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Four coplanar Griffith cracks moving in an infinitely long elastic strip under antiplane shear stress

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Abstract. This paper concerns with the problem of determining the anti-plane dynamic stress distributions around four coplanar finite length Griffith cracks moving steadily with constant velocity in an infinitely long finite width strip. The two-dimensional Fourier transforms have been used to reduce the mixed boundary value problem to the solution of five integral equations. These integral equations have been solved using the finite Hilbert transform technique to obtain the analytic form of crack opening displacement and stress intensity factors. Numerical results have also been depicted graphically.

Keywords. Moving Griffith cracks; elastic strip; Hilbert transform; stress intensity factor.

1. Introduction

In recent years, scattering of elastic waves by cracks of finite dimension in a strip of elastic material has been investigated by several investigators. The theory of cracks in 2-dimensional medium was first developed by Griffith [3]. Sih and Chen [7] solved the problem of a uniformly propagating finite crack in a strip of isotropic material under plane extension. Singh *et al* [8] also studied the problem of propagation for a finite length crack moving in a strip under anti-plane shear stress and gave the closed form solution. In the above analysis, the usual method of solving mixed boundary value problems by integral transforms is used to reduce the problem to a Fredholm integral equation of second kind and then proceed to its numerical solution.

As regards the crack problem research has been restricted mainly to the case of one or two cracks because of the severe mathematical complexity encountered in solving the problems of three or more cracks. Jain and Kanwal [6] solved the low frequency solution of diffraction of normally incident longitudinal waves by two co-planar Griffith cracks in an infinite isotropic elastic medium. Using a completely different technique Itou [5] solved the diffraction problem of elastic waves by two co-planar Griffith cracks in an infinite elastic medium. Problems on three coplanar Griffith cracks moving steadily in an elastic strip has been solved by Das and Sarkar [2].

To the best knowledge of the authors, the problem of stress distribution around four co-planar Griffith cracks in a strip has not been investigated so far. In this paper we have considered the problem of propagation of four co-planar Griffith cracks moving steadily in an infinitely long finite width strip under antiplane shear stress. Cracks are assumed to be moving steadily along a fixed direction with a constant speed V less than the shear wave velocity in the medium. The application of two-dimensional Fourier transforms reduced this problem to that of solving a set of five integral equations with cosine kernel and weight function. Employing finite Hilbert transform technique [9], the closed form solutions are obtained when the lateral boundaries are subjected to shearing stresses. The dynamic stress intensity factors and the crack opening displacement have been evaluated numerically for various values of crack velocity and distance between the cracks and the results have been presented by means of graphs.

2. Formulation of the problem

We first consider a strip of elastic material occupying the region $-h' \leqslant Y' \leqslant h'$ referred to a fixed co-ordinate system (X', Y', Z'). The strip extends from $-\infty$ to ∞ in X'-direction and contains four coplanar Griffith cracks such that these cracks are located in the region $-d' \leqslant X' \leqslant c'$, $-b' \leqslant X' \leqslant -a'$, $a' \leqslant X' \leqslant b'$, $c' \leqslant X' \leqslant d'$, $|Z'| < \infty$, $Y' = 0$ moving at a constant speed v in the X'-direction.

In dynamic problem of antiplane shear, there exists a single non-vanishing component of displacement $W = W(X', Y', t)$ in the Z'-direction. The corresponding stress components are

$$\sigma_{X'Z'} = \mu \frac{\partial W}{\partial X'}, \quad \sigma_{Y'Z'} = \mu \frac{\partial W}{\partial Y'} \tag{2.1}$$

where μ is the shear modulus of elastic material.

The two dimensional wave equation for $W(X', Y', t)$ is given by

$$\frac{\partial^2 W}{\partial X'^2} + \frac{\partial^2 W}{\partial Y'^2} = \frac{1}{c_2^2} \frac{\partial^2 W}{\partial t^2} \tag{2.2}$$

where $c_2 = (\mu/\rho)^{1/2}$ is the shear wave velocity and ρ is the density of the material.

Using Galilean transformation, $x' = X' - Vt$, $y' = Y'$, $z' = Z'$, $t' = t$ where (x', y', z') represents the translating co-ordinate system and also normalizing all the lengths with respect to d' so that $x' = d'x$, $y' = d'y$, $a' = ad'$, $b' = bd'$, $c' = cd'$, $h' = d'h$, $W = d'w$, (figure 1) equation (2.2) reduces to

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = 0 \tag{2.3}$$

with

$$s^2 = 1 - v^2/c_2^2. \tag{2.4}$$

Since the geometry of the problem is symmetric about the y-axis, introducing Fourier

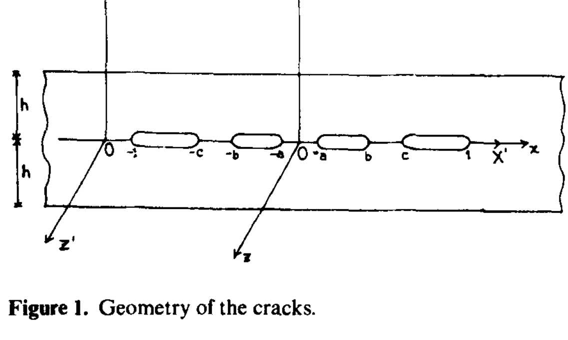

Figure 1. Geometry of the cracks.

cosine transform

$$A_1(\xi) = \int_0^\infty A(x)\cos(\xi x)\,dx$$

and

$$A(x) = \frac{2}{\pi}\int_0^\infty A_1(\xi)\cos(\xi x)\,d\xi$$

we obtain the solution of equation (2.3) as

$$w(x,y) = \pm\frac{2}{\pi}\int_0^\infty [A_1(\xi)\exp(-\xi|y|s) + A_2(\xi)\exp(\xi|y|s)]\cos(\xi x)\,d\xi \quad (2.5)$$

with ($y \gtrless 0$)

$$\sigma_{yz}(x,y) = -\frac{2\mu s}{\pi}\int_0^\infty [A_1(\xi)\exp(-\xi|y|s) - A_2(\xi)\exp(\xi|y|s)]\,\xi\cos(\xi x)\,d\xi \quad (2.6)$$

where s is the positive root of equation (2.4) and $A_1(\xi)$, $A_2(\xi)$ are the unknown functions to be determined.

In our case uniform shearing stress p is applied to the upper and lower boundaries $y = \pm h$ of the strip. The equivalent problem in our case involves the application of the shear stress $-p$ to the crack faces at $y = \phi$. Accordingly, the boundary conditions are

$$\sigma_{yz}(x, \pm h) = 0, \quad 0 < x < \infty \quad (2.7)$$

$$w(x,0) = 0, \quad x \in I_1, I_3, I_5 \quad (2.8a\text{–}c)$$

$$\sigma_{yz}(x,0) = -p, \quad x \in I_2, I_4 \quad (2.9a\text{–}b)$$

where $I_1 = (0,a)$, $I_2 = (a,b)$, $I_3 = (b,c)$, $I_4 = (c,1)$, $I_5 = (1,\infty)$.

3. Solution of the problem

Using the expression for $w(x,y)$ from (2.5) in (2.7) it has been found that

$$A_1(\xi) = \frac{A(\xi)}{1 + \exp(-2\xi hs)}$$

and

$$A_2(\xi) = \frac{A(\xi)\exp(-2\xi hs)}{1 + \exp(-2\xi hs)}$$

where $A(\xi)$ is to be determined from the boundary conditions. With the help of boundary conditions (2.8) and (2.9), $A(\xi)$ is found to satisfy the following set of five integral equations

$$\int_0^\infty A(\xi)\cos(\xi x)\,d\xi = 0, \quad x \in I_1, I_3, I_5 \quad (3.1a\text{–}c)$$

and

$$\int_0^\infty \xi H_1(\xi hs)A(\xi)\cos(\xi x)\,d\xi = \frac{\pi p}{2\mu s}, \quad x \in I_2, I_4 \quad (3.2a\text{–}b)$$

where

$$H_1(\xi hs) = \frac{1-\exp(-2\xi hs)}{1+\exp(-2\xi hs)} = \tanh(\xi hs). \tag{3.3}$$

In order to solve the set of five integral equations given by equations (3.1) and (3.2), let us take

$$A(\xi) = \frac{1}{\xi}\int_a^b g(u^2)\cosh(eu)\sin(\xi u)\mathrm{d}u + \frac{1}{\xi}\int_c^1 h(v^2)\cosh(ev)\sin(\xi v)\mathrm{d}v. \tag{3.4}$$

In (3.4), $g(u^2)$ and $h(v^2)$ are unknown functions to be determined from the boundary conditions and $e = \pi/2hs$.

Using the following result [4]

$$\int_0^\infty \frac{\sin(\xi u)\cos(\xi x)}{\xi}\mathrm{d}\xi = \begin{cases} \frac{\pi}{2}, & u > x > 0 \\ 0, & x > u > 0 \end{cases}$$

it is found that the choice of $A(\xi)$ satisfies equations (3.1a, c) if $g(u^2)$ and $h(v^2)$ satisfy

$$\int_a^b g(u^2)\cosh(eu)\mathrm{d}u = 0 \tag{3.5a}$$

and

$$\int_c^1 h(v^2)\cosh(ev)\mathrm{d}v = 0. \tag{3.5b}$$

Now (3.2a–b) may be written in the form

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_0^\infty \tanh(\xi hs)A(\xi)\sin(\xi x)\mathrm{d}\xi = \frac{\pi p}{2\mu s}, \quad x \in I_2, I_4. \tag{3.6a–b}$$

Substitution of (3.4) in (3.6a) and use of the following result [1]

$$\int_0^\infty \xi^{-1}\tanh(\xi hs)\sin(\xi u)\sin(\xi x)\mathrm{d}\xi = \frac{1}{2}\log\left|\frac{\sinh(ex)+\sinh(eu)}{\sinh(ex)-\sinh(eu)}\right|$$

yields

$$\int_A^B \frac{UG(U^2)}{U^2 - X^2}\mathrm{d}U = \frac{\pi}{2}F(x), \quad (A < X < B) \tag{3.7}$$

where $\cosh(eu) = U$, $\cosh(ev) = S$, $X = \cosh(ex)$, $A = \cosh(ea)$, $B = \cosh(eb)$, $C = \cosh(ec)$, $D = \cosh(e)$, $g(u^2) = G(U^2)$, $h(v^2) = H(S^2)$ and

$$F(X) = \frac{p}{\mu s X} - \frac{2}{\pi}\int_C^D \frac{SH(S^2)}{S^2 - X^2}\mathrm{d}S. \tag{3.8}$$

Using the finite Hilbert transform technique [9] the solution of equation (3.7) is

$$G(U^2) = \frac{2p}{\pi\mu s}\sqrt{\frac{U^2-A^2}{B^2-U^2}}\int_A^B \sqrt{\frac{B^2-X^2}{X^2-A^2}}\frac{\mathrm{d}X}{U^2-X^2} - \frac{2}{\pi}\sqrt{\frac{U^2-A^2}{B^2-U^2}}$$
$$\times \int_C^D \sqrt{\frac{S^2-B^2}{S^2-A^2}}\frac{SH(S^2)}{S^2-U^2}\mathrm{d}S + \frac{B_1}{\sqrt{(U^2-A^2)(B^2-U^2)}},$$
$$(A < U < B) \tag{3.9}$$

where B_1 is a constant to be determined from (3.5a). Substitution of expression for $A(\xi)$ from (3.4) in (3.6a) yields with aid of (3.9) the following singular integral equation involving $H(S^2)$

$$\int_C^D \sqrt{\frac{S^2-B^2}{S^2-A^2}}\frac{SH(S^2)}{S^2-U^2}\,dS = \frac{\pi}{2}\left[\frac{p}{\mu s X}\sqrt{\frac{X^2-B^2}{X^2-A^2}} - \frac{2pA^2}{\pi\mu s B X^2}\right.$$
$$\left.\times\left\{\left(\frac{X^2-B^2}{X^2-A^2}\right)\Pi\left(\frac{\pi}{2},\frac{X^2(B^2-A^2)}{B^2(X^2-A^2)},q\right) - \frac{B^2}{A^2}F\left(\frac{\pi}{2},q\right)\right\} + \frac{B_1}{X^2-A^2}\right] \tag{3.10}$$

where $q=(B^2-A^2)^{1/2}/B$ and $F(\phi,k)$, $\Pi(\phi,n,k)$ are elliptic integrals of first and third kind respectively.

Again, using finite Hilbert transform technique [9] it is found that

$$H(S^2) = -\frac{2}{\pi}\sqrt{\frac{(S^2-A^2)(S^2-C^2)}{(S^2-B^2)(D^2-S^2)}}\left[\frac{p}{\mu s}\left\{\int_C^D\sqrt{\frac{(D^2-X^2)(X^2-B^2)}{(X^2-C^2)(X^2-A^2)}}\right.\right.$$
$$\left.\times\frac{dX}{(X^2-S^2)} - \int_C^D\sqrt{\frac{(D^2-Y^2)(B^2-Y^2)}{(Y^2-A^2)(C^2-Y^2)}}\frac{dY}{(S^2-Y^2)}\right\}$$
$$\left.-\frac{\pi}{2}\sqrt{\frac{D^2-A^2}{C^2-A^2}}\frac{B_1}{(S^2-A^2)}\right] + \frac{B_2\sqrt{S^2-A^2}}{\sqrt{(S^2-B^2)(S^2-C^2)(D^2-S^2)}}$$
$$(C<S<D) \tag{3.11}$$

the constant B_2 occurring in (3.11) is to be determined using the condition given by equation (3.5b).

Next, substituting the value of $H(S^2)$ from equation (3.11) in equation (3.9) $G(U^2)$ may be written in the following form

$$G(U^2) = \frac{2}{\pi}\sqrt{\frac{U^2-A^2}{B^2-U^2}}\frac{p}{\mu s}\left[\frac{(B^2-U^2)}{BU^2(U^2-A^2)}\left\{A^2\Pi\left(\frac{\pi}{2},\frac{X^2(B^2-A^2)}{B^2(X^2-A^2)},q\right)\right.\right.$$
$$\left.+(U^2-A^2)F\left(\frac{\pi}{2},q\right)\right\} + \frac{1}{B}F\left(\frac{\pi}{2},q\right)$$
$$-\sqrt{\frac{C^2-U^2}{D^2-U^2}}\left\{\int_C^D\sqrt{\frac{(D^2-X^2)(X^2-B^2)}{(X^2-C^2)(X^2-A^2)}}\frac{dX}{(X^2-U^2)}\right.$$
$$\left.+\int_A^B\sqrt{\frac{(D^2-Y^2)(B^2-Y^2)}{(Y^2-A^2)(C^2-Y^2)}}\frac{dY}{(Y^2-U^2)}\right\}$$
$$\left.+\int_A^B\sqrt{\frac{(B^2-Y^2)}{(Y^2-A^2)}}\frac{dY}{(Y^2-U^2)}\right] + \left(\frac{(D^2-A^2)(C^2-U^2)}{(C^2-A^2)(D^2-U^2)}\right)^{1/2}$$
$$\times\frac{B_1}{[(U^2-A^2)(B^2-U^2)]^{1/2}} - \frac{B_2(U^2-A^2)^{1/2}}{[(B^2-U^2)(C^2-U^2)(D^2-U^2)]^{1/2}}$$
$$(A<U<B) \tag{3.12}$$

To determine the values of the unknown constants B_1 and B_2, we substitute $H(S^2)$ and

$G(U^2)$ given by (3.11) and (3.12) in (3.5a, b) and obtain

$$B_1 = \frac{p}{\mu s}\left\{\frac{K_3(K_{1,2} - K_{1,1}) - K_6(K_{1,3} + K_{2,3})}{RK_4K_6 + K_3K_5}\right\} \tag{3.13a}$$

$$B_2 = \frac{p}{\mu s}\left\{\frac{RK_4(K_{1,1} - K_{1,2}) - K_5(K_{1,3} + K_{2,3})}{RK_4K_6 + K_3K_5}\right\} \tag{3.13b}$$

where

$$K_{1,1} = \int_C^D M_1(X)\mathrm{d}X \int_C^D \frac{M_2(S)}{X^2 - S^2}\mathrm{d}S \tag{3.14}$$

$$K_{1,2} = \int_A^B M_1(Y)\mathrm{d}Y \int_C^D \frac{M_2(S)}{S^2 - Y^2}\mathrm{d}S \tag{3.15}$$

$$K_{1,3} = \int_C^D M_1(X)\mathrm{d}X \int_A^B \frac{M_2(U)}{X^2 - U^2}\mathrm{d}U \tag{3.16}$$

$$K_{2,3} = \int_A^B M_1(Y)\mathrm{d}Y \int_A^B \frac{M_2(U)}{Y^2 - U^2}\mathrm{d}U \tag{3.17}$$

$$K_3 = \frac{\pi}{2}\int_A^B \frac{M_2(U)}{C^2 - U^2}\mathrm{d}U, \quad K_4 = \int_A^B \frac{M_2(U)}{U^2 - A^2}\mathrm{d}U \tag{3.18}$$

$$K_5 = R\int_C^D \frac{M_2(S)}{S^2 - A^2}\mathrm{d}S, \quad K_6 = \frac{\pi}{2}\int_C^D \frac{M_2(S)}{S^2 - C^2}\mathrm{d}S \tag{3.19}$$

$$M_1(T) = \left[\frac{(D^2 - T^2)(T^2 - B^2)}{(T^2 - C^2)(T^2 - A^2)}\right]^{1/2},$$

$$M_2(T) = \left[\frac{(T^2 - A^2)(T^2 - C^2)}{(T^2 - B^2)(D^2 - T^2)}\right]^{1/2} \frac{T}{\sqrt{T^2 - 1}} \tag{3.20}$$

and

$$R = -\frac{\pi}{2}\sqrt{\frac{D^2 - A^2}{C^2 - A^2}}. \tag{3.21}$$

4. Stress intensity factors

The corresponding displacement and stress components in the plane of the cracks may be written as

$$w(x,0) = \frac{1}{e}\int_X^B \frac{UG(U^2)}{\sqrt{U^2 - 1}}\mathrm{d}U, \quad x \in I_2$$

$$= \frac{1}{e}\int_X^D \frac{SH(S^2)}{\sqrt{S^2 - 1}}\mathrm{d}S, \quad x \in I_4 \tag{4.1a, b}$$

and

$$[\sigma_{yz}(x,0)]_{0<x<a} = -\frac{2\mu s X}{\pi}\left[\int_A^B \frac{UG(U^2)}{U^2 - X^2}\mathrm{d}U + \int_C^D \frac{SH(S^2)}{S^2 - X^2}\mathrm{d}S\right] \tag{4.2a}$$

$$[\sigma_{yz}(x,0)]_{b<x<c} = \frac{2\mu s X}{\pi}\left[\int_A^B \frac{UG(U^2)}{X^2 - U^2}\mathrm{d}U + \int_C^D \frac{SH(S^2)}{S^2 - X^2}\mathrm{d}S\right] \tag{4.2b}$$

$$[\sigma_{yz}(x,0)]_{x>1}=\frac{2\mu sX}{\pi}\left[\int_A^B \frac{UG(U^2)}{X^2-U^2}\mathrm{d}U+\int_C^D \frac{SH(S^2)}{X^2-S^2}\mathrm{d}S\right]. \tag{4.2c}$$

Using the results given by (3.11) and (3.12) the expressions (4.2a–c) yield after some algebraic manipulation, the results

$$[\sigma_{yz}(x,0)]_{0<x<a}=\frac{2\mu s}{\pi}[F_1(X)-F_2(X)-F_3(X)+F_4(X)$$
$$-F_5(X)-F_6(X)-F_7(X)-F_8(X)] \tag{4.3a}$$

$$[\sigma_{yz}(x,0)]_{b<x<c}=\frac{2\mu s}{\pi}[F_1(X)-F_2(X)+F_3(X)+F_4(X)$$
$$-F_5(X)-F_6(X)-F_7(X)-F_8(X)] \tag{4.3b}$$

$$[\sigma_{yz}(x,0)]_{x>1}=\frac{2\mu s}{\pi}[F_1(X)-F_2(X)+F_3(X)+F_4(X)$$
$$-F_5(X)-F_6(X)-F_7(X)+F_8(X)] \tag{4.3c}$$

where

$$F_1(X)=\frac{2pX}{\pi\mu S}\int_C^D\left[\frac{(D^2-Y^2)(Y^2-B^2)}{(Y^2-C^2)(Y^2-A^2)}\right]^{1/2}\left[\frac{\pi}{2(Y^2-X^2)}\left\{\left[\frac{Y^2-A^2}{Y^2-B^2}\right]^{1/2}\right.\right.$$
$$\left.\left.-\left[\frac{A^2-X^2}{B^2-X^2}\right]^{1/2}\right\}\left[\frac{C^2-B^2}{D^2-B^2}\right]^{1/2}+I_{A,C}^{B,D}(X,Y)\right]\mathrm{d}Y \tag{4.4a}$$

$$F_2(X)=\frac{2pX}{\pi\mu s}\int_A^B\left[\frac{(D^2-Y^2)(B^2-Y^2)}{(C^2-Y^2)(Y^2-A^2)}\right]^{1/2}\left[\frac{\pi}{2(Y^2-X^2)}\right.$$
$$\left.\times\left[\frac{(C^2-B^2)(A^2-X^2)}{(D^2-B^2)(B^2-X^2)}\right]^{1/2}+L_{A,C}^{B,D}(X,Y)\right]\mathrm{d}Y \tag{4.4b}$$

$$F_3(X)=\frac{B_1X}{X_1}\left[\frac{\pi}{2}\left[\frac{(C^2-B^2)}{(D^2-B^2)}\right]^{1/2}+J_{A,C}^{B,D}(X)\right]\left[\frac{(D^2-A^2)}{(C^2-A^2)}\right]^{1/2} \tag{4.4c}$$

$$F_4(X)=B_2X\left[\frac{\pi}{2[(C^2-B^2)(D^2-B^2)]^{1/2}}\left\{1-\left[\frac{(A^2-X^2)}{(B^2-X^2)}\right]^{1/2}\right\}\right.$$
$$\left.+K_{A,C}^{B,D}(X)\right] \tag{4.4d}$$

$$F_5(X)=\frac{2pX}{\pi\mu s}\int_C^D\left[\frac{(D^2-Y^2)(Y^2-B^2)}{(Y^2-C^2)(Y^2-A^2)}\right]^{1/2}$$
$$\times\left[\frac{\pi}{2(Y^2-X^2)}\left[\frac{(D^2-A^2)(C^2-X^2)}{(D^2-B^2)(D^2-X^2)}\right]^{1/2}-L_{C,A}^{D,B}(X,Y)\right]\mathrm{d}Y \tag{4.4e}$$

$$F_6(X)=\frac{2pX}{\pi\mu s}\int_A^B\left[\frac{(D^2-Y^2)(B^2-Y^2)}{(C^2-Y^2)(Y^2-A^2)}\right]^{1/2}\left[\frac{\pi}{2(Y^2-X^2)}\left\{\left[\frac{C^2-X^2}{D^2-X^2}\right]^{1/2}\right.\right.$$
$$\left.\left.-\left[\frac{C^2-Y^2}{D^2-Y^2}\right]^{1/2}\right\}\left[\frac{D^2-A^2}{D^2-B^2}\right]^{1/2}+I_{C,A}^{D,B}(X,Y)\right]\mathrm{d}Y \tag{4.4f}$$

$$F_7(X)=\frac{B_1X}{(A^2-X^2)}\left[\frac{\pi}{2}\left[\frac{(D^2-A^2)}{(D^2-B^2)}\right]^{1/2}\left\{\left[\frac{C^2-X^2}{D^2-X^2}\right]^{1/2}-\left[\frac{C^2-A^2}{D^2-A^2}\right]^{1/2}\right\}\right.$$

$$\left.+I^{D,B}_{C,A}(X,A)\right]\left[\frac{(D^2-A^2)}{(C^2-A^2)}\right]^{1/2} \quad (4.4g)$$

$$F_8(X)=\frac{B_1X}{X_2}\left[\frac{\pi}{2}\left[\frac{(D^2-A^2)}{(D^2-B^2)}\right]^{1/2}-J^{B,D}_{C,A}(X)\right] \quad (4.4h)$$

$$I^{Q,S}_{P,R}(X,Y)=\int_P^Q\left(\frac{S^2-R^2}{Y^2-X^2}\right)\left\{\left[\frac{(Y^2-P^2)}{(Y^2-Q^2)}\right]^{1/2}\tan^{-1}\left[\frac{(U^2-P^2)(Y^2-Q^2)}{(Q^2-U^2)(Y^2-P^2)}\right]^{1/2}\right.$$

$$\left.-\left[\frac{(P^2-X^2)}{(Q^2-X^2)}\right]^{1/2}\tan^{-1}\left[\frac{(U^2-P^2)(Q^2-X^2)}{(Q^2-U^2)(P^2-X^2)}\right]^{1/2}\right\}$$

$$\times\frac{U\,\mathrm{d}U}{[(R^2-U^2)(S^2-U^2)^3]^{1/2}} \quad (4.4i)$$

$$L^{Q,S}_{P,R}(X,Y)=\int_P^Q\left(\frac{S^2-R^2}{Y^2-X^2}\right)\left\{\left[\frac{(P^2-X^2)}{(Q^2-X^2)}\right]^{1/2}\tan^{-1}\left[\frac{(U^2-P^2)(Q^2-X^2)}{(Q^2-U^2)(P^2-X^2)}\right]^{1/2}\right.$$

$$+\frac{1}{2}\left[\frac{(Y^2-P^2)}{(Q^2-Y^2)}\right]^{1/2}$$

$$\left.\times\log\left|\frac{[(U^2-P^2)(Q^2-Y^2)]^{1/2}-[(Q^2-U^2)(Y^2-P^2)]^{1/2}}{[(U^2-P^2)(Q^2-Y^2)]^{1/2}-[(Q^2-U^2)(Y^2-P^2)]^{1/2}}\right|\right\}$$

$$\times\frac{U\,\mathrm{d}U}{[(R^2-U^2)(S^2-U^2)^3]^{1/2}} \quad (4.4j)$$

$$J^{Q,S}_{P,R}(X)=\int_P^Q\left\{\tan^{-1}\left[\frac{(U^2-P^2)(Q^2-X^2)}{(Q^2-U^2)(P^2-X^2)}\right]^{1/2}\right\}\frac{U(S^2-R^2)\,\mathrm{d}U}{[(R^2-U^2)(S^2-U^2)^3]^{1/2}} \quad (4.4k)$$

$$K^{Q,S}_{P,R}(X)=\int_P^Q\left\{\tan^{-1}\left[\frac{(U^2-P^2)}{(Q^2-U^2)}\right]^{1/2}-\left[\frac{(P^2-X^2)}{(Q^2-X^2)}\right]^{1/2}\right.$$

$$\left.\times\tan^{-1}\left[\frac{(U^2-P^2)(Q^2-X^2)}{(Q^2-U^2)(P^2-X^2)}\right]^{1/2}\right\}\frac{U(2U^2-R^2-S^2)\,\mathrm{d}U}{[(R^2-U^2)^3(S^2-U^2)^3]^{1/2}} \quad (4.4l)$$

$$X_1=[(A^2-X^2)(B^2-X^2)]^{1/2},\quad X_2=[(C^2-X^2)(D^2-X^2)]^{1/2}. \quad (4.4m)$$

The dynamic stress intensity factors are given by

$$N_a=\underset{x\to a-}{\mathrm{Lt}}\,[2(a-x)]^{1/2}\left|\frac{\sigma_{yz}(x,0)}{P}\right|_{0<x<a} \quad (4.5a)$$

$$N_b=\underset{x\to b+}{\mathrm{Lt}}\,[2(x-b)]^{1/2}\left|\frac{\sigma_{yz}(x,0)}{P}\right|_{b<x<c} \quad (4.5b)$$

$$N_c=\underset{x\to c-}{\mathrm{Lt}}\,[2(c-x)]^{1/2}\left|\frac{\sigma_{yz}(x,0)}{P}\right|_{b<x<c} \quad (4.5c)$$

$$N_1 = \underset{x\to 1+}{\mathrm{Lt}} [2(x-1)]^{1/2} \left| \frac{\sigma_{yz}(x,0)}{P} \right|_{x>1} \tag{4.5d}$$

With the aid of the results given by (4.3) in (4.5) it follows that

$$N_a = -\frac{\mu s\sqrt{A}}{[e(A^2-1)^{1/2}(B^2-A^2)]^{1/2}} B_1 \tag{4.6a}$$

$$\begin{aligned} N_b = &-\frac{\mu s\sqrt{B}}{\sqrt{e(B^2-1)^{1/2}}}\left[-\frac{2p}{\pi\mu s}\left[\frac{(B^2-A^2)(C^2-B^2)}{(D^2-B^2)}\right]^{1/2}\left\{\int_A^B G_1(Y)\mathrm{d}Y\right.\right. \\ &\left.+\int_C^D G_1(Y)\mathrm{d}Y\right\} + \left[\frac{(C^2-B^2)(D^2-A^2)}{(B^2-A^2)(C^2-A^2)(D^2-B^2)}\right]^{1/2} B_1 \\ &\left.-\left[\frac{(B^2-A^2)}{(C^2-B^2)(D^2-B^2)}\right]^{1/2} B_2\right] \end{aligned} \tag{4.6b}$$

$$N_c = -\frac{\mu s[C(C^2-A^2)]^{1/2}}{[e(C^2-1)^{1/2}(C^2-B^2)(D^2-C^2)]^{1/2}} B_2 \tag{4.6c}$$

$$\begin{aligned} N_1 = &-\frac{\mu s\sqrt{D}}{\sqrt{e(D^2-1)^{1/2}}}\left[-\frac{2p}{\pi\mu s}\left[\frac{(D^2-A^2)(D^2-C^2)}{(D^2-B^2)}\right.\right. \\ &\times\left\{\int_A^B G_2(Y)\mathrm{d}Y + \int_C^D G_2(Y)\mathrm{d}Y\right\} + \left[\frac{(D^2-C^2)}{(D^2-B^2)(C^2-A^2)}\right]^{1/2} B_1 \\ &\left.+\left[\frac{(D^2-A^2)}{(D^2-C^2)(D^2-B^2)}\right]^{1/2} B_2\right] \end{aligned} \tag{4.6d}$$

where

$$G_1(Y) = \frac{[(D^2-Y^2)]^{1/2}}{[(Y^2-A^2)(Y^2-B^2)(Y^2-C^2)]^{1/2}} \tag{4.7a}$$

$$G_2(Y) = \frac{[(B^2-Y^2)]^{1/2}}{[(Y^2-A^2)(C^2-Y^2)(D^2-Y^2)]^{1/2}}. \tag{4.7b}$$

The crack opening displacements are obtained by using the expressions for $G(U^2)$ and $H(S^2)$ from (3.12) and (3.11) in (4.1a, b).

Again letting $a \to 0$ and simplifying, it may be noted that the results (4.6b), (4.6c) and (4.6d) become those given by equations (3.14) of Das [1].

5. Numerical results

The numerical values of stress intensity factors (SIF) N_a, N_b, N_c and N_1 given by (4.6a–d) at the tips of the crack have been plotted against crack speed (V/c_2) for different values of crack lengths, separating distances of the cracks and strip width (h). Keeping the length of the outer cracks and distance between inner and outer cracks fixed ($b = 0{\cdot}6$, $c = 0{\cdot}8$) SIFs at the tips of the cracks have been plotted against crack speed ($0{\cdot}1 \leqslant V/c_2 < 1$) for different lengths of the inner cracks ($a = 0{\cdot}2, 0{\cdot}4$) and strip width ($h = 1, 3, 5$). It is found from the graphs (figures 2–5) that SIFs increase

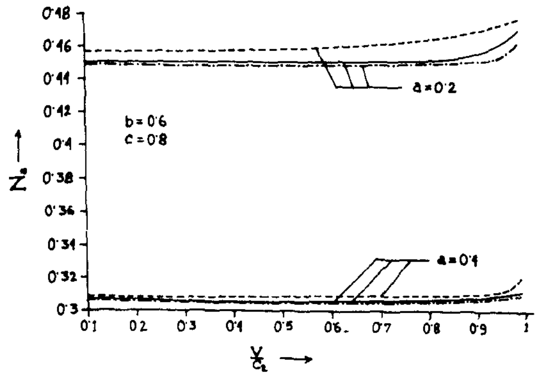

Figure 2. Stress intensity factor N_a vs. V/c_2.
(--- $h = 1$, —— $h = 2$, -·-·- $h = 5$)

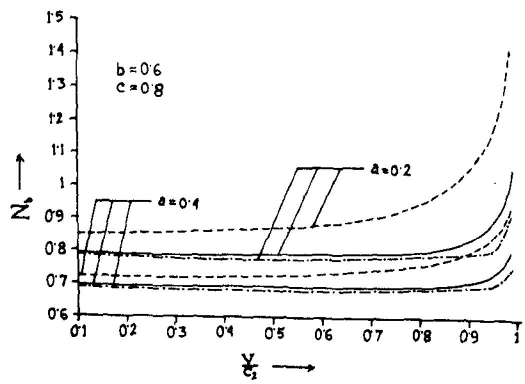

Figure 3. Stress intensity factor N_b vs. V/c_2.
(--- $h = 1$, —— $h = 2$, -·-·- $h = 5$)

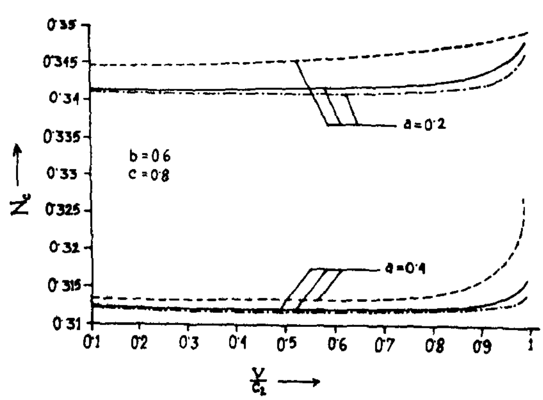

Figure 4. Stress intensity factor N_c vs. V/c_2.
(--- $h = 1$, —— $h = 2$, -·-·- $h = 5$)

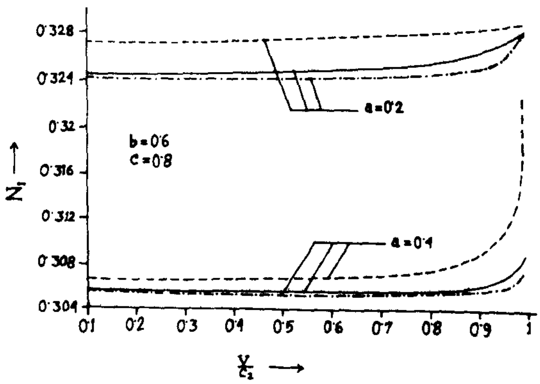

Figure 5. Stress intensity factor N_1 vs. V/c_2.
(--- $h = 1$, —— $h = 2$, -·-·- $h = 5$)

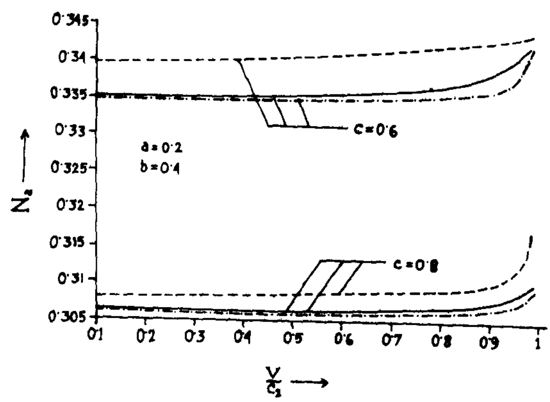

Figure 6. Stress intensity factor N_a vs. V/c_2.
(--- $h = 1$, —— $h = 2$, -·-·- $h = 5$)

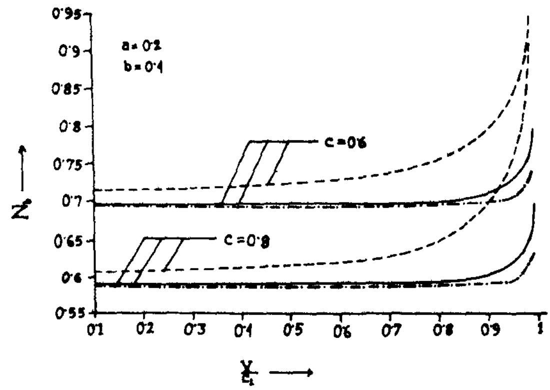

Figure 7. Stress intensity factor N_b vs. V/c_2.
(--- $h = 1$, —— $h = 2$, -·-·- $h = 5$)

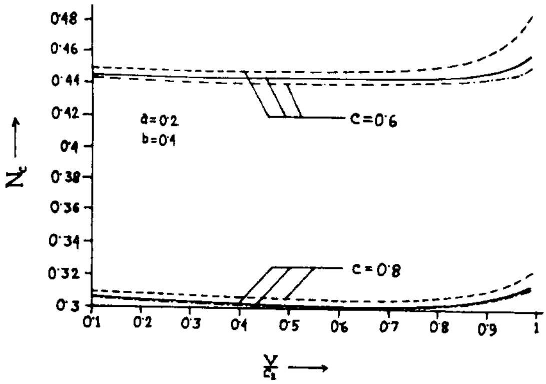

Figure 8. Stress intensity factor N_c vs. V/c_2.
(--- $h = 1$, —— $h = 2$, -·-·- $h = 5$)

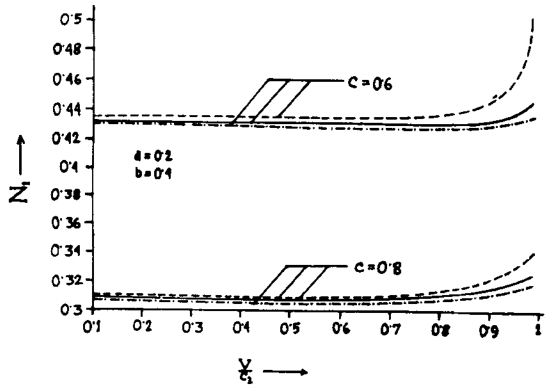

Figure 9. Stress intensity factor N_1 vs. V/c_2.
(--- $h = 1$, —— $h = 2$, -·-·- $h = 5$)

rapidly as $V/c_2 \to 1$ and with the decrease in the value of inner crack length i.e. with the increase in the value of the distance between inner cracks the value of SIF decreases.

Similar effect on SIFs can be found with the increase in the value of b when lengths of the outer cracks and the distance between inner cracks are kept fixed.

Next, keeping the lengths of the inner cracks fixed ($a = 0{\cdot}2$, $b = 0{\cdot}4$) it is seen from the graphs (figures 6–9) that the value of SIF N_b is higher for higher values of c (0·6, 0·8). But the nature is opposite in case of N_a, N_b and N_1.

In all the cases mentioned above the SIFs increase with the increase in the value of V/c_2 gradually at a slow rate in the beginning but increase rapidly as $V/c_2 \to 1$. Also the value of SIFs are higher for lower values of h in these cases.

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