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Erratum

Quasi-parabolic Siegel formula

NITIN NITSURE

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Abstract. The main result of the above paper is mistaken, because of a defective lemma. Here we replace the defective lemma, and derive the corrected quasi-parabolic analogue of the Siegel formula.

In my paper 'Quasi-parabolic Siegel formula' (see [N]), the main result is the following formula (Theorem 3.4).

$$\sum_{E\in J(r,L)} \frac{1}{|\mathrm{ParAut}(E)|} = f(q, r_{i,j})(q-1)^{s-1} q^{(r^2-1)(g-1)-s} Z_{X-S}(q^{-2}) \cdots Z_{X-S}(q^{-r}).$$

The above formula is incorrect. The mistake occurs in Lemma 3.2 of the paper, which asserted that

$$\lim_{m\to\infty} \frac{|\mathrm{Hom}^S_{\mathrm{inj}}(\mathcal{O}^r_X, E(m))|}{q^{r\chi(E(m))}} = 1.$$

What follows is the correction to Lemma 3.2 (and its proof), and the resulting correction to the main result (Theorem 3.4 of [N]). We follow the notations and the numbering of equations and statements in the original paper.

Lemma (Corrected version of Lemma 3.2 and eq. (10))

$$\lim_{m\to\infty} \frac{|\mathrm{Hom}^S_{\mathrm{inj}}(\mathcal{O}^r_X, E(m))|}{q^{r\chi(E(m))}} = \frac{(q^r-1)^s(q^r-q)^s\cdots(q^r-q^{r-1})^s}{q^{r^2 s}}. \tag{10}$$

If S is non-empty, the limit is already attained for all large enough m (where 'large enough' depends on E).

Proof. If S is empty, the above lemma reduces to lemma 3 in [G-L]. If S is nonempty, then any morphism of locally free sheaves on X which is injective when restricted to S is injective. Let m be large enough, so that $E(m)$ is generated by global sections, $H^1(X, E(m)) = 0$, and $h^0(X, E(m)) = \chi(E(m)) \geqslant rs$. Then $H^0(X, E(m))$ has a basis consisting of sections σ_{i,P_j}, τ_l for $i = 1,\ldots,r$, $j = 1,\ldots,s$, and $l = 1,\ldots,\chi(E(m)) - rs$, such that

(1) the sections τ_l are zero on S,
(2) the sections σ_{i,P_j}, are zero at all other points of S except P_j (and hence σ_{i,P_j} restrict at P_j to a basis of the fiber of $E(m)$ at P_j.

Any element of $\mathrm{Hom}_{\mathcal{O}_X}(\mathcal{O}^r_X, E(m)) = \mathrm{Hom}_{\mathbf{F}_q}(\mathbf{F}^r_q, H^0(X, E(m)))$ is given in terms of this basis by a $r \times q^{\chi(E(m))}$ matrix A. The condition that this lies in

$$\mathrm{Hom}^S_{\mathrm{inj}}(\mathcal{O}^r_X, E(m)) \subset \mathrm{Hom}(\mathcal{O}^r_X, E(m))$$

is the condition that each of the s disjoint $r \times r$-minors, corresponding to the part $\sigma_{1,P_j},\ldots,\sigma_{r,P_j}$ of the basis, has nonzero determinant. This contributes the factor

$$\frac{|GL_r(\mathbf{F}_q)|}{|M_r(\mathbf{F}_q)|} = \frac{(q^r-1)(q^r-q)\cdots(q^r-q^{r-1})}{q^{r^2}}$$

for each P_j, which proves the lemma.

Now using this new value for $\lim_{m\to\infty}|\mathrm{Hom}^S_{\mathrm{inj}}(\mathcal{O}^r_X, E(m))|/q^{r\chi(E(m))}$ in place of the earlier mistaken value 1 in the equation (10) of [N], but keeping the rest of [N] as it is, we immediately get the folllowing corrected form of the main result (Theorem 3.4 of [N]).

Theorem. (Quasi-parabolic Siegel formula – Corrected form)

$$\sum_{E\in J(r,L)} \frac{1}{|\mathrm{ParAut}(E)|} = f(q,r_{i,j})\frac{q^{(r^2-1)(g-1)}}{q-1} Z_X(q^{-2})\cdots Z_X(q^{-r}).$$

This corrected formula has the following essential feature, which the mistaken formula lacked. If the quasi-parabolic structure at each point of S is trivial (that is, each flag consists only of the zero subspace and the whole space), then on one hand $\mathrm{ParAut}(E) = \mathrm{Aut}(E)$, and on the other hand each flag variety is a point, and so $f(q,r_{i,j}) = 1$. Hence in this situation (which includes the case when S is empty) the above formula reduces to the original Siegel formula

$$\sum_{E\in J(r,L)} \frac{1}{|\mathrm{Aut}(E)|} = \frac{q^{(r^2-1)(g-1)}}{q-1} Z_X(q^{-2})\cdots Z_X(q^{-r}).$$

References

[G-L] Ghione F and Letizia M, Effective divisors of higher rank on a curve and the Siegel formula *Compos. Math.* **83** (1992) 147–159

[N] Nitsure N, Quasi-parabolic Siegel formula, *Proc. Indian Acad. Sci.* **106** (1996) 133–137