau))\mathrm{d}\tau\right)\mathrm{d}s. \quad (4)$$

Thus (4) is well defined and it is called a mild solution of the system (1).

DEFINITION

The system (1) is said to be controllable on the interval J if for every $x_0, x_1 \in E$, there exists a control $u \in L^2(J, U)$ such that the solution $x(.)$ of (1) satisfies $x(b) = x_1$.

We further consider the following additional conditions:

(vii) There exists a constant $K_1 > 0$ such that $\int_0^b \nu(t)\mathrm{d}t \leq K_1$.

(viii) The linear operator W from $L^2(J, U)$ into E defined by

$$Wu = \int_0^b [T(b-s)\sigma - AT(b-s)]Bu(s)\mathrm{d}s$$

induces an invertible operator $\tilde{W}$ defined on $L^2(J, U)/\ker W$ and there exists a positive constant $K_2 > 0$ such that $\|\tilde{W}^{-1}\| \leq K_2$. The construction of the bounded inverse operator $\tilde{W}^{-1}$ in general Banach space is outlined in the Remark.

(ix) $M\|x_0\| + [bM\|\sigma B\| + K_1]\ K_2[\|x_1\| + M\|x_0\| + N] + N \leq r$, where $N = bM[M_1[r + b(L_1 r + L_2)] + M_2]$.

(x) Let $q = bMM_1K_2[1 + bL_1](bM\|\sigma B\| + K_1)$ be such that $0 \leq q < 1$.

3. Main result

Theorem. *If the hypotheses* (i)–(x) *are satisfied, then the boundary control integrodifferential system* (1) *is controllable on J.*

Proof. Using the hypothesis (viii), for an arbitrary function $x(.)$ define the control

$$u(t) = \tilde{W}^{-1}\left[x_1 - T(b)x_0 - \int_0^b T(b-s)f\left(s, x(s), \int_0^s g(s,\tau,x(\tau))\mathrm{d}\tau\right)\mathrm{d}s\right](t). \tag{5}$$

Let $Y = C(J, B_r)$. Using this control, we shall show that the operator Φ defined by

$$\begin{aligned}\Phi x(t) = T(t)x_0 &+ \int_0^t [T(t-s)\sigma - AT(t-s)]B\tilde{W}^{-1}[x_1 - T(b)x_0 \\ &- \int_0^b T(b-\tau)f\left(\tau, x(\tau), \int_0^\tau g(\tau,\theta,x(\theta))\mathrm{d}\theta\right)\mathrm{d}\tau](s)\mathrm{d}s \\ &+ \int_0^t T(t-s)f\left(s, x(s), \int_0^s g(s,\theta,x(\theta))\mathrm{d}\theta\right)\mathrm{d}s\end{aligned}$$

has a fixed point. First we show that Φ maps Y into itself. For $x \in Y$,

$$\begin{aligned}\|\Phi x(t)\| \leq \|T(t)x_0\| &+ \Bigg\|\int_0^t [T(t-s)\sigma - AT(t-s)]B\tilde{W}^{-1}\Bigg[x_1 - T(b)x_0 \\ &- \int_0^b T(b-\tau)f\left(\tau, x(\tau), \int_0^\tau g(\tau,\theta,x(\theta))\mathrm{d}\theta\right)\mathrm{d}\tau\Bigg](s)\mathrm{d}s\Bigg\| \\ &+ \left\|\int_0^t T(t-s)f\left(s, x(s), \int_0^s g(s,\theta,x(\theta))\mathrm{d}\theta\right)\mathrm{d}s\right\| \\ \leq \|T(t)x_0\| &+ \int_0^t \|T(t-s)\|\|\sigma B\|\|\tilde{W}^{-1}\|\Bigg[\|x_1\| + \|T(b)x_0\| \\ &+ \int_0^b \|T(b-\tau)\|\Bigg[\Bigg\|f\left(\tau, x(\tau), \int_0^\tau g(\tau,\theta,x(\theta))\mathrm{d}\theta\right) \\ &- f(\tau,0,0)\Bigg\| + \|f(\tau,0,0)\|\Bigg]\mathrm{d}\tau\Bigg]\mathrm{d}s \\ &+ \int_0^t \|AT(t-s)B\|\|\tilde{W}^{-1}\|\Bigg[\|x_1\| + \|T(b)x_0\| \\ &+ \int_0^b \|T(b-\tau)\|\Bigg[\Bigg\|f\left(\tau, x(\tau), \int_0^\tau g(\tau,\theta,x(\theta))\mathrm{d}\theta\right) \\ &- f(\tau,0,0)\Bigg\| + \|f(\tau,0,0)\|\Bigg]\mathrm{d}\tau\Bigg]\mathrm{d}s \\ &+ \int_0^t \|T(t-s)\|\Bigg[\Bigg\|f\left(s, x(s), \int_0^s g(s,\theta,x(\theta))\mathrm{d}\theta\right) \\ &- f(s,0,0)\Bigg\| + \|f(s,0,0)\|\Bigg]\mathrm{d}s \\ \leq M\|x_0\| &+ bM\|\sigma B\|K_2[\|x_1\| + M\|x_0\| \\ &+ bM[M_1[r + b(L_1 r + L_2)] + M_2] \\ &+ K_1K_2[\|x_1\| + M\|x_0\| + bM[M_1[r + b(L_1 r + L_2)] + M_2] \\ &+ bM[M_1[r + b(L_1 r + L_2)] + M_2] \\ \leq M\|x_0\| &+ [bM\|\sigma B\| + K_1]K_2[\|x_1\| + M\|x_0\| + N] + N \\ \leq r.&\end{aligned}$$

Thus Φ maps Y into itself. Now, for x_1, $x_2 \in Y$ we have

$$\begin{aligned}\|\Phi x_1(t) - \Phi x_2(t)\| \le & \int_0^t [\|T(t-s)\|\|\sigma B\| + \|AT(t-s)B\|]\|\tilde{W}^{-1}\| \\ & \left[\int_0^b \|T(b-\tau)\| \left\| f(\tau, x_1(\tau), \int_0^\tau g(\tau,\theta,x_1(\theta))\mathrm{d}\theta) \right.\right. \\ & \left.\left. -f(\tau, x_2(\tau), \int_0^\tau g(\tau,\theta,x_2(\theta))\mathrm{d}\theta) \right\| \mathrm{d}\tau \right] \mathrm{d}s \\ & + \int_0^t \|T(t-s)\| \left\| f(s, x_1(s), \int_0^s g(s,\theta,x_1(\theta))\mathrm{d}\theta) \right. \\ & \left. -f(s, x_2(s), \int_0^s g(s,\theta,x_2(\theta))\mathrm{d}\theta) \right\| \mathrm{d}s \\ \le & \int_0^t [M\|\sigma B\| + \nu(t)]K_2[bMM_1(\|x_1(\tau) - x_2(\tau)\| \\ & + bL_1\|x_1(\theta) - x_2(\theta)\|)]\mathrm{d}s \\ & + bMM_1(\|x_1(s) - x_2(s)\| + bL_1\|x_1(\theta) - x_2(\theta)\|) \\ \le & \, q\|x_1(t) - x_2(t)\|.\end{aligned}$$

Therefore, Φ is a contraction mapping and hence there exists a unique fixed point $x \in Y$ such that $\Phi x(t) = x(t)$. Any fixed point of Φ is a mild solution of (1) on J which satisfies $x(b) = x_1$. Thus the system (1) is controllable on J.

4. Applications

Example 1. Let Ω be a bounded and open subset of R^n and let Γ be a sufficiently smooth boundary of Ω (say of class C^∞).

Consider the boundary control integrodifferential system,

$$\begin{aligned}\frac{\partial y(t,x)}{\partial t} - \Delta y(t,x) &= \mu(t, y(t,x), \int_0^t \eta(t,s,y(s,x))\mathrm{d}s), \quad \text{in} \quad Y = (0,b)\times\Omega, \\ y(t,0) &= u(t,0), \quad \text{on} \quad \Sigma = (0,b)\times\Gamma, \quad t \in [0,b], \\ y(0,x) &= y_0(x), \quad \text{for} \quad x \in \Omega, \end{aligned} \tag{6}$$

where $u \in L^2(\Sigma)$, $y_0 \in L^2(\Omega)$, $\mu \in L^2(Y)$ and $\eta \in Y$.

The above problem can be formulated as a boundary control problem of the form (1) by suitably taking the spaces E, X, U and the operators B_1, σ and τ as follows:

Let $E = L^2(\Omega)$, $X = H^{-\frac{1}{2}}(\Gamma)$, $U = L^2(\Gamma)$, $B_1 = I$, the identity operator and $D(\sigma) = \{y \in L^2(\Omega);\ \Delta y \in L^2(\Omega)\}$, $\sigma = \Delta$. The operator τ is the 'trace' operator such that $\tau y = y|_\Gamma$ is well defined and belongs to $H^{-\frac{1}{2}}(\Gamma)$ for each $y \in D(\sigma)$ (see [5]) and the operator A is given by

$$A = \Delta, \quad D(A) = H_0^1(\Omega) \cup H^2(\Omega).$$

(Here $H^k(\Omega), H^\alpha(\Gamma)$ and $H_0^1(\Omega)$ are usual Sobolev spaces on Ω, Γ.)

Let us assume that the nonlinear functions μ and η satisfy the following Lipschitz condition:

$$\|\mu(t, v_1, w_1) - \mu(t, v_2, w_2)\| \leq K_1[\|v_1 - v_2\| + \|w_1 - w_2\|],$$
$$\|\eta(t, s, v_1) - \eta(t, s, v_2)\| \leq K_2\|v_1 - v_2\|,$$

where K_1, $K_2 > 0$, v_1, $v_2 \in B_r$ and w_1, $w_2 \in \Omega$.

Define the linear operator $B : L^2(\Gamma) \to L^2(\Omega)$ by $Bu = w_u$ where w_u is the unique solution to the Dirichlet boundary value problem,

$$\Delta w_u = 0 \quad \text{in} \quad \Omega,$$
$$w_u = u \quad \text{in} \quad \Gamma.$$

In other words (see [9])

$$\int_\Omega w_u \Delta\psi \mathrm{d}x = \int_\Gamma u \frac{\partial\psi}{\partial n}\mathrm{d}x, \quad \text{for all} \quad \psi \in H_0^1(\Omega) \cup H^2(\Omega), \tag{7}$$

where $\partial\psi/\partial n$ denotes the outward normal derivative of ψ which is well defined as an element of $H^{\frac{1}{2}}(\Gamma)$. From (7), it follows that,

$$\|w_u\|_{L^2(\Omega)} \leq C_1\|u\|_{H^{-\frac{1}{2}}(\Gamma)}, \quad \text{for all} \quad u \in H^{-\frac{1}{2}}(\Gamma)$$

and

$$\|w_u\|_{H^1(\Omega)} \leq C_2\|u\|_{H^{\frac{1}{2}}(\Gamma)}, \quad \text{for all} \quad u \in H^{\frac{1}{2}}(\Gamma),$$

where C_i, $i = 1, 2$ are positive constants independent of u.

From the above estimates it follows by an interpolation argument [12] that

$$\|AT(t)B\|_{L(L^2(\Gamma),L^2(\Gamma))} \leq Ct^{-\frac{3}{4}}, \quad \text{for all} \quad t > 0 \quad \text{with} \quad \nu(t) = Ct^{-\frac{3}{4}}.$$

Further assume that the bounded invertible operator $\tilde{W}$ exists. Choose b and other constants such that the conditions (ix) and (x) are satisfied. Hence, we see that all the conditions stated in the theorem are satisfied and so the system (6) is controllable on $[0, b]$.

Example 2. Consider the boundary control system,

$$\begin{aligned} &\frac{\partial y(t,x)}{\partial t} - \Delta y(t,x) = f(t, y(t,x), \int_0^t g(t,s,y(s,x))\mathrm{d}s \quad \text{in} \quad Q = (0,b) \times \Omega, \\ &\frac{\partial y(t,0)}{\partial n} + \beta y(t,0) = u(t,0), \quad \text{in} \quad (0,b) \times \Gamma, \quad t \in [0,b], \\ &\qquad y(0,x) = y_0(x), \quad x \in \Omega, \end{aligned} \tag{8}$$

where $y_0 \in L^2(\Omega)$, $f \in L^2(Q)$, $g \in Q$ and $u \in L^2(\Gamma)$. Here β is a nonnegative constant. Let us assume that the nonlinear functions f and g satisfy the Lipschitz condition:

$$\|f(t, v_1, w_1) - f(t, v_2, w_2)\| \leq M_1[\|v_1 - v_2\| + \|w_1 - w_2\|],$$
$$\|g(t, s, v_1) - g(t, s, v_2)\| \leq M_2\|v_1 - v_2\|,$$

where $M_1, M_2 > 0$, $v_1, v_2 \in B_r$ and $w_1, w_2 \in \Omega$.

Take $E = L^2(\Omega)$, $U = X = L^2(\Gamma)$, $B_1 = I$, $\sigma y = \Delta y$, $\tau y = \beta y + (\partial y/\partial n)$ and $D(\sigma) = H^2(\Omega)$.

The operator A is given by

$$Ay = \Delta y, D(A) = \left\{ y \in H^2(\Omega); \quad \beta y + \frac{\partial y}{\partial n} = 0 \right\}.$$

Now the problem (8) becomes an abstract formulation of (1).

Define the linear operator $B : L^2(\Gamma) \to L^2(\Omega)$ by $Bu = z_u$ where $z_u \in H^1(\Omega)$ is the unique solution to the Neumann boundary value problem,

$$z_u - \Delta z_u = 0 \quad \text{in} \quad \Omega,$$

$$\beta z_u + \frac{\partial z_u}{\partial n} = u \quad \text{in} \quad \Gamma.$$

Consider on the product space $H^1(\Omega) \times H^1(\Omega)$, the bilinear functional

$$h(y, \psi) = \int_\Omega (y\psi + \operatorname{grad} y \operatorname{grad} \psi) \mathrm{d}x - \int_\Gamma (u - \beta y)\psi \mathrm{d}\sigma, \tag{9}$$

where $u \in H^{-\frac{1}{2}}(\Gamma)$ (here $\int_\Gamma u\psi \, \mathrm{d}\sigma$ is the value of u at $\psi \in H^{\frac{1}{2}}(\Gamma)$. Since h is coercive, there is a $z_u \in H^1(\Omega)$ satisfying $h(z_u, \psi) = 0$ for all $\psi \in H^1(\Omega)$. Hence $z_u = Bu$ is the solution to (8). From (9) we see that

$$\|w_u\|_{H^1(\Omega)} \leq C_1 \|u\|_{H^{-\frac{1}{2}}(\Gamma)}.$$

Since the operator $-A$ is self-adjoint and positive, we have

$$\int_0^b \|AT(t)y_0\|^2_{L^2(\Omega)} \mathrm{d}t \leq C\|y_0\|^2_{D((-A)^{\frac{1}{2}})} \quad \text{for all} \quad y_0 \in D((-A)^{\frac{1}{2}}) = H^1(\Omega). \tag{10}$$

Let δ be the scalar function defined by

$$\delta(t) = \lim_{n \to \infty} \inf \|A_n T(t)\|_{L(H^1(\Omega), L^2(\Omega))}, \quad t \in [0, b],$$

where $A_n = A(I + n^{-1}A)^{-1}$ for $n = 1, 2, \ldots$. Obviously,

$$\|AT(t)\|_{L(H^1(\Omega), L^2(\Omega))} \leq \delta(t) \quad \text{for} \quad t \in (0, b]. \tag{11}$$

Also we find that (10) implies that

$$\int_0^b \|A_n T(t) y_0\|^2_{L(H^1(\Omega), L^2(\Omega))} \mathrm{d}t \leq C \quad \text{for all } n.$$

Therefore by Fatou's lemma it follows that $\delta \in L^2(0, b)$ and hence from (10) and (11) we have

$$\|AT(t)Bu\|_{L^2(\Omega)} \leq C\delta(t)\|u\|_{L^2(\Gamma)}, \quad \text{for all} \quad t \in (0, b), \quad u \in L^2(\Gamma)$$

with $\nu(t) = C\delta(t) \in L^2(0, b)$. Further assume that the bounded invertible operator $\tilde{W}$ exists. Choose b and other constants in such a way that the conditions (ix) and (x) are satisfied. Thus we find that all the conditions stated in the theorem are satisfied. Hence the system (8) is controllable on $[0, b]$.

Remark (see also [11]). *Construction of* $\tilde{W}^{-1}$.

Let $Y = L^2[J, U]/\ker W$. Since ker W is closed, Y is a Banach space under the norm

$$\|[u]\|_Y = \inf_{u \in [u]} \|u\|_{L^2[J,U]} = \inf_{W\hat{u}=0} \|u + \hat{u}\|_{L^2[J,U]},$$

where $[u]$ are the equivalence classes of u.

Define $\tilde{W} : Y \to X$ by

$$\tilde{W}[u] = Wu, \quad u \in [u].$$

Now $\tilde{W}$ is one-to-one and

$$\|\tilde{W}[u]\|_X \leq \|W\| \|[u]\|_Y.$$

We claim that V = Range W is a Banach space with the norm

$$\|v\|_V = \|\tilde{W}^{-1} v\|_Y.$$

This norm is equivalent to the graph norm on $D(\tilde{W}^{-1})$ = Range W, $\tilde{W}$ is bounded and since $D(\tilde{W}) = Y$ is closed, $\tilde{W}^{-1}$ is closed and so the above norm makes Range $W = V$, a Banach space.

Moreover,

$$\begin{aligned}\|Wu\|_V &= \|\tilde{W}^{-1} Wu\|_Y = \|\tilde{W}^{-1} \tilde{W}[u]\| \\ &= \|[u]\| = \inf_{u \in [u]} \|u\| \leq \|u\|,\end{aligned}$$

so

$$W \in \mathcal{L}(L^2[J, U], V).$$

Since $L^2[J, U]$ is reflexive and ker W is weakly closed, so that the infimum is actually attained. For any $v \in V$, we can therefore choose a control $u \in L^2[J, U]$ such that $u = \tilde{W}^{-1} v$.

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