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Scattering of antiplane shear wave by a propagating crack at the interface of two dissimilar elastic media

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Abstract. An analysis of the scattering of horizontally polarized shear wave by a semi-infinite crack running with uniform velocity along the interface of two dissimilar semi-infinite elastic media has been carried out. The mixed boundary value problem has been solved completely by the Wiener–Hopf technique. The effect of different values of the material parameter, the angle of incidence of incident wave and the crack propagation velocity on the stress intensity factor have been illustrated graphically.

Keywords. Diffraction of elastic waves; propagating crack; SH-wave; stress intensity factor.

1. Introduction

It is well known that the problems of diffraction of elastic wave by cracks or inclusions are of considerable importance in view of their application in seismology and geophysics. If the cracks or inclusions are located at the interface of layered media, the study becomes more relevant. The extensive use of composite materials in modern technology has also evoked interest in the wave propagation problems in layered media with interfacial discontinuities. Onder *et al* [5] studied the diffraction of monochromatic plane SH-waves obliquely incident on a rigid half plane between the two different semi-infinite media.

In this paper we have considered the problem of the diffraction of a plane harmonic SH-wave by a semi-infinite crack running uniformly along the interface of two dissimilar semi-infinite elastic media. The problem of scattering of plane harmonic polarized shear wave by a half plane crack in an infinite isotropic medium extending under antiplane strain was studied earlier by Jahanshahi [3]. Chen and Sih [1, 2] also solved the in plane problem of the diffraction of stress waves by a running crack in an incident wave field in an infinite elastic medium. We have applied Fourier transform and Wiener–Hopf technique [4] to solve the mixed boundary value problem. The resulting integrals have been evaluated asymptotically to obtain the displacement and stress field near about the crack tip. It is found that the stress intensity factor depends sensitively upon the speed of crack propagation, the angle of incidence of the incoming wave and on the material properties of the elastic media. Quantitative assessment of the effect of the aforementioned parameters on the stress intensity factor has been made by displaying the numerical results graphically for two pairs of different materials.

2. Formulation of the problem and its solution

Let a plane crack move at a constant velocity V on the interface of two bonded dissimilar elastic semi-infinite medium due to the incidence of the plane harmonic SH-wave

$$v_1^0 = V_1 \exp[-i\{\Lambda_1(X\cos\Theta_1 + Y\sin\Theta_1) + \Omega T\}] \tag{1}$$

in the medium where the co-efficient of rigidity, density and shear-wave velocity respectively are given by μ_1, ρ_1 and C_1. The crack lies on the bimaterial interface along $Y=0$ with respect to the fixed rectangular co-ordinate system (X, Y, Z).

We assume that the displacement and stress due to the scattered field are

$$v_j = v_j(X, Y, T) \tag{2}$$

and

$$(\tau_{xz})_j = \mu_j \frac{\partial v_j}{\partial X}, \quad (\tau_{yz})_j = \mu_j \frac{\partial v_j}{\partial Y} \tag{3}$$

where the subscript $j = 1, 2$ refers to the upper and lower half-planes and T the time.

The equations of SH-wave motion in either elastic half-space are given by

$$\frac{\partial^2 v_j}{\partial X^2} + \frac{\partial^2 v_j}{\partial Y^2} = \frac{1}{C_j^2}\frac{\partial^2 v_j}{\partial T^2} \quad (j = 1, 2) \tag{4}$$

where $C_j = (\mu_j/\rho_j)^{1/2}$ is the shear-wave velocity. Without any loss of generality, we further assume that $C_1 > C_2$.

Due to the incident wave given in (1), the reflected and transmitted waves in the absence of the crack may be written in the form

$$v_1^r(X, Y, T) = V_1^r \exp[-i\{\Lambda_1(X\cos\Theta_1 - Y\sin\Theta_1) + \Omega T)\}]$$

and

$$v_2^T(X, Y, T) = V_2^T \exp[-i\{\Lambda_2(X\cos\Theta_2 + Y\sin\Theta_2) + \Omega T)\}] \tag{5}$$

where

$$V_1^r = \frac{\mu_1\Lambda_1\sin\Theta_1 - \mu_2\Lambda_2\sin\Theta_2}{\mu_1\Lambda_1\sin\Theta_1 + \mu_2\Lambda_2\sin\Theta_2} V_1 = A_1^R V_1 \quad \text{(say)}$$

and

$$V_2^T = \frac{2\mu_1\Lambda_1\sin\Theta_1}{\mu_1\Lambda_1\sin\Theta_1 + \mu_2\Lambda_2\sin\Theta_2} V_1 = A_2^T V_1 \quad \text{(say)} \tag{6}$$

with

$$\Lambda_1\cos\Theta_1 = \Lambda_2\cos\Theta_2.$$

V_1, V_1^r and V_2^T are the incident, reflected and transmitted wave amplitude respectively, Λ_j the wave number, $\Omega = \Lambda_j C_j$ the circular frequency and Θ_1, Θ_2 the angles of incidence and refraction respectively.

Assume that the crack has been moving in the horizontal direction along the interface for a sufficiently long time and that a steady state has been reached in the neighbourhood of the crack.

A set of moving co-ordinate systems (x, y, z, t) attached to the crack tip moving at

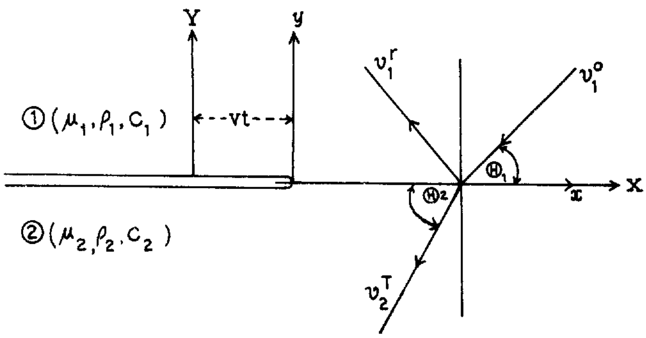

Figure 1. Geometry of the propagating crack.

a constant velocity V is introduced in accordance with

$$x = X - Vt, \quad y_j = s_j Y, \quad z = Z, \quad t = T \tag{7}$$

where $s_j = (1 - M_j^2)^{1/2}$ and $M_j = V/C_j$ is the Mach number.

In terms of the moving co-ordinate system (x, y, t), (4) becomes

$$\frac{\partial^2 v_j}{\partial x^2} + \frac{\partial^2 v_j}{\partial y_j^2} + \frac{1}{C_j^2 s_j^2}\frac{\partial}{\partial t}\left(2M_j C_j \frac{\partial v_j}{\partial x} - \frac{\partial v_j}{\partial t}\right) = 0. \tag{8}$$

It is convenient to define an apparent circular frequency $\omega = \alpha\Omega$ and the angles of reflection ϕ_1 and refraction ϕ_2 are given by

$$\cos\phi_j = M_j + (\Lambda_j/\lambda_j)\cos\Theta_j, \quad \sin\phi_j = (s_j/\alpha)\sin\Theta_j,$$

where

$$\alpha = (1 + M_j \cos\Theta_j) \quad \text{and} \quad \lambda_j = (\Lambda_j/s_j^2)\alpha. \tag{9}$$

Using these relations in a moving system, (1) and (5) take the form

$$\begin{bmatrix} v_1^0 \\ v_1^r \\ v_2^T \end{bmatrix} = \begin{bmatrix} w_1^0(x, y_1) \\ w_1^r(x, y_1) \\ w_2^T(x, y_2) \end{bmatrix} \exp\{i(M_1\lambda_1 x - \omega t)\} \tag{10}$$

where

$$w_1^0(x, y_1) = V_1 \exp\{-i\lambda_1(x\cos\phi_1 + y_1\sin\phi_1)\}$$
$$w_1^r(x, y_1) = A_1^R V_1 \exp\{-i\lambda_1(x\cos\phi_1 - y_1\sin\phi_1)\}$$
$$w_2^T(x, y_2) = A_2^T V_1 \exp[-i\{(\beta_2 + \lambda_2\cos\phi_2)x + \lambda_2 y_2 \sin\phi_2\}] \tag{11}$$

and

$$\beta_2 = M_1\lambda_1\left(1 - \frac{\lambda_2}{\lambda_1}\frac{C_1}{C_2}\right) < 0 \quad \text{since} \quad C_1 > C_2.$$

Using (10), we assume the solution of the governing equation (8) as

$$v_j(x, y_j, t) = w_j(x, y_j)\exp[i(M_j\lambda_j x - \omega t)]. \tag{12}$$

Substitution of (12) in (8) yields the Helmholtz equation

$$\frac{\partial^2 w_j}{\partial x^2}+\frac{\partial^2 w_j}{\partial y_j^2}+\lambda_j^2 w_j=0 \quad (j=1,2). \tag{13}$$

Applying Fourier transform, (13) can be solved and the result is

$$w_1(x,y_1)=\frac{1}{2\pi}\int_{-\infty}^{\infty} A_1(\xi)\exp\{-i\xi x-(\xi^2-\lambda_1^2)^{1/2}y_1\}\,\mathrm{d}\xi, \quad (y_1>0)$$

and

$$w_2(x,y_2)=\frac{1}{2\pi}\int_{-\infty}^{\infty} A_2(\xi)\exp\{-i\xi x-(\xi^2-\lambda_2^2)^{1/2}y_2\}\,\mathrm{d}\xi, \quad (y_2<0) \tag{14}$$

where $A_1(\xi)$ and $A_2(\xi)$ are the unknown functions to be determined. From (12) and (14) we obtain the displacement components due to scattered field as

$$v_1=\exp[i(M_1\lambda_1 x-\omega t]\frac{1}{2\pi}\int_{-\infty}^{\infty} A_1(u)\exp[-iux-\gamma_1 y_1]\,\mathrm{d}u, \quad (y_1>0)$$

and

$$v_2=\exp[i(M_1\lambda_1 x-\omega t]\frac{1}{2\pi}\int_{-\infty}^{\infty} A_2(u)\exp[-iux+\gamma_2 y_2]\,\mathrm{d}u, \quad (y_2<0) \tag{15}$$

where

$$\gamma_1=(u^2-\lambda_1^2)^{1/2} \quad \text{and} \quad \gamma_2=[(u-\beta_2)^2-\lambda_2^2]^{1/2}. \tag{16}$$

Therefore, the expressions for the stresses are

$$(\tau_{xz})_1=-i\mu_1\exp[i(M_1\lambda_1 x-\omega t)]$$
$$\times\frac{1}{2\pi}\int_{-\infty}^{\infty}(u-M_1\lambda_1)A_1(u)\exp[-iux-\gamma_1 y_1]\,\mathrm{d}u$$

$$(\tau_{xz})_2=-i\mu_2\exp[i(M_1\lambda_1 x-\omega t)]$$
$$\times\frac{1}{2\pi}\int_{-\infty}^{\infty}(u-M_1\lambda_1)A_2(u)\exp[-iux+\gamma_2 y_2]\,\mathrm{d}u$$

and

$$(\tau_{y_1z})_1=-\mu_1 s_1\exp[i(M_1\lambda_1 x-\omega t)]\frac{1}{2\pi}\int_{-\infty}^{\infty}\gamma_1 A_1(u)\exp[-iux-\gamma_1 y_1]\,\mathrm{d}u$$

$$(\tau_{y_2z})_2=\mu_2 s_2\exp[i(M_1\lambda_1 x-\omega t)]\frac{1}{2\pi}\int_{-\infty}^{\infty}\gamma_2 A_2(u)\exp[-iux+\gamma_2 y_2]\,\mathrm{d}u. \tag{17}$$

The unknown functions $A_1(u)$ and $A_2(u)$ are to be determined from the following boundary conditions at the interface $y=0$

(i) $$v_1(x,0)=v_2(x,0), \quad x>0$$

(ii) $$\mu_1 s_1\frac{\partial v_1}{\partial y_1}=\mu_2 s_2\frac{\partial v_2}{\partial y_2}, \quad -\infty<x<\infty$$

and

(iii) $$\frac{\partial v_1^0}{\partial y_1}+\frac{\partial v_1^r}{\partial y_1}+\frac{\partial v_1}{\partial y_1}=0, \quad x<0, \quad y\to 0+.$$

From the boundary condition (ii) we obtain

$$\mu_1 s_1 \gamma_1 A_1(u)+\mu_2 s_2 \gamma_2 A_2(u)=0 \tag{18}$$

and from other two boundary conditions, we get

$$\int_{-\infty}^{\infty} B_1(u)\exp(-iux)\mathrm{d}u=0 \quad (x>0)$$

and

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} M(u)B_1(u)\exp(-iux)\mathrm{d}u=N\exp[-i\lambda_1 x\cos\phi_1], \quad (x<0) \tag{19}$$

where

$$B_1(u)=\frac{\mu_1 s_1\gamma_1+\mu_2 s_2\gamma_2}{\mu_2 s_2\gamma_2}A_1(u)$$

$$M(u)=\gamma_1\frac{\mu_2 s_2\gamma_2}{(\mu_1 s_1\gamma_1+\mu_2 s_2\gamma_2)} \tag{20}$$

and

$$N=-\frac{i\Lambda_1 v_1\sin\Theta_1}{s_1}(1-A_1^R).$$

The solution of the dual integral equation may be obtained by a method based on the Wiener–Hopf technique. The first part of (19) can be satisfied if we choose

$$B_1(u)=L_-(u) \tag{21}$$

where $L_-(u)$ is a function of u, analytic in the lower half of the complex u-plane. The second part is satisfied if we take

$$M(u)B_1(u)=\frac{N}{i(u-\alpha_1)}\frac{U_+(u)}{U_+(\alpha_1)} \tag{22}$$

where $\alpha_1=\lambda_1\cos\phi_1$ and $U_+(u)$ is a function free from zeros and singularities in the upper half of the complex u-plane. Thus (22) is a solution of the second part of (19) can easily be shown by completing the path from $-\infty$ to ∞ by a semi-circle of infinite radius in the upper u-plane and then applying the residue theorem and Jordan's Lemma. The path of integration is chosen to avoid possible branch points and is indented below the pole $u=\alpha_1$.

Eliminating $B_1(u)$ from (21) and (22) we obtain

$$\frac{L_-(u)}{U_+(u)}=\frac{N}{i(u-\alpha_1)M(u)}\frac{1}{U_+(\alpha_1)} \tag{23}$$

and

$$M(u)=\frac{\mu_2 s_2}{\mu_1 s_1+\mu_2 s_2}(u^2-\lambda_1^2)^{1/2}F(u) \tag{24}$$

where

$$F(u)=\frac{(\mu_1 s_1+\mu_2 s_2)\gamma_2}{(\mu_1 s_1\gamma_1+\mu_2 s_2\gamma_2)}$$

and

$$F(u)\to 1 \quad \text{as} \quad |u|\to\infty.$$

The function $F(u)$ can be expressed as the product of two functions such that

$$F(u)=F_+(u)\cdot F_-(u) \tag{25}$$

where $F_+(u)$ and $F_-(u)$ are analytic in the upper and lower half of the complex u-plane respectively. The expressions for $F_+(u)$ and $F_-(u)$ have been derived in the appendix.

In view of (25), (24) assumes the form

$$\frac{U_+(u)}{(u+\lambda_1)^{1/2}F_+(u)}=\frac{L_-(u)}{\dfrac{N}{iU_+(\alpha_1)}\dfrac{\mu_1 s_1+\mu_2 s_2}{\mu_2 s_2(u-\alpha_1)(u-\lambda_1)^{1/2}F_-(u)}} \tag{26}$$

where

$$U_+(u)=(u+\lambda_1)^{1/2}F_+(u). \tag{27}$$

So

$$L_-(u)=\frac{N}{i(\alpha_1+\lambda_1)^{1/2}F_+(\alpha_1)}\frac{\mu_1 s_1+\mu_2 s_2}{\mu_2 s_2(u-\alpha_1)(u-\lambda_1)^{1/2}F_-(u)}. \tag{28}$$

Hence the functions $A_1(u)$ and $A_2(u)$ are

$$A_1(u)=\frac{N}{i(\alpha_1+\lambda_1)^{1/2}F_+(\alpha_1)}\frac{\gamma_2(\mu_1 s_1+\mu_2 s_2)}{(\mu_1 s_1\gamma_1+\mu_2 s_2\gamma_2)(u-\alpha_1)(u-\lambda_1)^{1/2}F_-(u)}$$

and

$$A_2(u)=\frac{-N}{i(\alpha_1+\lambda_1)^{1/2}F_+(\alpha_1)}\frac{\mu_1 s_1\gamma_1(\mu_1 s_1+\mu_2 s_2)}{\mu_2 s_2(\mu_1 s_1\gamma_1+\mu_2 s_2\gamma_2)(u-\alpha_1)(u-\lambda_1)^{1/2}F_-(u)}. \tag{29}$$

The singular behaviour of the stress components for the scattered waves at the crack-tip is due to the divergence of the integrals around $x=y_j=0$ in (17). Making use of (29) and asymptotic expressions of the integrands of (17) for large values of u, we obtain near about the crack-tip,

$$(\tau_{xz})_1=\frac{B(1+i)}{s_1}\int_0^\infty u^{-1/2}\exp[-s_1 uY](\cos ux-\sin ux)du$$

$$(\tau_{xz})_2=\frac{-B(1+i)}{s_2}\int_0^\infty u^{-1/2}\exp[-s_2 u|Y|](\cos ux-\sin ux)du$$

$$(\tau_{yz})_1=-B(1+i)\int_0^\infty u^{-1/2}\exp[-s_1 uY](\cos ux+\sin ux)du$$

$$(\tau_{yz})_2=-B(1+i)\int_0^\infty u^{-1/2}\exp[-s_2 u|Y|](\cos ux+\sin ux)du \tag{30}$$

where

$$B = -\frac{N\mu_1 s_1}{2\pi(\alpha_1 + \lambda_1)^{1/2} F_+(\alpha_1)}; \quad y_j = s_j Y \quad (j = 1, 2).$$

Using the results

$$\int_0^\infty u^{-1/2} \exp[-s_1 u Y] \cos ux \, dx = (\pi/2)^{1/2} \left[\frac{(s_1^2 y^2 + x^2)^{1/2} + s_1 y}{s_1^2 y^2 + x^2}\right]^{1/2}$$

$$\int_0^\infty u^{-1/2} \exp[-s_1 u Y] \sin ux \, dx = (\pi/2)^{1/2} \left[\frac{(s_1^2 y^2 + x^2)^{1/2} - s_1 y}{s_1^2 y^2 + x^2}\right]^{1/2} \tag{31}$$

the stresses near about the crack tip given by (30) can be evaluated. The displacement near about the crack tip can be obtained from the crack tip stresses by integration.

Now introducing the factor $\exp[i(M_1 \lambda_1 x - \omega t)]$ and taking the real part, the stresses and displacements near about the moving crack-tip are found to be equal to

$$\begin{bmatrix} (\tau_{yz})_j \\ (\tau_{xz})_j \\ v_j \end{bmatrix} = \mathrm{Re} \begin{bmatrix} K_1 \left[\dfrac{(s_j^2 Y^2 + x^2)^{1/2} + x}{s_j^2 Y^2 + x^2}\right]^{1/2} \\ (-1)^j \dfrac{K_1}{s_1} \left[\dfrac{(s_j^2 Y^2 + x^2)^{1/2} - x}{s_j^2 Y^2 + x^2}\right]^{1/2} \\ (-1)^{j+1} \dfrac{2K_1}{\mu_j s_j} [(x^2 + s_j^2 Y^2)^{1/2} - x]^{1/2} \end{bmatrix} \exp\left[i\left(M_1 \lambda_1 x - \omega t - \frac{\pi}{4}\right)\right] \tag{32}$$

where

$$K_1 = (2/\pi)^{1/2} \frac{\mu_1 \mu_2 \Lambda_1 \Lambda_2 V_1 \sin\Theta_1 \sin\Theta_2}{(\alpha_1 + \lambda_1)^{1/2} F_+(\alpha_1)(\mu_1 \Lambda_1 \sin\Theta_1 + \mu_2 \Lambda_2 \sin\Theta_2)}. \tag{33}$$

In the case of crack propagation in an isotropic elastic medium using the result $\mu_1 = \mu_2$, $\rho_1 = \rho_2$ and $F_+(\alpha_1) = 1$, we obtain

$$K_1 = (1/\pi)^{1/2} \mu_1 \Lambda_1^{1/2} V_1 (1 - M_1)^{1/2} \sin(\Theta_1/2). \tag{34}$$

Putting $r = (x^2 + y^2)^{1/2}$, $\tan\phi = |Y|/x$, the expression of displacements and stresses given by (32) near about the moving crack-tip is found to be equal to

$$v_1 = \frac{2K_1}{\mu_1 s_1} r^{1/2} \{(1 - M_1^2 \sin^2\phi)^{1/2} - \cos\phi\}^{1/2} \cos(\omega t + (\pi/4)) + 0(r^{3/2})$$

$$v_2 = -\frac{2K_1}{\mu_2 s_2} r^{1/2} \{(1 - M_2^2 \sin^2\phi)^{1/2} - \cos\phi\}^{1/2} \cos(\omega t + (\pi/4)) + 0(r^{3/2})$$

$$(\tau_{yz})_1 = \frac{K_1}{r^{1/2}} \left\{\frac{(1 - M_1^2 \sin^2\phi)^{1/2} + \cos\phi}{1 - M_1^2 \sin^2\phi}\right\}^{1/2} \cos(\omega t + (\pi/4)) + 0(r^{1/2})$$

$$(\tau_{yz})_2 = \frac{K_1}{r^{1/2}}\left\{\frac{(1-M_2^2\sin^2\phi)^{1/2}+\cos\phi}{1-M_2^2\sin^2\phi}\right\}^{1/2}\cos(\omega t+(\pi/4))+0(r^{1/2})$$

$$(\tau_{xz})_1 = -\frac{K_1}{s_1}\frac{1}{r^{1/2}}\left\{\frac{(1-M_1^2\sin^2\phi)^{1/2}-\cos\phi}{1-M_1^2\sin^2\phi}\right\}^{1/2}\cos(\omega t+(\pi/4))+0(r^{1/2})$$

$$(\tau_{xz})_2 = \frac{K_1}{s_2}\frac{1}{r^{1/2}}\left\{\frac{(1-M_2^2\sin^2\phi)^{1/2}-\cos\phi}{1-M_2^2\sin^2\phi}\right\}^{1/2}\cos(\omega t+(\pi/4))+0(r^{1/2}). \tag{35}$$

Taking the value of K_1 given by (34), the results given by (35) agree with the results of the crack propagation in an isotropic elastic medium as given by Jahanshahi [3].

When the crack is stationary, the corresponding results of stresses and displacements near about the crack-tip can be derived by making M_1 and M_2 approach zero and are given by

$$(\tau_{yz})_1 = K_1^*(2/r)^{1/2}\cos\tfrac{1}{2}\phi\cos(\Omega t+\tfrac{1}{4}\pi)+0(r^{1/2})$$

$$(\tau_{yz})_2 = K_1^*(2/r)^{1/2}\cos\tfrac{1}{2}\phi\cos(\Omega t+\tfrac{1}{4}\pi)+0(r^{1/2})$$

$$(\tau_{xz})_1 = -K_1^*(2/r)^{1/2}\sin\tfrac{1}{2}\phi\cos(\Omega t+\tfrac{1}{4}\pi)+0(r^{1/2})$$

$$(\tau_{xz})_2 = K_1^*(2/r)^{1/2}\sin\tfrac{1}{2}\phi\cos(\Omega t+\tfrac{1}{4}\pi)+0(r^{1/2}) \tag{36}$$

and

$$v_1 = \frac{2\sqrt{2}K_1^*}{\mu_1}(r)^{1/2}\sin\tfrac{1}{2}\phi\cos(\Omega t+\tfrac{1}{4}\pi)+0(r^{3/2})$$

$$v_2 = \frac{-2\sqrt{2}K_1^*}{\mu_2}(r)^{1/2}\sin\tfrac{1}{2}\phi\cos(\Omega t+\tfrac{1}{4}\pi)+0(r^{3/2}) \tag{37}$$

where

$$K_1^* = \sqrt{2/\pi}\,\frac{\mu_1\mu_2\Lambda_1\Lambda_2 V_1\sin\Theta_1\sin\Theta_2}{(\Lambda_1\cos\phi_1+\Lambda_1)^{1/2}F_+^*(\Lambda_1\cos\phi_1)(\mu_1\Lambda_1\sin\phi_1+\mu_2\Lambda_2\sin\phi_2)} \tag{38}$$

and

$$F_+^*(\Lambda_1\cos\phi_1) = \exp\left[\frac{1}{\pi}\int_{\Lambda_1}^{\Lambda_2}\tan^{-1}\left\{\frac{\mu_1(s^2-\Lambda_1^2)^{1/2}}{\mu_2(\Lambda_2^2-s^2)^{1/2}}\right\}\frac{ds}{s+\Lambda_1\cos\phi_1}\right]. \tag{39}$$

If we put $\mu_1=\mu_2$, $\rho_1=\rho_2$, the corresponding results of the stationary crack in an isotropic elastic medium are found to be

$$(\tau_{yz})_{1,2} = V_1(\sin\tfrac{1}{2}\Theta_1)(\cos\tfrac{1}{2}\phi)\cos(\Omega t+\tfrac{1}{4}\pi)\left[\frac{2\Lambda_1\mu_1^2}{\pi r}\right]^{1/2}+0(r^{1/2})$$

$$(\tau_{xz})_{1,2} = \mp V_1(\sin\tfrac{1}{2}\Theta_1)(\cos\tfrac{1}{2}\phi)\cos(\Omega t+\tfrac{1}{4}\pi)\left[\frac{2\Lambda_1\mu_1^2}{\pi r}\right]^{1/2}+0(r^{1/2})$$

and

$$v_{1,2} = \pm V_1(\sin\tfrac{1}{2}\Theta_1)(\sin\tfrac{1}{2}\phi)\cos(\Omega t+\tfrac{1}{4}\pi)\left[\frac{8\Lambda_1 r}{\pi}\right]^{1/2}+0(r^{3/2}) \tag{40}$$

which are same as given by Jahanshahi [3].

3. Results and discussion

K_1 given by (33) is the dynamic stress intensity factor at the moving crack-tip and K_1^* given by (38) is the value of the corresponding quantity when the crack is stationary. The variation of K_1/K_1^* with the values of V/C_2 where V is the crack speed has been depicted graphically for the following two sets of materials.

First set:

Wrought iron	$\rho_1 = 7{\cdot}8\,\mathrm{g/cm^3}$,	$\mu_1 = 7{\cdot}7 \times 10^{11}\,\mathrm{dyn/cm^2}$
Copper	$\rho_2 = 8{\cdot}96\,\mathrm{g/cm^3}$,	$\mu_2 = 4{\cdot}5 \times 10^{11}\,\mathrm{dyn/cm^2}$

Second set:

Steel	$\rho_1 = 7{\cdot}6\,\mathrm{g/cm^3}$,	$\mu_1 = 8{\cdot}32 \times 10^{11}\,\mathrm{dyn/cm^2}$
Aluminium	$\rho_2 = 2{\cdot}7\,\mathrm{g/cm^3}$,	$\mu_2 = 2{\cdot}63 \times 10^{11}\,\mathrm{dyn/cm^2}$.

It is found that in both the cases the stress intensity factor gradually decreases with an increase in the value of V/C_2 and approaches zero as $V/C_2 \to 1$; the decrease in the value of K_1/K_1^* for the second set being more rapid than for the first set. We also find that in both the cases for any fixed value of C_1/C_2, K_1/K_1^* decreases with decrease in the value of Θ.

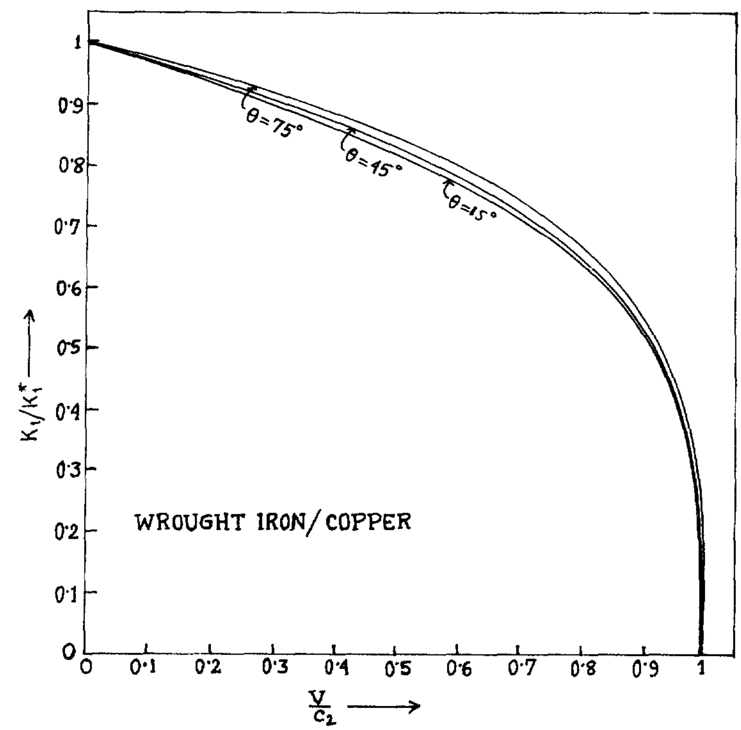

Figure 2. Stress intensity factor vs dimensionless crack speed (wrought iron/copper).

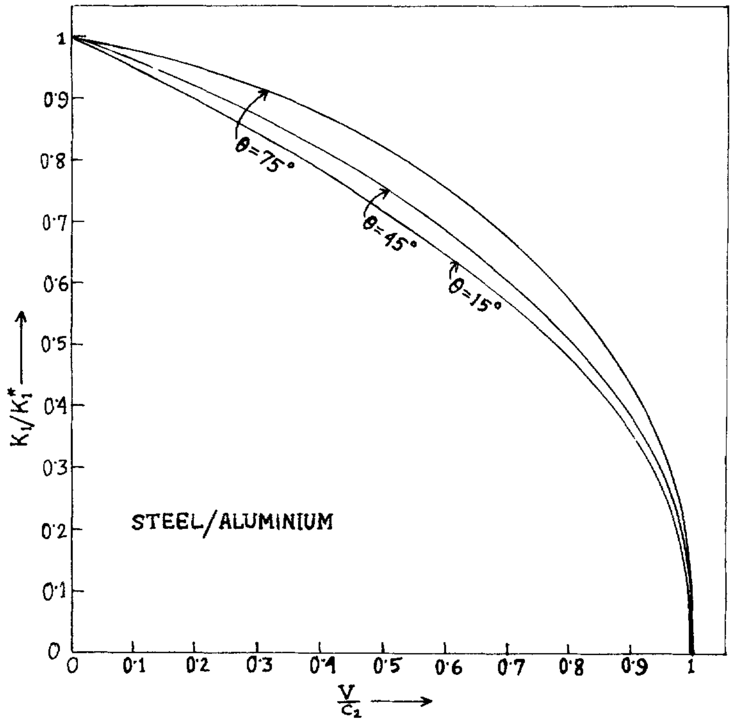

Figure 3. Stress intensity factor vs dimensionless crack speed (steel/aluminium).

Appendix

Factorization of $F(\xi)$ into $F_+(\xi)$ and $F_-(\xi)$

Consider

$$F(\xi) = \frac{(\mu_1 s_1 + \mu_2 s_2)\gamma_2}{(\mu_1 s_1 \gamma_1 + \mu_2 s_2 \gamma_2)} \tag{A1}$$

The branch points of $F(\xi)$ are at $\xi = \lambda_1,\ -\lambda_1,\ \lambda_2 + \beta_2,\ -(\lambda_2 - \beta_2)$ where

$$-(\lambda_2 - \beta_2) < -\lambda_1 < \lambda_1 < \lambda_2 + \beta_2 \text{ since } C_2 < C_1.$$

Since $F(\xi) \to 1$ as $|\xi| \to \infty$, $F(\xi)$ possesses no singularity within the rectangular contour (shown in figure 4), by Cauchy's residue theorem we can write

$$\log F(\xi) = \frac{1}{2\pi i}\int_{c_+ + c_-} \frac{\log F(s)}{s - \xi}\mathrm{d}s \tag{A2}$$

$$= \frac{1}{2\pi i}\int_{c_+} \frac{\log F(s)}{s-\xi}\mathrm{d}s + \frac{1}{2\pi i}\int_{c_-} \frac{\log F(s)}{s-\xi}\mathrm{d}s$$

$$\log F(\xi) = \log F_+(\xi) + \log F_-(\xi), \tag{A3}$$

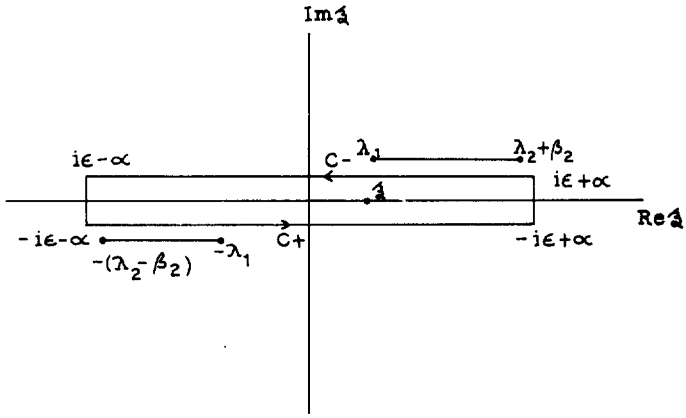

Figure 4. Rectangular contour in the complex ξ-plane.

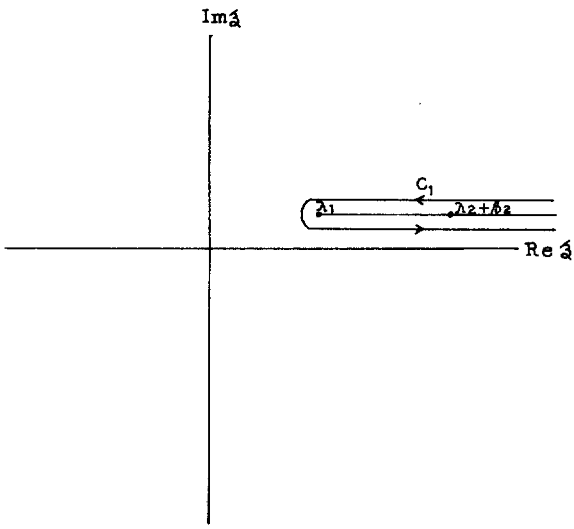

Figure 5. Path of integration C_1 round the branch cut.

where $F_+(\xi)$ and $F_-(\xi)$ are analytic in the upper and lower half of the complex ξ-plane respectively and can be expressed as

$$F_+(\xi) = \exp\left[\frac{1}{2\pi i}\int_{c+} \frac{\log F(s)}{s-\xi}\mathrm{d}s\right]$$

and

$$F_-(\xi) = \exp\left[\frac{1}{2\pi i}\int_{c-} \frac{\log F(s)}{s-\xi}\mathrm{d}s\right]. \tag{A4}$$

In order to evaluate $F_-(\xi)$ the path of integration C_- can be deformed to the path C_1 round the branch cut through λ_1 and $\lambda_2 + \beta_2$ as shown in figure 5.

After a little algebraic manipulation it can be shown that

$$F_-(\xi) = \exp\left[\frac{1}{2\pi i}\int_{\lambda_1}^{\lambda_2+\beta_2} \frac{1}{s-\xi}\log\left\{1 + i\frac{m_1}{m_2}\frac{(s^2-\lambda_1^2)^{1/2}}{[\lambda_2^2-(\beta_2-s)^2]^{1/2}}\right\}ds\right.$$
$$\left. - \frac{1}{2\pi i}\int_{\lambda_1}^{\lambda_2+\beta_2} \frac{1}{s-\xi}\log\left\{1 - i\frac{m_1}{m_2}\frac{(s^2-\lambda_1^2)^{1/2}}{[\lambda_2^2-(\beta_2-s)^2]^{1/2}}\right\}ds\right] \tag{A5}$$

which after simplification becomes

$$F_-(\xi) = \exp\left[\frac{1}{\pi}\int_{\lambda_1}^{\lambda_2+\beta_2} \frac{1}{s-\xi}\tan^{-1}\left\{\frac{m_1}{m_2}\frac{(s^2-\lambda_1^2)^{1/2}}{[\lambda_2^2-(\beta_2-s)^2]^{1/2}}\right\}ds\right] \tag{A6}$$

where

$$m_1 = \frac{\mu_1 s_1}{\mu_1 s_1 + \mu_2 s_2} \quad \text{and} \quad m = \frac{\mu_2 s_2}{\mu_1 s_1 + \mu_2 s_2}. \tag{A7}$$

Similarly it can be shown that

$$F_+(\xi) = \exp\left[\frac{1}{\pi}\int_{\lambda_1}^{\lambda_2-\beta_2} \frac{1}{s+\xi}\tan^{-1}\left\{\frac{m_1}{m_2}\frac{(s^2-\lambda_1^2)^{1/2}}{[\lambda_2^2-(\beta_2+s)^2]^{1/2}}\right\}ds\right] \tag{A8}$$

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