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Some characterization theorems in rotatory magneto thermohaline convection

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Abstract. The present paper extends the results of Banerjee *et al* [2] for the hydromagnetic thermohaline convection problems of Veronis' [9] and Stern's [8] types to include the effect of a uniform vertical rotation.

Keywords. Hydromagnetic thermohaline convection; uniform vertical rotation.

1. Introduction

The establishment of non-occurrence of any slow oscillatory motions which may be neutral or unstable imply the validity of the principle of exchange of stabilities (PES). The validity of PES in a certain class of stability problems eliminates the unsteady terms from the linearized perturbation equations which results in notable mathematical simplicity since the transition from stability to instability occurs via a marginal state which is characterized by the vanishing of both real and imaginary parts of the complex time eigenvalue associated with the perturbation. Pellew and Southwell [5] proved the validity of PES for the classical Rayleigh–Bénard convection problem (RBCP). Chandrasekhar [3] in his investigations of hydromagnetic RBCP conjectured that if the total kinetic energy associated with a perturbation exceeds the total magnetic energy associated with it, then PES is valid. Sherman and Ostrach [7] established the above conjecture of Chandrasekhar for a more general problem when the fluid is confined in an arbitrary region and the uniform magnetic field is applied in an arbitrary direction. However, the result of Sherman and Ostrach is of limited value since one cannot *a priori* be certain when their criterion will be satisfied. Banerjee *et al* [1] established that for the hydromagnetic RBCP if $Q\sigma_1/\pi^2 \leqslant 1$, where Q is the Chandrasekhar number and σ_1 is the magnetic Prandtl number, then the total kinetic energy associated with an arbitrary perturbation which may be neutral or unstable is greater than the total magnetic energy associated with it and consequently PES is valid in this parameter regime. Banerjee *et al* [2] further extended these energy considerations to the hydromagnetic thermohaline convection problems of Veronis' [9] and Stern's [8] types. The aim of the present paper is to extend the results of Banerjee *et al* for the hydromagnetic thermohaline convection problems of Veronis' and Stern's types to include the effect of a uniform vertical rotation.

2. Basic equations and boundary conditions

The non-dimensional linearized perturbation equations governing thermohaline convection problem in the presence of a uniform vertical rotation and magnetic field are given by (cf. Gupta *et al* [4]).

$$(D^2-a^2)(D^2-a^2-p/\sigma)w = Ra^2\theta - R_s a^2\phi - QD(D^2-a^2)h_z + TDZ \quad (1)$$

$$(D^2-a^2-p)\theta = -w \quad (2)$$

$$(D^2-a^2-p/\tau)\phi = -w/\tau \quad (3)$$

$$(D^2-a^2-p\sigma_1/\sigma)h_z = -Dw \quad (4)$$

$$(D^2-a^2-p/\sigma)Z = -Dw - QDX \quad (5)$$

$$(D^2-a^2-p\sigma_1/\sigma)X = -DZ \quad (6)$$

together with the boundary conditions

$$w = 0 = \theta = \phi = Dw = Z = DX = h_z \quad \text{at } z = 0, 1. \quad (7)$$

The various symbols occurring in the above equations are defined as follows:

z is real independent variable such that $0 \leqslant z \leqslant 1$ and stands for vertical coordinate, $D = \mathrm{d}/\mathrm{d}z$ denotes the derivatives with respect to z. a^2 is the square of the wave number, σ is the thermal Prandtl number, τ is the Lewis number, σ_1 is the magnetic Prandtl number, R is the thermal Rayleigh number, R_s is the thermohaline concentration Rayleigh number, Q is the Chandrasekhar number, T is the Taylor number and $p = p_r + ip_i$ is a complex constant in general representing the complex growth rate. Further w, θ, ϕ, Z, X and h_z are complex valued functions of z and stand respectively for the vertical velocity, temperature, concentration, vertical vorticity, vertical current density and vertical magnetic field. We note that $R > 0$ and $R_s > 0$ for Veronis' configuration whereas for Stern's configuration, we have $R < 0$ and $R_s < 0$.

System of eqs (1)–(7), constitute an eigenvalue problem for p for given values of $a^2, R, R_s, Q, T, \sigma$ and σ_1 and a given state of the system is stable, neutral or unstable according to $p_r < 0$ or $p_r = 0$ or $p_r > 0$. Further, if $p_r = 0$ implies $p_i = 0$ for all wave numbers a^2. then the principle of exchange of stabilities (PES) is valid, otherwise we will have overstability at least when instability sets in certain modes.

3. Mathematical analysis

We prove the following theorems:

Theorem 1. *A necessary condition for the existence of a nontrivial solutions* $(p, w, \theta, \phi, h_z, X, Z)$ *of eqs* (1)–(7) *with* $R > 0, R_s > 0$ *and* $p = p_r + ip_i, p_i \neq 0$ *is that*

$$J_1 < (J_2 + J_3 + J_4), \quad (8)$$

where

$$J_1 = \int_0^1 (|Dw|^2 + a^2|w|^2)\mathrm{d}z,$$

$$J_2 = Q\sigma_1 \int_0^1 (|Dh_z|^2 + a^2|h_z|^2)\mathrm{d}z, \quad (9)$$

$$J_3 = R_s a^2 \sigma \int_0^1 |\phi|^2 \mathrm{d}z, \quad (10)$$

and

$$J_4 = T\int_0^1 |Z|^2 \mathrm{d}z. \tag{11}$$

Proof. Multiplying eq. (1) by w^* (the complex conjugate of w) integrating the resulting equation over the range of z, we have

$$\int_0^1 w^*(D^2 - a^2)(D^2 - a^2 - p/\sigma)w\,\mathrm{d}z = Ra^2 \int_0^1 w^*\theta\,\mathrm{d}z - R_s a^2 \int_0^1 w^*\phi\,\mathrm{d}z$$
$$+ T\int_0^1 w^* DZ\,\mathrm{d}z - Q\int_0^1 w^* D(D^2 - a^2)h_z\,\mathrm{d}z. \tag{12}$$

Using eqs (2)–(6) and boundary conditions (7), we can write

$$Ra^2 \int_0^1 w^*\theta\,\mathrm{d}z = -Ra^2 \int_0^1 \theta(D^2 - a^2 - p^*)\theta^*\,\mathrm{d}z, \tag{13}$$

$$-R_s a^2 \int_0^1 w^*\phi\,\mathrm{d}z = R_s a^2 \int_0^1 \phi(D^2 - a^2 - p^*/\tau)\phi^*\,\mathrm{d}z, \tag{14}$$

$$-\int_0^1 w^* D(D^2 - a^2)h_z = \int_0^1 Dw^*(D^2 - a^2)h_z\,\mathrm{d}z$$
$$= -\int_0^1 h_z(D^2 - a^2)(D^2 - a^2 - p^*\sigma_1/\sigma)h_z^*\,\mathrm{d}z, \tag{15}$$

$$\int_0^1 w^* DZ\,\mathrm{d}z = -\int_0^1 Dw^* Z\,\mathrm{d}z = \int_0^1 Z(D^2 - a^2 - p^*/\sigma)Z^*\,\mathrm{d}z$$
$$+ Q\int_0^1 ZDX^*\,\mathrm{d}z = \int_0^1 Z(D^2 - a^2 - p^*/\sigma)Z^*\,\mathrm{d}z - Q\int_0^1 DZX^*\,\mathrm{d}z$$
$$= \int_0^1 Z(D^2 - a^2 - p^*/\sigma)Z^*\,\mathrm{d}z + Q\int_0^1 X(D^2 - a^2 - p\sigma_1/\sigma)X^*\,\mathrm{d}z. \tag{16}$$

It follows from eqs (12)–(16) that

$$\int_0^1 w^*(D^2 - a^2)(D^2 - a^2 - p/\sigma)w\,\mathrm{d}z = -Ra^2 \int_0^1 \theta(D^2 - a^2 - p^*)\theta^*\,\mathrm{d}z$$
$$+ R_s a^2 \tau \int_0^1 \phi(D^2 - a^2 - p^*/\tau)\phi^*\,\mathrm{d}z - Q\int_0^1 h_z(D^2 - a^2)$$
$$\times (D^2 - a^2 - p^*\sigma_1/\sigma)h_z^*\,\mathrm{d}z + T\int_0^1 Z(D^2 - a^2 - p^*/\sigma)Z^*\,\mathrm{d}z$$
$$+ QT\int_0^1 X(D^2 - a^2 - p\sigma_1/\sigma)X^*\,\mathrm{d}z. \tag{17}$$

Integrating various terms of eq. (17) by parts for an appropriate number of times and making use of boundary conditions (7), we have

$$I_1 + I_2 + I_3 + I_4 + I_5 + I_6 = 0, \tag{18}$$

where

$$I_1 = \int_0^1 (|D^2 w|^2 + 2a^2|Dw|^2 + a^4|w|^2)\mathrm{d}z + p/\sigma \int_0^1 (|Dw|^2 + a^2|w|^2)\mathrm{d}z,$$

$$I_2 = -Ra^2 \int_0^1 (|D\theta|^2 + a^2|\theta|^2 + p^*|\theta|^2)\mathrm{d}z,$$

$$I_3 = R_s a^2 \tau \int_0^1 (|D\phi|^2 + a^2|\phi|^2 + p^*/\tau|\phi|^2)\mathrm{d}z,$$

$$I_4 = T\int_0^1 (|DZ|^2 + a^2|Z|^2 + p^*/\sigma|Z|^2)\mathrm{d}z,$$

$$I_5 = TQ\int_0^1 (|DX|^2 + a^2|X|^2 + p\sigma_1/\sigma|X|^2)\mathrm{d}z,$$

and

$$I_6 = Q\left[\int_0^1 (|D^2 h_z|^2 + 2a^2|Dh_z|^2 + a^4|h_z|^2)\mathrm{d}z\right] + Qp^*\sigma_1/\sigma\left[\int_0^1 (|Dh_z|^2 + a^2|h_z|^2)\mathrm{d}z\right].$$

Equating the imaginary parts of both sides of eq. (18) and cancelling $p_i(\neq 0)$ throughout, we have

$$\int_0^1 (|Dw|^2 + a^2|w|^2)\mathrm{d}z + Ra^2\sigma\int_0^1 |\theta|^2\mathrm{d}z + QT\sigma_1\int_0^1 |X|^2\mathrm{d}z - \left\{R_s a^2\sigma\int_0^1 |\phi|^2\mathrm{d}z + T\int_0^1 |Z|^2\mathrm{d}z + Q\sigma_1\int_0^1 (|Dh_z|^2 + a^2|h_z|^2)\mathrm{d}z\right\} = 0, \tag{19}$$

or

$$[J_1 - \{J_2 + J_3 + J_4\}] + Ra^2\sigma\int_0^1 |\theta|^2\mathrm{d}z + QT\sigma_1\int_0^1 |X|^2\mathrm{d}z = 0. \tag{20}$$

Equation (20) clearly implies that

$$J_1 < (J_2 + J_3 + J_4).$$

This completes the proof of the theorem.

We note that expressions for J_1, J_2, J_3 and J_4 as given by eqs (8)–(11) respectively, represent the total kinetic energy, magnetic energy, concentration energy and rotational energy. In view of this, Theorem 1 can be restated as follows:

A necessary condition for the existence of oscillatory motion which may be stable, neutral or unstable for Veronis' thermohaline convection problem in the presence of a uniform vertical rotation and magnetic field is that the sum total of magnetic, concentration and rotational energies must exceed the total kinetic energy or, equivalently, if the total kinetic energy exceeds the sum total of magnetic, concentration and rotational energies, then the oscillatory motions are not allowed.

The above result, no doubt yields us a condition in terms of energies of the system for the non-occurrence of oscillatory motions, however, it is of limited value, since one can not *a priori* be certain when this condition will be satisfied as it involves the unknown eigen functions of the problem. It will therefore be more useful to express this condition in terms of the parameters of the problem prescribed by the fluid properties. We establish this in the following theorem.

Theorem 2. *If* $(p, w, \theta, \phi, h_z, X, Z)$, $p = p_r + ip_i, p_i \neq 0$, $p_r \geqslant 0$, $R > 0$ *and* $R_s > 0$ *is a solution of eqs* (1)–(7) *and* $\left[\frac{Q\sigma_1}{\pi^2} + \frac{R_s\sigma}{2\tau^2\pi^4} + \frac{T}{\pi^4}\right] \leqslant 1$, *then*, $J_1 > (J_2 + J_3 + J_4)$.

Proof. Multiplying eq. (3) by its complex conjugate, integrating over the range of z by parts a suitable number of times and making use of boundary conditions (7), we have

$$\int_0^1 (|D^2\phi|^2 + 2a^2|D\phi|^2 + a^4|\phi|^2)\mathrm{d}z + 2p_r/\tau \int_0^1 (|D\phi|^2 + a^2|\phi|^2)\mathrm{d}z$$

$$+ |p|^2/\tau^2 \int_0^1 |\phi|^2\mathrm{d}z = \frac{1}{\tau^2}\int_0^1 |w|^2\mathrm{d}z. \tag{21}$$

Since, $p_r \geqslant 0$, therefore eq. (21) gives

$$2a^2 \int_0^1 |D\phi|^2\mathrm{d}z < \frac{1}{\tau^2}\int_0^1 |w|^2\mathrm{d}z$$

which upon using Poincare inequality [6]

$$\pi^2 \int_0^1 |\phi|^2\mathrm{d}z \leqslant \int_0^1 |D\phi|^2\mathrm{d}z \quad (\text{since } \phi(0) = 0 = \phi(1))$$

yields that

$$a^2 \int_0^1 |\phi|^2\mathrm{d}z < \frac{1}{2\tau^2}\pi^2 \int_0^1 |w|^2\mathrm{d}z. \tag{22}$$

Further, since $w(0) = 0 = w(1)$ also, therefore

$$\int_0^1 |w|^2\mathrm{d}z \leqslant \frac{1}{\pi^2}\int_0^1 |Dw|^2\mathrm{d}z. \tag{23}$$

Combining inequalities (22)–(23), we have

$$a^2 \int_0^1 |\phi|^2\mathrm{d}z < \frac{1}{2\tau^2}\pi^4 \int_0^1 |Dw|^2\mathrm{d}z$$

$$< \frac{1}{2\tau^2}\pi^4 \int_0^1 (|Dw|^2 + a^2|w|^2)\mathrm{d}z. \tag{24}$$

Multiplying eq. (4) by h_z^* (the complex conjugate of h_z), integrating the resulting equation by parts a suitable number of times in the range of z, making use of boundary conditions (7) and then equating the real parts from both sides of the resulting equation, we have

$$\int_0^1 (|Dh_z|^2 + a^2|h_z|^2)\mathrm{d}z + p_r\sigma_1/\sigma \int_0^1 |h_z|^2\mathrm{d}z$$

$$= \text{real part of}\left[\int_0^1 wDh_z^* \mathrm{d}z\right]$$

$$\leqslant \left|\int_0^1 wDh_z^* \mathrm{d}z\right|$$

$$\leqslant \int_0^1 |w||Dh_z| \mathrm{d}z$$

$$\leqslant \left[\int_0^1 |w|^2 \mathrm{d}z\right]^{1/2}\left[\int_0^1 |Dh_z|^2 \mathrm{d}z\right]^{1/2}$$

(by Schwartz inequality). (25)

Since $p_r \geqslant 0$, therefore inequality (25) implies that

$$\int_0^1 |Dh_z|^2 \mathrm{d}z < \left[\int_0^1 |w|^2 \mathrm{d}z\right]. \tag{26}$$

Combining inequalities (23), (25) and (26), we get

$$\int_0^1 (|Dh_z|^2 + a^2|h_z|^2)\mathrm{d}z < \frac{1}{\pi^2}\int_0^1 |Dw|^2 \mathrm{d}z$$

$$< \frac{1}{\pi^2}\int_0^1 (|Dw|^2 + a^2|w|^2)\mathrm{d}z. \tag{27}$$

Now, multiplying eq. (5) by Z^* (the complex conjugate of Z), integrating by parts a suitable number of times, using boundary conditions (7) and equating the real parts of the resulting equation, we have

$$\int_0^1 (|DZ|^2 + a^2|Z|^2 + p_r/\sigma|Z|^2)\mathrm{d}z + Q\int_0^1 (|DX|^2 + a^2|X|^2 + p_r\sigma_1/\sigma|X|^2)\mathrm{d}z$$

$$= \text{real part of}\left(\int_0^1 Z^* Dw \mathrm{d}z\right)$$

$$= \text{real part of}\left(-\int_0^1 wDZ^* \mathrm{d}z\right)$$

$$\leqslant \left|-\int_0^1 wDZ^* \mathrm{d}z\right| = \left|\int_0^1 wDZ^* \mathrm{d}z\right|$$

$$\leqslant \int_0^1 |w||DZ| \mathrm{d}z$$

$$\leqslant \left[\int_0^1 |w|^2 \mathrm{d}z\right]^{1/2}\left[\int_0^1 |DZ|^2 \mathrm{d}z\right]^{1/2},$$

(by Schwartz inequality)

which by virtue of inequality (23) and the fact that $p_r \geqslant 0$ gives

$$\int_0^1 |DZ|^2 < \frac{1}{\pi^2}\int_0^1 |Dw|^2 \mathrm{d}z$$

$$< \frac{1}{\pi^2} \int_0^1 (|Dw|^2 + a^2|w|^2)\mathrm{d}z. \tag{28}$$

Inequality (28) together with the Poincare inequality

$$\int_0^1 |Z|^2 \mathrm{d}z \leqslant \frac{1}{\pi^2} \int_0^1 |DZ|^2 \mathrm{d}z$$

leads to the inequality

$$\int_0^1 |Z|^2 \mathrm{d}z < \frac{1}{\pi^4} \int_0^1 (|Dw|^2 + a^2|w|^2)\mathrm{d}z. \tag{29}$$

Combining inequalities (24), (27) and (29), we have

$$(J_2 + J_3 + J_4) < \left[\frac{Q\sigma_1}{\pi^2} + \frac{R_s\sigma}{2\tau^2\pi^4} + \frac{T}{\pi^4}\right] J_1. \tag{30}$$

Inequality (30) clearly implies that if

$$\left[\frac{Q\sigma_1}{\pi^2} + \frac{R_s\sigma}{2\tau^2\pi^4} + \frac{T}{\pi^4}\right] \leqslant 1,$$

then

$$J_1 > (J_2 + J_3 + J_4).$$

This completes the proof of the theorem.

Theorem 2 implies that if $\left[\frac{Q\sigma_1}{\pi^2} + \frac{R_s\sigma}{2\tau^2\pi^4} + \frac{T}{\pi^4}\right] \leqslant 1$, then the total kinetic energy associated with an arbitrary oscillatory ($p_i \neq 0$) perturbation which may be neutral ($p_r = 0$) or unstable ($p_r > 0$) exceeds the sum total of its magnetic, concentration and rotational energies. In particular, it follows that, in the parameter regime $\left[\frac{Q\sigma_1}{\pi^2} + \frac{R_s\sigma}{2\tau^2\pi^4} + \frac{T}{\pi^4}\right] \leqslant 1$, the principle of exchange of stabilities is valid for the problem under consideration.

Theorem 3. *A necessary condition for the existence of a nontrivial solutions* $(p, w, \theta, \phi, h_z, X, Z)$ *of eqs* (1)–(7) *with* $R < 0$, $R_s < 0$ *and* $p = p_r + ip_i$, $p_i \neq 0$ *is that*

$$J_1 < (J_2 + J_4 + J_5),$$

where J_1, J_2 *and* J_4 *are as given by eqs* (8), (9), *and* (11) *and*

$$J_5 = |R|a^2\sigma \int_0^1 |\theta|^2 \mathrm{d}z. \tag{31}$$

Proof. Putting $R = -|R|$ and $R_s = -|R_s|$ in eq. (18) and proceeding exactly as in Theorem 1, we get the desired result. Keeping in view the fact that J_5 represents the thermal energy, Theorem 3 can be restated as follows:

A necessary condition for the existence of oscillatory motions which may be stable, neutral or unstable for Stern's thermohaline convection problem in the presence of a uniform vertical rotation and magnetic field is that the sum total of magnetic, thermal and rotational energies must exceed that total kinetic energy, or, equivalently, if the total kinetic energy exceeds the sum total of magnetic, thermal and rotational energies then the oscillatory motions are not allowed. Further, Theorem 3 is qualitatively of the same form as Theorem 1 and possesses the same drawback. We remedy this in the following theorem analogous to Theorem 2.

Theorem 4. *If* $(p, w, \theta, \phi, h_z, X, Z), p = p_r + ip_i, p_i \neq 0, p_r \geqslant 0, R < 0$ *and* $R_s < 0$ *is the solution of the eqs* (1)–(7) *and* $\left[\frac{Q\sigma_1}{\pi^2} + \frac{|R|\sigma}{2\pi^4} + \frac{T}{\pi^4}\right] \leqslant 1$, *then*

$$J_1 > (J_2 + J_4 + J_5).$$

Proof. Multiplying eq. (2) by its complex conjugate, integrating by parts a suitable number of times over the range of z, using boundary condition (7) and equating the real parts of the resulting equation, we have

$$\int_0^1 (|D^2\theta|^2 + 2a^2|D\theta|^2 + a^4|\theta|^2)\mathrm{d}z + 2p_r \int_0^1 (|D\theta|^2 + a^2|\theta|^2)\mathrm{d}z$$
$$+ |p|^2 \int_0^1 |\theta|^2 \mathrm{d}z = \int_0^1 |w|^2 \mathrm{d}z. \tag{32}$$

Since, $p_r \geqslant 0$, it follows from eq. (32) that

$$2a^2 \int_0^1 |D\theta|^2 \mathrm{d}z < \int_0^1 |w|^2 \mathrm{d}z,$$

which upon using the Poincare inequality

$$\int_0^1 |\theta|^2 \mathrm{d}z \leqslant \frac{1}{\pi^2} \int_0^1 |D\theta|^2 \mathrm{d}z$$

and inequality (23) gives

$$a^2 \int_0^1 |\theta|^2 \mathrm{d}z < \frac{1}{2\pi^4} \int_0^1 |Dw|^2 \mathrm{d}z$$
$$< \frac{1}{2\pi^4} \int_0^1 (|Dw|^2 + a^2|w|^2)\mathrm{d}z. \tag{33}$$

It follows from inequalities (24), (29) and (33) that

$$(J_2 + J_4 + J_5) < \left[\frac{Q\sigma_1}{\pi^2} + \frac{|R|\sigma}{2\pi^4} + \frac{T}{\pi^4}\right] J_1 \tag{34}$$

Inequality (34) clearly implies that if

$$\left[\frac{Q\sigma_1}{\pi^2} + \frac{|R|\sigma}{2\pi^4} + \frac{T}{\pi^4}\right] \leqslant 1,$$

then

$$J_1 > (J_2 + J_4 + J_5).$$

This completes the proof of the theorem.

Theorem 4 implies that if $\left[\frac{Q\sigma_1}{\pi^2}+\frac{|R|\sigma}{2\pi^4}+\frac{T}{\pi^4}\right]\leqslant 1$, then the total kinetic energy associated with an arbitrary oscillatory perturbation which may be neutral or unstable exceeds the sum total of its magnetic, thermal and rotational energies. In particular it follows that in the parameter regime $\left[\frac{Q\sigma_1}{\pi^2}+\frac{|R|\sigma}{2\pi^4}+\frac{T}{\pi^4}\right]\leqslant 1$, the PES is valid for the problem under consideration. Theorems 1–4 clearly provide a natural extension of the results of Banerjee *et al* [12] as could be easily seen by putting $T=0$.

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References

[1] Banerjee M B and Gupta J R, *Studies in hydrodynamic and hydromagnetic stability*, (Shimla, India: Silverline Publ.) (1991)
[2] Banerjee M B, Gupta J R and Katyal S P, A characterization theorem in magneto thermohaline convection, *J. Math. Anal. Appl.* **144** (1989) 141–146
[3] Chandrasekhar S. *Hydrodynamic and hydromagnetic stability*, (Oxford: Clarendon Press) (1961)
[4] Gupta J R, Sood S K and Bhardwaj U D, The characterization of non-oscillatory motions in rotatory hydromagnetic thermohaline convection. *Indian J. Pure Appl. Math.* **17** (1986) 100–107
[5] Pellew A and Southwell R V, On the maintained convective motion in a fluid heated from below, *Proc. R. Soc. London* **A176** (1940) 312–343
[6] Schultz M H, *Spline Analysis*, (New Jersey: Prentice Hall) (1973)
[7] Sherman M and Ostrach S, On the principle of exchange of stabilities for the magnetohydrodynamic thermal stability problem in completely confined fluids, *J. Fluid Mech.* **23** (1966) 661–671
[8] Stern M E, The salt fountain and thermohaline convection, *Tellus* **12** (1960) 172–175
[9] Veronis G, On finite amplitude instability in thermohaline convection, *J. Mar. Res.* **23** (1965) 1–17