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Singular pencils of quadrics and compactified Jacobians of curves

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Abstract. Let Y be an irreducible nodal hyperelliptic curve of arithmetic genus g such that its nodes are also ramification points (char $\neq 2$). To the curve Y, we associate a family of quadratic forms which is dual to a singular pencil of quadrics in $\mathbb{P}^{2g+1}$ with Segre symbol $[2\ldots 2\,1\ldots 1]$, where the number of 2's is equal to the number of nodes. We show that the compactified Jacobian of Y is isomorphic to the space R of $(g-1)$ dimensional linear subspaces of $\mathbb{P}^{2g+1}$ which are contained in the intersection Q of quadrics of the pencil. We also prove that (under this isomorphism) the generalized Jacobian of Y is isomorphic to the open subset of R consisting of the $(g-1)$ dimensional subspaces not passing through any singular point of Q.

Keywords. Nodal hyperelliptic curve; compactified Jacobian.

1. Introduction

The geometry of a nonsingular pencil of quadrics Q is intimately connected with some moduli spaces of vector bundles on the nonsingular hyperelliptic curve associated to Q([2], [3] etc). A few years back Newstead [5] studied the geometry of singular pencils of quadrics in $\mathbb{P}^5$. As he has remarked, a very interesting problem will be to extend the above connection between the geometry of a pencil and vector bundles on the associated hyperelliptic curve to the singular case. We have taken here one step in this direction.

Let A and B be matrices corresponding to two nonsingular members of a pencil Q. If the equation $\det(A - xB) = 0$ does not have all roots distinct, then Q is a singular pencil. The simplest case is when this equation has at most double roots. Then one can associate to Q a singular curve Y with the following properties: (i) Y has at the most ordinary double points as singularities; (2) Y is a two sheeted covering over $\mathbb{P}^1$ with the set of singular points contained in the set of ramification points.

In this paper, we start with an irreducible curve Y with the above two properties and construct a pencil of quadrics Q in $\mathbb{P}^{2g+1}$, g being the arithmetic genus of Y(see §3). We denote by R the space of $(g-1)$-dimensional subspaces of $\mathbb{P}^{2g+1}$ which are isotropic for all the members of the pencil Q. Then our main theorem is:

Theorem. *The compactified Jacobian of Y is isomorphic to R.*

We also describe explicitly the subvariety of R corresponding to the generalized Jacobian of Y(Theorem 1, §4). The moduli of rank two bundles on Y will be studied in a subsequent paper.

2. Notations and Preliminaries

Let Y be an irreducible reduced curve with ordinary double points and F a torsion free coherent sheaf of rank one on Y.

DEFINITION 2.1

The degree of F ($\deg F$) is the integer $\chi(F) - \chi(\theta_Y)$ (I.4.1, [4]).

2.2. Since Y is Gorenstein, the dualising sheaf ω_Y is a locally free sheaf of rank one, of degree $2g-2$. (I.4.2, [4]).
2.3. (a) $\deg(F) < 0$ implies that $H^0(F) = 0$,
(b) $\deg(F) > 2g-2$ implies that $H^1(F) = 0$ (I.4.6[4]).
2.4. Let $\bar{J}$ denote the compactified Jacobian of Y. Tensorising by a line bundle of degree d, $\bar{J}$ can be identified with the space of torsion free coherent sheaves of rank one and degree d.

By Proposition 10.1, [6], $\bar{J} = \amalg \operatorname{Pic} Y_i$ where $\amalg$ denotes disjoint union over all partial normalizations Y_i of Y. Thus, if Y has only one singular point, $\bar{J} = \operatorname{Pic} Y \amalg \operatorname{Pic} \tilde{Y}$, $\tilde{Y}$ being the normalization of Y.

Henceforth we make the following assumptions. (1) Y is hyperelliptic i.e. there exists a finite degree two map $p: Y \to \mathbb{P}^1$ (2) The unique node w_0 of Y is a ramification point of p. Let $W_0 = \{w_0, w_1, \ldots, w_{2g}\}$ be the set of ramification points of Y. Let i denote the hyperelliptic involution on Y. We define an i-action on a torsion free coherent sheaf E on Y as a $\mathbb{Z}/2$-action on E which lifts the involution i. We call E i-invariant if $E \approx i^*E$.

Lemma 2.5. (a) *There is a one-one correspondence between i-invariant line bundles of even degree (respectively odd degree) on Y and the set of partitions of* $W_0 = \{w_0, w_1, \ldots, w_{2g}\}$ *into two subsets S, T such that* $S \cap \{w_1, \ldots, w_{2g}\}$ *and* $T \cap \{w_1, \ldots, w_{2g}\}$ *have even cardinality (respectively odd cardinality).*
(b) *There is a one-one correspondence between i-invariant nonlocally free torsion free sheaves of rank* 1 *and even degree (respectively odd degree) and partitions of* $\{w_1, \ldots, w_{2g}\} = W_0 - w_0$ *into two subsets* S', T' *of odd (respectively even) cardinality.*

Proof. This lemma can be proved as in the case when Y is nonsingular, so we omit the details.
(a) The correspondence is given by (zero degree case)

$$L = \Big\{ \bigotimes_{\omega \in S}' \theta_Y(\omega) \otimes \theta_Y\left(-\frac{\#s}{2}\right) \quad \text{if } w_0 \notin S,$$

$$L = \bigotimes_{\omega \in S \setminus \omega_0} \theta_Y(\omega) \otimes \theta_Y\left(\frac{1-\#S}{2}\right) \otimes M \quad \text{if } w_0 \in S, \text{ where } \theta_Y(d) = p^*\theta_{\mathbb{P}^1}(d)$$

and M is the unique squareroot of θ_Y which becomes trivial on pulling back to $\tilde{Y}$, the normalization of Y. The set S is characterized as the set of points w in W_0 such that i-action on the fibre of L at w is (-identity). The odd degree case can be dealt with similarly.
(b) Let $\pi: \tilde{Y} \to Y$ be the normalization map. The map $\operatorname{Pic}(\tilde{Y}) \to T(Y) = \{$nonlocally free torsion free sheaves of rank 1 on $Y\}$ defined by $L \to \pi_* L$ is an isomorphism;

moreover L is i-invariant iff $\pi_* L$ is so. Thus i-invariants in Pic $\tilde{Y}$ are in bijective correspondence with i-invariants in $T(Y)$. Using this and the correspondence between i-invariants in Pic $\tilde{Y}$ and partitions of $W_0 - w_0$, we get the result. In case degree $=0$, one has

$$L = \bigotimes_{\omega \in S'} \theta_Y(w) \otimes \theta_Y\left(\frac{-1-\#S'}{2}\right) \otimes \pi_*(\theta_{\tilde{Y}}), \text{ for } s' \subset \{w_1, \dots, w_{2g}\}, \#S' \text{ odd.}$$

S' is the set of points w in $W_0 - w_0$ such that i-action on the fibre of ($\pi^* L$/torsion) at w is negative of the identity.

3. Quadratic form associated to a singular curve

3.1. Let Y be an irreducible curve with a single node at w_0. Let $p\colon Y \to \mathbb{P}^1$ be a double cover with $w_0, w_1, \dots, w_{2g}$ the ramification points. The image of w_i in $\mathbb{P}^1$ will again be denoted by w_i, for all i. Consider the divisor $W = 2w_0 + w_1 + \cdots + w_{2g}$ in $\mathbb{P}^1$. We have $\theta_W = (\theta/m_{w_0}^2) \oplus \left(\bigoplus_{i=1}^{2g} \theta/m_{w_i}\right)$. The multiplication in θ induces quadratic maps A_i on θ/m_{w_i} and A_0 on $\theta/m_{w_0}^2 \approx \theta/m_{w_0} \oplus m_{w_0}/m_{w_0}^2$ given by $A_i(X_i) = X_i^2$, $A_0(X_0, Y_0) = (X_0^2, 2X_0Y_0)$. Identifying θ_W with k^d, $A = \bigoplus_{i=0}^{2g} A_i$ is a quadratic map from k^d to k^d $(d = 2g+2)$. Now, given an element $b = (b_0, b_{00}, b_1, \dots, b_{2g})$ of k^d, there exists a polynomial f_b of degree $2g+1$ such that $f_b(w_i) = b_i$ for $i \geqslant 0$, and the derivative $f_b'(w_0) = b_{00}$. In fact

$$f_b(x) = \sum_{i \geqslant 0} b_i \mu_{w_i}(x) + \lambda\left(\prod_{i \geqslant 0} (x - w_i)\right)$$

where

$$\mu_{w_i}(x) = \prod_{j \neq i} (x - w_j) / \prod_{j \neq i} (w_i - w_j) \text{ and } \lambda = \left(b_{00} - \sum_{i \geqslant 0} b_i \mu'_{w_i}(w_0) / \prod_{i > 0} (w_0 - w_i\right).$$

Then

$$f_b(x) = \sum_{i \geqslant 0} b_i \lambda_i(x) + \tfrac{1}{2} b_{00} \lambda_{00}(x),$$

where

$$\lambda_i(w_j) = \delta_{ij},$$

$\lambda_{00}(w_j) = 0$ for all i. Define $\phi_x : k^d \to k$ by $\phi_x(b) = f_b(x)$. Let $Q_x = \phi_x \circ A$. Then $\{Q_x\}_x$ is a family of quadratic forms on k^d parametrized by $\mathbb{P}^1$ with values in $\theta_{p^1}(2g+1)$ or equivalently a quadratic form Q on the trivial bundle $\mathbb{P}^1 \times k^d$ with values in $\theta_{\mathbb{P}^1}(2g+1)$. Tracing the above maps, one gets

$$(*)\quad Q_x(X_0, Y_0, X_1, \dots, X_{2g}) = \sum_{i=0}^{2g} \lambda_i(x) X_i^2 + \lambda_{00}(x) X_0 Y_0$$

$$\text{with } \lambda_i(w_j) = \delta_{ij}, \quad \lambda_i'(w_0) = 0, \quad \lambda_{00}(w_i) = 0 \text{ for all } i.$$

3.2. We make a few observations regarding Q. The space $(0, Y_0, 0, \dots, 0)$ is isotropic for Q_x for all x, this is the canonical one dimensional subspace corresponding to $m_{w_0}/m_{w_0}^2$. Also, $Q_{w_i} = X_i^2$, $i = 0, 1, \dots, 2g$; and these are precisely the singular members of the family. In fact (*) shows that Q is dual (adjoint) to a singular pencil of the form

$$Q_1 = \sum_{i=1}^{2g} X_i^2 + 2X_0Y_0$$

$$Q_2 = \sum_{i=1}^{2g} a_iX_i^2 + X_0^2 + 2a_0X_0Y_0,$$

with $\{a_i\}$ distinct scalars. This is a pencil with Segre symbol [11....12]. We recall that the adjoint of a quadratic form Q is given by the adjoint of the matrix of Q.

3.3. We now give an alternative way of defining Q. We will interpret the maps A_j to have values in $(\theta_{\mathbb{P}^1}(d-1))_{w_i}$, $i = 1,\ldots,2g$, and A_0 to have values in $(\theta_{\mathbb{P}^1}(d-1))\otimes\theta/m_{w_0}^2$. The evaluation map $\psi: H^0(\theta_{\mathbb{P}^1}(d-1))\to\theta_{\mathbb{P}^1}(d-1)\otimes\theta_W$ can easily be checked to be an isomorphism. Let e_x be the evaluation map $H^0(\theta_{\mathbb{P}^1}(d-1))\to(\theta_{\mathbb{P}^1}(d-1))_x$. Then $\phi_x = e_x\circ\psi^{-1}$; so that $e_x\circ\psi^{-1}\circ A$ gives Q_x.

3.4. *The general case.* We have restricted ourselves to the case of a single node merely for the simplicity of exposition. The construction in 3.3 works in the case of many ordinary double points too. In that case, for W, we have to take the divisor $\sum_i 2v_i + \sum_j w_j$, where v_i varies over all ordinary double points of Y and $\{v_i, w_j\}_{i,j}$ is the set of ramification points for the double cover $Y\to\mathbb{P}^1$. The quadratic form Q is then adjoint to the pencil of the form

$$Q_1 = \sum_{i=1}^{n} X_i^2 + \sum_{j=1}^{m} Z_jY_j$$

$$Q_2 = \sum_{i=1}^{n} a_iX_i^2 + \sum_{j=1}^{m} (Z_j^2 + 2b_jZ_jY_j)$$

where a_i, b_j are distinct scalars, m equals the number of ordinary double points; $2m+n = 2g+2$, g being the arithmetic genus of Y.

3.5. In this and the next article we want to study a subvariety of the variety of g dimensional linear subspaces of k^{2g+2} which are isotropic for the singular pencil

$$Q_1 = \sum_{i=1}^{2g} X_i^2 + 2X_0Y_0$$

$$Q_2 = \sum_{i=1}^{2g} a_iX_i^2 + X_0^2 + 2a_0X_0Y_0$$

where a_i's are all distinct scalars. We have a splitting $k^{2g+2} = \bigoplus_{i=1}^{2g} C_i + C_0$ where X_i is the coordinate in C_i for $i = 1,\ldots,2g$ and X_0, Y_0 are the coordinates in C_0. The subspace of C_0 defined by $X_0 = 0$ will be denoted by C_0^0. For simplicity of notation we will often denote $\bigoplus_{i=1}^{2g} C_i$ by k^{2g}.

Let Grass$_r$( ) denote the grassmannian of r-dimensional subspaces. Define P, P_0 by

$$P = \left\{ V_0 \in \mathrm{Grass}_g(k^d) \;\middle|\; \begin{array}{l} V_0 \text{ isotropic for the pencil} \\ V_0 = V_1 \oplus C_0^0,\ V_1 \subset k^{2g} \end{array} \right\}$$

$$P_0 = \left\{ V_1 \in \mathrm{Grass}_{g-1}(k^{2g}) \,\middle|\, \begin{array}{l} V_1 \text{ isotropic for the pencil} \\ \text{restricted to } k^{2g} \end{array} \right\}.$$

and
Clearly, $P \approx P_0$.

Remark. It is well known that the pencil on k^d induces a dual family on $(k^d)^*$, if a quadratic form on k^d is given by matrix B, the corresponding form on $(k^d)^*$ is given by the adjoint of B. Under the natural correspondence between $\mathrm{Grass}_g(k^d)$ and $\mathrm{Grass}_{g+2}(k^d)^*$ which maps a subspace $V_0 \subset k^d$ to the subspace V of linear forms on k^d vanishing on V_0, P corresponds to the subvariety P' of $\mathrm{Grass}_{g+2}(k^d)^*$ consisting of subspaces of the form $V = V_1 \oplus C_0^0$ (C_0^0 being the unique subspace of $(C_0)^*$ isotropic for the dual family) such that any generic member of the (dual) family restricted to V has rank 2. Also, if we identify (k^d) with $(k^d)^*$ using the isomorphism given by a generic member of the pencil, say Q_1, then V gets identified to the orthogonal complement of V_0 with respect to Q_1.

Lemma 3.6. *If V_0 is isotropic for the pencil but V_0 does not belong to P, then*

$$V_0' + k^{2g} = k^d$$

where V_0' denotes the orthogonal complement of V_0 with respect to a generic form of the pencil.

Proof. Since V_0 is isotropic and the only subspace of C_0 isotropic for a generic form is C_0^0 it follows that $V_0 \cap C_0 = C_0^0$ or $\{0\}$. Hence V_0 is not in P iff $V_0 \cap C_0 = \{0\}$. Taking orthogonal complements with respect to a generic form, the latter is equivalent to $V_0' + k^{2g} = k^d$.

Remark 3.7. Lemma 3.6 says that V is not in P' iff $\dim V \cap k^{2g} = g$. For V in P', $\dim V \cap k^{2g} = g+1$.

4. The generalized Jacobian

With the notations of 3.1, and 3.5, the main result of the section is the following.

Theorem 1. *The generalized Jacobian of Y is isomorphic to the complement of P in the space R of g-dimensional vector subspaces isotropic for the pencil.*

Since the proof is on similar lines as in §4, [3], we only sketch it briefly with necessary modifications. We will actually show that the generalized Jacobian of Y is isomorphic to the complement of P'(3.5) in the variety R' of $(g+2)$-dimensional spaces on which Q has rank two generically. We take J as the variety of line bundles of degree g on Y.

4.1. The map $f: J \to R' - P'$. Let W be the ramification divisor in Y, $\theta_Y(1) = p^*\theta_{P^1}(1)$. Fix a line bundle ξ of degree $2g+1$ on Y. Take α in J. Define $N = (\alpha^2 \otimes \xi)(-g)$. Using 2.3 and the Riemann–Roch theorem, one sees that $h^0(N) = g+2$ and $H^0(N)$ gets embedded in $H^0(N \otimes \theta_W)$ under the restriction map. Now $\xi \otimes \theta_W = \oplus \xi_{w_i} \oplus (\xi \otimes \theta/I) =$

$\bigoplus_{i=1}^{2g} C_i \oplus C_0$, length $\theta/I=2$, support $\theta/I=w_0$. Using $\alpha\otimes i^*\alpha=\theta_Y(g)$ and $\alpha_{w_i}=(i^*\alpha)_{w_i}$, one gets $H^0(N\otimes\theta_W)\approx\oplus C_i\oplus C_0$. Define $f(\alpha)=H^0(N)$ embedded in $\oplus C_i\oplus C_0$ as above. As in §4, [3] one can check that $f(\alpha)$ is in R'. If L is the kernel of the map $N\to N\otimes\theta/I$, $H^0(N)\cap\left(\bigoplus_{i=1}^{2g} C_i\right)=H^0(L)$ and the latter is of dimension g, so that $f(\alpha)$ is in $R'-P'$.

4.2. *Definition of* f^{-1}. Define a map $h: Y\times k^{2g+2}\to\theta_W$ by $h=0$ outside W, h/w_i is given by mapping C_i isomorphically onto θ/m_{w_i} and h/w_0 is given by mapping C_0 isomorphically onto θ/I. The kernel K of this map is a vector bundle (Lemma 2.2, [1]). Since Q/K vanishes identically on W, it induces a form q on K with values in $\theta_Y(2g+1)(-2W)=\theta_Y(-1)$ which is everywhere nondegenerate. Take V in $R'-P'$. Let V' be the fibre product of K and $Y\times V$ over $Y\times k^{2g+2}$. As the form restricted to $\bigoplus_{i>0} C_i$ is dual to a nondegenerate pencil, as in the case when Y is nonsingular, one has $V+\left(\bigoplus_{i\neq j} C_j\right)+C_0=k^{2g+2}$ for $i>0$. By Remark 3.7, $V+\left(\bigoplus_{i>0} C_i\right)=k^{2g+2}$. Therefore the composite of h with the inclusion of $Y\times V$ in $Y\times k^{2g+2}$ is onto, showing that V' is locally free. As in the nonsingular case [3], q restricted to V' has rank two everywhere. Let V'' be the kernel of q on V'. Define $F=(V'/V'')(g+1)$. The quadratic form on F gives an isomorphism $q:F\to F^*(2g+1)$. There is also an isomorphism $\psi:F\to F^*(2g+1)$ induced by the canonical alternating form on F with values in det F. Define $N=\mathrm{Ker}((q^{-1}\circ\psi)-Id)$. Then $F=N\oplus i^*N$ with $i^*N\approx\mathrm{Im}((q^{-1}\circ\psi)-id)$) (3.7, [3]). Let $L=(N\otimes\xi^{-1})(g)$. As in [3], to give the inverse of f, we have only to choose a canonical square root of the even degree line bundle L which can be done using Proposition 2.5.

Remark 4.3. The desingularization $\tilde{Y}$ of Y is a nonsingular hyperelliptic curve with ramification points $w_1, w_2,\ldots,w_{2g}$. So, by the theorem in the nonsingular case, the Jacobian of Y is isomorphic to P_0 (defined in 3.5) and hence to P'.

5. The compactified Jacobian

We shall first prove the following theorem in the case of a single node and then indicate the general proof.

Theorem 2. *Let Y be an irreducible curve of arithmetic genus g with only ordinary double points as singularities. Let $p: Y\to\mathbb{P}^1$ be a double cover such that the set of ramification points contains all double points. Then the compactified Jacobian $\bar{J}$ of Y is isomorphic to the variety R of g-dimensional subspaces of k^{2g+2} which are isotropic for the singular pencil of quadrics of the form*

$$Q_1=\sum_i X_i^2+\sum_j 2Y_jZ_j$$

$$Q_2=\sum_i a_iX_i^2+\sum_j (Y_j^2+2b_jY_jZ_j)$$

associated to the double cover. Here a_i, b_j are all distinct scalars, j varies over the ordinary double points of Y and i varies over the remaining ramification points.

Proof. We first show that the morphism f^{-1} defined on $R' - P'$ (§4.2) can be extended to whole of R'. Starting with a V in R' we get a rank two coherent sheaf F as in 4.2. If V is in P', F is not locally free, nor does it split. However, we do have a nondegenerate quadratic form on F with values in $\mathcal{O}_Y(2g+1)\otimes m_{w_0}$ in this case, giving an isomorphism $q: F \tilde{\to} F^*(2g+1)\otimes m_{w_0}$. Also, one has det F modulo torsion $= \mathcal{O}_Y(2g+1)\otimes m_{w_0}$. So the canonical alternating form on F gives a morphism (and *not* an isomorphism) Ψ: $F \to F^*(2g+1)\otimes m_{w_0}$. Define $N = \ker((q^{-1}\circ\psi) - Id)$. Notice that this definition of N agrees with that in the case V is in $R' - P'$, in that case we in fact have $F = \mathrm{Ker}((q^{-1}\circ\Psi) - Id)\oplus \mathrm{Im}(q^{-1}\circ\Psi - Id) = N \oplus i^*N$. The rest of the procedure to define f^{-1} works as before, i.e. one can choose canonically a torsion free rank 1 sheaf L such that $L\otimes L$ modulo torsion $= (N\otimes\xi^{-1})(g)$ and define $f'(V) = L$. Thus one gets a morphism $f': R' \to \bar{J}$ extending f^{-1}. One can check by similar methods that f' restricted to P' is an isomorphism onto Pic $\tilde{Y}$ embedded in $\bar{J}$ as the set J_1 of torsion free rank one sheaves which are not locally free, the embedding being given by the map $L \to \pi_* L$. Thus we have a bijective birational morphism $f': R' \to \bar{J}$. Let $\tilde{R}$ and $\tilde{J}$ denote the normalizations of R' and $\bar{J}$ respectively. $\tilde{J}$ is the disjoint union of Pic Y and two copies of Pic $\tilde{Y}$, so $\tilde{J} \neq R'$. The morphism f' induces an isomorphism of normalizations $\tilde{R}$ and $\tilde{J}$, let $\tilde{f}$ be the composite of the inverse of this isomorphism with the natural map from $\tilde{R}'$ to R'. Since $\tilde{f}$ respects the identification of two copies of Pic $\tilde{Y}$, it factors through $\bar{J}$ giving $\bar{f}: \bar{J} \to R'$, the inverse of f'. This finishes the proof of the theorem in the one node case.

6. The general case

In the case of m nodes, we have a decomposition $k^{2g+2} = \bigoplus_{i=1}^{n} C_i \oplus \bigoplus_{j=1}^{m} D_j$, dim $C_i = 1$, dim $D_j = 2$, $n + 2m = 2g+2$. It can be proved (in a similar way as in the one node case) that the generalized Jacobian of Y is isomorphic to the open subset of R' consisting of those subspaces which intersect $\bigoplus_i C_i \bigoplus_{j\neq k} D_j$, in subspaces of dimension g, for $i \leqslant k \leqslant m$. As before, we can also get a birational bijective morphism $\bar{f}': R' \to \bar{J}$ inducing an isomorphism of normalizations. We identify the normalizations $\tilde{R}'$ and $\tilde{J}$ via this isomorphism. Thus we are again in the situation $\tilde{J} \to R' \to \bar{J}$. At a singular point L of $\bar{J}$, the complete local ring has the form $T\otimes \prod_i k[[X_i, Y_i]]/(X_iY_i)$, i varying over the p number of nodes at which L is not locally free, T being a regular ring. Thus, the singularities of $\bar{J}$ being product singularities, we can prove $R' = \bar{J}$ using induction on p, i.e. can assume the result for $p < m$. Let U be the complement in $\bar{J}$ of the closed subset S corresponding to torsion free sheaves which are not locally free at all m nodes. By induction, $O_{R'}/U \simeq O_{\bar{J}}/U$. Since $\bar{J}$ is Cohen-Macaulay and $\bar{J} - U$ is not a divisor ($m \geqslant 2$) we have $H^0(V, O_{\bar{J}}) \simeq H^0(U\cap V, O_{\bar{J}})$, for any open set V. The commutative diagram

$$\begin{array}{ccc} H^0(V, O_{\bar{J}}) & \to & H^0(U\cap V, O_{\bar{J}}) \\ \downarrow & & \downarrow \approx \\ H^0(V, R') & \to & H^0(U\cap V, O_{R'}) \end{array}$$

shows that $H^0(V, O_{\bar{J}}) = H^0(V, O_{R'})$ i.e. $O_{\bar{J}} = O_{R'}$. Thus $\bar{J} = R'$. Thus we have proved the general theorem.

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