Proc. Indian Acad. Sci. (Math. Sci.), Vol. 95, No. 2, December 1986, pp. 109–120.

On torsional loading in an axisymmetric micropolar elastic medium

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MS received 21 December 1985; revised 28 October 1986

Abstract. Effects of torsional loading in an axisymmetric micropolar elastic half-space are studied. The components of microrotation displacement, force stress and couple stress are obtained for a half-space subjected to an arbitrary load produced by shearing stress. A special case of a particular type of twist has been discussed in detail for a specific model and the micropolar effects have been shown graphically.

Keywords. Torsional loading; axisymmetric micropolar elastic medium; microrotation; displacement; force stress; couple stress.

1. Introduction

Modern engineering structures are often made up of materials possessing an internal structure. Polycrystalline materials, materials with fibrous or coarse grain structure come in this category. Classical elasticity is inadequate to represent the behaviour of such materials. The analysis of such materials requires incorporating the theory of oriented media. Describing the behaviour of oriented media, the micropolar theory is one of these theories.

The basic equations of the linear micropolar theory of elasticity have been given by Kuvchinski and Aero [9], Palmov [14], Eringen and Suhubi [3].

The first and second axial-symmetric problems in micropolar elasticity have been investigated by Nowacki [11,12]. Khan and Dhaliwal [4] also discussed the axisymmetric problem for a half-space in micropolar elastic medium. Nowacki and Nowacki [13] studied the axial symmetric Lamb's problem in micropolar elasticity. The present authors have already discussed the axisymmetric problem, the plane problem and Lamb's plane problem in micropolar elastic half-space with strech [5, 6, 7]. Kumar *et al* [8] also discussed Lamb's plane problem in thermoelastic micropolar medium with stretch.

Eason [1] discussed the problem of torsional loading of an elastic half-space. Wave motion due to impulsive twist on the surface has been investigated by Sarkar [15]. Sengupta [16] studied the problem of torsional vibration of a semi-infinite elastic medium by a particular type of twist. Tiwari [17] discussed the problem of effect of couple stress in a semi-infinite elastic medium due to impulsive twist on the surface.

In this paper, we first obtain the general solution for the problem of torsional loading in an axisymmetric micropolar elastic half-space under the action of an arbitrary load on its boundary and then the case of a particular type of twist has been discussed in detail.

2. Formulation of the problem

We consider a homogeneous, isotropic micropolar elastic half-space. We take cylindrical polar coordinate (r, θ, z), where z is pointing into the medium. Due to symmetry about z-axis, all quantities are independent of θ. We take

$$\boldsymbol{\omega} = (\omega_r, 0, \omega_z), \ \mathbf{u} = (0, u_\theta, 0) \tag{1}$$

and assume that on the plane boundary $z = 0$ of the half-space $z \geqslant 0$, the only non-vanishing stress is the shearing stress which is given as $t_{z\theta} = -f(r)$, where $f(r)$ is the prescribed function of r. Moreover the displacement, stress and couple stress components tend to zero as $z \to \infty$.

3. Basic equations, boundary conditions and solutions

The constitutive equations and field equations in the absence of body forces and body moments for static case for micropolar elastic medium are given by Nowacki [10]:

$$t_{ji} = \lambda u_{k,k} \ \delta_{ji} + (\mu - \alpha) \ (u_{i,j} + u_{j,i}) + 2\alpha(u_{i,j} - \varepsilon_{kji}\omega_k), \tag{2}$$

$$m_{ji} = \beta\omega_{k,k}\delta_{ji} + (\gamma + \varepsilon)\omega_{i,j} + (\gamma - \varepsilon)\omega_{j,i}, \tag{3}$$

$$(\mu + \alpha)\nabla^2\mathbf{u} + (\lambda + \mu - \alpha) \text{ grad div } \mathbf{u} + 2\alpha \text{ rot } \boldsymbol{\omega} = 0, \tag{4}$$

$$(\gamma + \varepsilon)\nabla^2\boldsymbol{\omega} + (\gamma + \beta - \varepsilon) \text{ grad div } \boldsymbol{\omega} + 2\alpha \text{ rot } \mathbf{u} - 4\alpha\boldsymbol{\omega} = 0, \tag{5}$$

where λ, μ, α, β, γ, ε are material constants, $\mathbf{u}$ the displacement vector, $\boldsymbol{\omega}$ the rotation vector, t_{ji} the force stress tensor and m_{ji} the couple stress tensor. Using (1), (4) and (5) reduce to

$$\left[(\gamma + \varepsilon)\left(\nabla^2 - \frac{1}{r^2}\right) - 4\alpha\right]\omega_r + (\beta + \gamma - \varepsilon)\frac{\partial e'}{\partial r} - 2\alpha\frac{\partial u_0}{\partial z} = 0, \tag{6}$$

$$[(\gamma + \varepsilon)\nabla^2 - 4\alpha]\omega_z + (\beta + \gamma - \varepsilon)\frac{\partial e'}{\partial z} + \frac{2\alpha}{r}\frac{\partial}{\partial r}(ru_\theta) = 0, \tag{7}$$

$$(\mu + \alpha)\left(\nabla^2 - \frac{1}{r^2}\right)u_\theta + 2\alpha\left(\frac{\partial \omega_r}{\partial z} - \frac{\partial \omega_z}{\partial r}\right) = 0, \tag{8}$$

where

$$e' = \frac{1}{r}\frac{\partial}{\partial r}(r\omega_r) + \frac{\partial \omega_z}{\partial z}, \ \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}. \tag{9}$$

Since, there acts a torsional loading on the plane boundary $z = 0$, mathematically the boundary conditions are

$$t_{z\theta} = -f(r), \; m_{zz} = m_{zr} = 0, \text{ at } z = 0. \tag{10}$$

To solve the problem, we introduce potential functions Φ, Ψ and v as

$$\omega_r = \frac{\partial\Phi}{\partial r} + \frac{\partial^2\Psi}{\partial r\partial z}, \tag{11}$$

$$\omega_z = \frac{\partial\Phi}{\partial z} - \left(\nabla^2 - \frac{\partial^2}{\partial z^2}\right)\Psi, \tag{12}$$

$$u_\theta = \partial v/\partial r. \tag{13}$$

Substituting (11) to (13) in (6) to (8), we get

$$\frac{\partial}{\partial r}[(\beta+2\gamma)\nabla^2 - 4\alpha]\Phi + \frac{\partial^2}{\partial r\partial z}[\{(\gamma+\varepsilon)\nabla^2 - 4\alpha\}\Psi - 2\alpha v] = 0, \tag{14}$$

$$\frac{\partial}{\partial z}[(\beta+2\gamma)\nabla^2 - 4\alpha]\Phi - \left(\nabla^2 - \frac{\partial^2}{\partial z^2}\right)[\{(\gamma+\varepsilon)\nabla^2 - 4\alpha\}\Psi - 2\alpha v] = 0, \tag{15}$$

$$\frac{\partial}{\partial r}(\nabla^2 v) + \frac{2\alpha}{(\mu+\alpha)}\frac{\partial}{\partial r}(\nabla^2\Psi) = 0. \tag{16}$$

We use Hankel transforms defined by

$$\left.\begin{aligned} \bar{g}(\zeta, z) &= \int_0^\infty g(r, z) r J_0(\zeta r)\,\mathrm{d}r, \\ \bar{g}(\zeta, z) &= \int_0^\infty g(r, z) r J_1(\zeta r)\,\mathrm{d}r, \end{aligned}\right\} \tag{17}$$

Applying transforms on (14) to (16), we get

$$[(\beta+2\gamma)(D_z^2 - \zeta^2) - 4\alpha]\bar{\Phi} + D_z[\{(\gamma+\varepsilon)(D_z^2 - \zeta^2) - 4\alpha\}\bar{\Psi} - 2\alpha\bar{v}] = 0, \tag{18}$$

$$D_z[(\beta+2\gamma)(D_z^2 - \zeta^2) - 4\alpha]\bar{\Phi} + \zeta^2[\{(\gamma+\varepsilon)(D_z^2 - \zeta^2) - 4\alpha\}\bar{\Psi} - 2\alpha\bar{v}] = 0, \tag{19}$$

$$\bar{v} = -\frac{2\alpha}{(\mu+\alpha)}\bar{\Psi}, \tag{20}$$

where $D_z = \mathrm{d}/\mathrm{d}z$. From (18) to (20), we get

$$(D_z^2 - \zeta^2)(D_z^2 - \zeta_1^2)\bar{\Phi} = 0, \tag{21}$$

$$(D_z^2 - \zeta^2)(D_z^2 - \zeta_2^2)\bar{\Psi} = 0, \tag{22}$$

$$(D_z^2 - \zeta^2)\bar{\Phi} = -\frac{(\gamma+\varepsilon)}{(\beta+2\gamma)} D_z(D_z^2 - \zeta^2)\bar{\Psi}, \tag{23}$$

where

$$\left.\begin{aligned} &\zeta_1^2 = \zeta^2+m_1^2,\ \zeta_2^2 = \zeta^2+m_2^2,\\ &m_1^2 = 4\alpha/(\beta+2\gamma),\ m_2^2 = 4\alpha\mu/(\mu+\alpha)\ (\gamma+\varepsilon). \end{aligned}\right\} \tag{24}$$

Since displacements, stress and couple stress components tend to zero as $z \to \infty$, Φ, Ψ and ν tend to zero as $z \to \infty$, the solutions of (21) and (22) may be written as

$$\bar{\Phi} = A\exp(-\zeta z)+B\exp(-\zeta_1 z), \tag{25}$$

$$\bar{\Psi} = C\exp(-\zeta z)+D\exp(-\zeta_2 z). \tag{26}$$

Substituting (25) and (26) in (23), we obtain

$$A = \frac{\mu}{(\mu+\alpha)}\ \zeta C. \tag{27}$$

Thus

$$\bar{\Phi} = \frac{\mu}{(\mu+\alpha)}\ \zeta C\ \exp(-\zeta z)+B\ \exp(-\zeta_1 z). \tag{28}$$

Taking the inverse transform of (26) and (28)

$$\Phi = \int_0^\infty \left[\frac{\mu}{\mu+\alpha}\ \zeta C\ \exp(-\zeta z)+B\exp(-\zeta_1 z)\right]\zeta J_0(\zeta r)\,\mathrm{d}\zeta, \tag{29}$$

$$\Psi = \int_0^\infty [C\ \exp(-\zeta z)+D\ \exp(-\zeta_2 z)]\zeta J_0(\zeta r)\,\mathrm{d}\zeta. \tag{30}$$

Substituting the values of Φ and Ψ in the boundary conditions (10), we get the following system of equations

$$2\alpha\zeta B+\frac{2\alpha\mu}{(\mu+\alpha)}\ \zeta^2 C = \bar{f}(\zeta), \tag{31}$$

$$(2\gamma\zeta^2+4\alpha)B-\frac{2\alpha\gamma}{(\mu+\alpha)}\ \zeta^3 C-2\gamma\zeta^2\zeta_2 D = 0, \tag{32}$$

$$2\gamma\zeta_1 B-\frac{2\alpha\gamma}{(\mu+\alpha)}\ \zeta^2 C-\left\{2\gamma\zeta^2-\frac{4\alpha\mu}{(\mu+\alpha)}\right\}D = 0, \tag{33}$$

where

$$\bar{f}(\zeta) = \int_0^\infty f(r)rJ_1(\zeta r)\,\mathrm{d}r. \tag{34}$$

Solving (31) to (33), we get

$$\left.\begin{aligned} &B = \zeta F_1(\zeta),\ C = \frac{(\mu+\alpha)}{\alpha}F_2(\zeta),\ D = F_3(\zeta)\\ &\text{where}\\ &F_1(\zeta) = (\zeta^2-\zeta\zeta_2+\varepsilon_1)\ \frac{\bar{f}(\zeta)}{2(\mu+\alpha)\Delta}, \end{aligned}\right\} \tag{35}$$

$$\left.\begin{aligned} F_2(\zeta) &= \left[\zeta^2 - \zeta_1\zeta_2 + \left(\frac{(2\mu+\alpha)}{\mu}\right)\varepsilon_1 + \frac{2\alpha}{\gamma}\frac{\varepsilon_1}{\zeta^2}\right]\frac{\bar{f}(\zeta)}{2(\mu+\alpha)\Delta},\\ F_3(\zeta) &= \left[\zeta(\zeta_1-\zeta) - \frac{2\alpha}{\gamma}\right]\frac{\bar{f}(\zeta)}{2(\mu+\alpha)\Delta},\\ \varepsilon_1 &= 2\alpha\mu/\gamma(\mu+\alpha),\ \Delta = (\zeta^2+\varepsilon_1)^2 - \frac{(\alpha\zeta+\mu\zeta_1)}{(\mu+\alpha)}\zeta^2\zeta_2. \end{aligned}\right\} \tag{35}$$

and

Thus, we get the solutions of Φ and Ψ as

$$\Phi = \int_0^\infty \zeta^2\left[\frac{\mu}{\alpha}F_2(\zeta)\exp(-\zeta z) + F_1(\zeta)\exp(-\zeta_1 z)\right]J_0(\zeta r)\,d\zeta, \tag{36}$$

$$\Psi = \int_0^\infty \zeta\left[\frac{(\mu+\alpha)}{\alpha}F_2(\zeta)\exp(-\zeta z) + F_3(\zeta)\exp(-\zeta_2 z)\right]J_0(\zeta r)\,d\zeta, \tag{37}$$

Now, we obtain the components of displacement, microrotation, force stress and couple stress as

$$u_\theta = 2\int_0^\infty \zeta^2\left[F_1(\zeta)\exp(-\zeta z) + \frac{\alpha}{(\mu+\alpha)}F_3(\zeta)\exp(-\zeta_2 z)\right]J_1(\zeta r)\,d\zeta, \tag{38}$$

$$\omega_r = \int_0^\infty \zeta^3\left[F_2(\zeta)\exp(-\zeta z) + \frac{\zeta_2}{\zeta}F_3(\zeta) \times\exp(-\zeta_2 z) - F_1(\zeta)\exp(-\zeta_1 z)\right]J_1(\zeta r)\,d\zeta. \tag{39}$$

$$\omega_z = \int_0^\infty \zeta^3\left[F_2(\zeta)\exp(-\zeta z) - \frac{\zeta_1}{\zeta}F_1(\zeta) \times\exp(-\zeta_1 z) + F_3(\zeta)\exp(-\zeta_2 z)\right]J_0(\zeta r)\,d\zeta, \tag{40}$$

$$t_{z\theta} = -2\mu\int_0^\infty \zeta^3\left[F_2(\zeta)\exp(-\zeta z) + \frac{\alpha}{\mu}F_1(\zeta)\exp(-\zeta_1 z)\right]J_1(\zeta r)\,d\zeta, \tag{41}$$

$$m_{zz} = -2\gamma\int_0^\infty \zeta^4\left[F_2(\zeta)\exp(-\zeta z) - F_1(\zeta)\left(1+\frac{2\alpha}{\gamma}\frac{1}{\zeta^2}\right)\exp(-\zeta_1 z) + \frac{\zeta_2}{\zeta}F_3(\zeta)\exp(-\zeta_2 z)\right]J_0(\zeta r)\,d\zeta, \tag{42}$$

$$m_{zr} = -2\gamma\int_0^\infty \zeta^4\left[F_2(\zeta)\exp(-\zeta z) - \frac{\zeta_1}{\zeta}F_1(\zeta)\exp(-\zeta_1 z) + F_3(\zeta)\right.$$

$$\times\left(1+\frac{\varepsilon_1}{\zeta^2}\right)\exp(-\zeta_2 z)\Bigg]J_1(\zeta r)\mathrm{d}\zeta. \tag{43}$$

If, we let the micropolar constants tend to zero, we obtain the displacement and stress components for the case of classical elasticity as

$$u_\theta = \frac{1}{\mu}\int_0^\infty \bar{f}(\zeta)\exp(-\zeta z)\ J_1(\zeta r)\mathrm{d}\zeta, \tag{44}$$

$$t_{z\theta} = -\int_0^\infty \bar{f}(\zeta)\zeta\exp(-\zeta z)\ J_1(\zeta r)\mathrm{d}\zeta. \tag{45}$$

These results agree with the results of Sarkar [15] for a static case.

4. Particular case

We consider the twist of the type given by

$$f(r) = \frac{r}{4a^4}\exp(-r^2/4a^4) \tag{46}$$

where r is the distance of the point from the origin of the co-ordinate system.

Applying the transform (17) on (46), we get

$$\bar{f}(\zeta) = \zeta\ \exp(-a^2\zeta^2). \tag{47}$$

Thus, from (38) to (43) and (47), we get

$$u_\theta = 2\int_0^\infty \zeta^3\exp(-a^2\zeta^2)\Bigg[F_1(\zeta)\exp(-\zeta z)+\frac{\alpha}{(\mu+\alpha)}F_3(\zeta)$$

$$\times\exp(-\zeta_2 z)\Bigg]J_1(\zeta r)\mathrm{d}\zeta, \tag{48}$$

$$\omega_r = \int_0^\infty \zeta^4\ \exp(-a^2\zeta^2)\ \Bigg[F_2(\zeta)\exp(-\zeta z)+\frac{\zeta_2}{\zeta}F_3(\zeta)$$

$$\times\exp(-\zeta_2 z)-F_1(\zeta)\ \exp(-\zeta_1 z)\Bigg]J_1(\zeta r)\mathrm{d}\zeta, \tag{49}$$

$$\omega_z = \int_0^\infty \zeta^4\ \exp(-a^2\zeta^2)\ \Bigg[F_2(\zeta)\ \exp(-\zeta z)-\frac{\zeta_1}{\zeta}F_1(\zeta)$$

$$\times\exp(-\zeta_1 z)+F_3(\zeta)\ \exp(-\zeta_2 z)\Bigg]J_0(\zeta r)\mathrm{d}\zeta, \tag{50}$$

$$t_{z\theta} = -2\mu\int_0^\infty \zeta^4\ \exp(-a^2\zeta^2)\Bigg[F_2(\zeta)\ \exp(-\zeta z)+\frac{\alpha}{\mu}F_1(\zeta)$$

$$\times \exp(-\zeta_1 z)\Bigg] J_1(\zeta r)\,d\zeta, \tag{51}$$

$$m_{zz} = -2\gamma \int_0^\infty \zeta^5 \exp(-a^2\zeta^2)\left[F_2(\zeta)\exp(-\zeta z) - F_1(\zeta)\left(1+\frac{2\alpha}{\gamma}\frac{1}{\zeta^2}\right)\right.$$

$$\left.\times \exp(-\zeta_1 z) + \frac{\zeta_2}{\zeta} F_3(\zeta)\ \exp(-\zeta_2 z)\right] J_0(\zeta r)\,d\zeta, \tag{52}$$

$$m_{zr} = -2\gamma \int_0^\infty \zeta^5 \exp(-a^2\zeta^2)\left[F_2(\zeta)\ \exp(-\zeta z)\ - \frac{\zeta_1}{\zeta} F_1(\zeta)\right.$$

$$\left.\times \exp(-\zeta_1 z) + \left(1+\frac{\varepsilon_1}{\zeta^2}\right) F_3(\zeta)\ \exp(-\zeta_2 z)\right] J_1(\zeta r)\,d\zeta. \tag{53}$$

By neglecting the micropolar effect, we get the corresponding results for the classical elasticity as

$$u_\theta = \frac{1}{\mu}\int_0^\infty \zeta\ \exp(-a^2\zeta^2)\ \exp(-\zeta z)\ J_1(\zeta r)\,d\zeta, \tag{54}$$

$$t_{z\theta} = -\int_0^\infty \zeta^2\ \exp(-a^2\zeta^2)\ \exp(-\zeta z)\ J_1(\zeta r)\,d\zeta. \tag{55}$$

5. Approximate evaluation of the integrals

The integrals involved in the expressions (48) to (53) are difficult to evaluate analytically, but can be evaluated either numerically or approximately. We evaluate them approximately.

Assuming α, m_1^2 and m_2^2 to be very small as compared to unity we expand ζ_1, ζ_2, $1/\Delta$ in an infinite series to obtain

$$\left.\begin{aligned} \zeta_1 &= \zeta + \frac{m_1^2}{2\zeta} + O(m_1^4),\\ \zeta_2 &= \zeta + \frac{m_2^2}{2\zeta} + O(m_2^4),\\ \Delta &= \gamma\varepsilon_1 A_1 \zeta^2, \end{aligned}\right\} \tag{56}$$

where

$$A_1 = \left[\frac{2}{\gamma} - \frac{1}{(\gamma+\varepsilon)} - \frac{1}{(\beta+2\gamma)}\right].$$

Hence, from (48) to (56), we get

$$u_\theta = \frac{1}{(\mu+\alpha)} \int_0^\infty \zeta\left(1+\frac{2\alpha P_1}{A_1\zeta^2}\right) \exp\,[(-a^2\zeta+z)\zeta] J_1(\zeta r)\,d\zeta. \tag{57}$$

$$\omega_r = \frac{\alpha}{\gamma(\mu+\alpha)A_1} \int_0^\infty (P_4+P_2\zeta z) \exp\,[(-a^2\zeta+z)\zeta] J_1(\zeta r)\,d\zeta, \tag{58}$$

$$\omega_z = \frac{\alpha}{\gamma(\mu+\alpha)A_1} \int_0^\infty (P_3+P_2\zeta z) \exp\,[(-a^2\zeta+z)\zeta] J_0(\zeta r)\,d\zeta, \tag{59}$$

$$t_{z\theta} = -\int_0^\infty \left(\zeta^2+\gamma\varepsilon_1 \frac{P_1}{A_1}\right) \exp\,[(-a^2\zeta+z)\zeta] J_1(\zeta r)\,d\zeta, \tag{60}$$

$$m_{zz} = -\frac{2\alpha P_2}{(\mu+\alpha)A_1}\, z \int_0^\infty \zeta^2 \exp\,[(-a^2\zeta+z)\zeta] J_0(\zeta r)\,d\zeta, \tag{61}$$

$$m_{zr} = -\frac{2\alpha P_2}{(\mu+\alpha)A_1}\, z \int_0^\infty \zeta^2 \exp\,[(-a^2\zeta+z)\zeta] J_1(\zeta r)\,d\zeta, \tag{62}$$

where

$$P_1 = \frac{1}{\gamma^2} - \frac{1}{(\gamma+\varepsilon)(\beta+2\gamma)},\quad P_2 = \frac{1}{(\gamma+\varepsilon)} + \frac{1}{(\beta+2\gamma)} - \frac{2}{(\gamma+\varepsilon)(\beta+2\gamma)},$$

$$P_3 = \frac{1}{\gamma} - \frac{1}{(\beta+2\gamma)},\quad P_4 = \frac{1}{\gamma} - \frac{1}{(\gamma+\varepsilon)}. \tag{63}$$

We expand $\exp(-a^2\zeta^2)$ in an infinite series occurring in the expressions (57) to (62). Assuming that $a\zeta$ is so small that its fourth order terms are negligible and using the results of Erdelyi [2], we get

$$u_\theta = \frac{r}{(\mu+\alpha)}\left[\frac{1}{\rho_1^3} + \frac{3a^2}{\rho_1^5}\left(1-\frac{5z^2}{\rho_1^2}\right) + 2\alpha\frac{P_1}{A_1}\right.$$

$$\left.\times\left\{\frac{1}{\rho_1+z} - \frac{a^2}{\rho_1^3} + \frac{3a^4}{2\rho_1^5}\left(\frac{5z^2}{\rho_1^2}-1\right)\right\}\right], \tag{64}$$

$$\omega_r = \frac{\alpha}{\gamma(\mu+\alpha)}\frac{r}{A_1\rho_1}\left[P_4\left\{\frac{1}{\rho_1+z} - \frac{3a^2z}{\rho_1^4} + \frac{15a^4z}{2\rho_1^6}\left(\frac{7z^2}{\rho_1^2}-3\right)\right\}\right.$$

$$\left.+P_2\frac{z}{\rho_1^2}\left\{1+\frac{3a^2}{\rho_1^2}\left(1-\frac{5z^2}{\rho_1^2}\right)\right\}\right], \tag{65}$$

$$\omega_z = \frac{\alpha}{\gamma(\mu+\alpha)A_1\rho_1}\left[P_3\left\{1+\frac{a^2}{\rho_1^2}\left(1-\frac{3z^2}{\rho_1^2}\right\} + \frac{9a^4}{2\rho_1^4}\left(1+\frac{5z^2}{\rho_1^2}\left(\frac{3z^2}{\rho_1^2}-2\right)\right)\right\}\right.$$

$$+P_2 \frac{z^2}{\rho_1^2}\left\{1+\frac{3a^2}{\rho_1^2}\left(3-\frac{5z^2}{\rho_1^2}\right)\right\}\Bigg], \tag{66}$$

$$t_{z\theta} = -\frac{r}{\rho_1}\left[\frac{3z}{\rho_1^4}+\frac{15a^2z}{\rho_1^6}\left(3-\frac{7z^2}{\rho_1^2}\right)+\gamma\varepsilon_1\frac{P_1}{A_1}\right.$$

$$\left.\times\left\{\frac{1}{\rho_1+z}-\frac{3a^2z}{\rho_1^4}+\frac{15a^4z}{2\rho_1^6}\left(\frac{7z^2}{\rho_1^2}-3\right)\right\}\right], \tag{67}$$

$$m_{zz} = \frac{2\alpha z P_2}{(\mu+\alpha)A_1}\frac{1}{\rho_1^3}\left[1-\frac{3z^2}{\rho_1^2}+\frac{9a^2}{\rho_1^2}\left\{1+\frac{5z^2}{\rho_1^2}\left(\frac{3z^2}{\rho_1^2}-2\right)\right\}\right], \tag{68}$$

$$m_{zr} = \frac{6\alpha}{(\mu+\alpha)}\frac{P_2}{A_1}\frac{z^2 r}{\rho_1^5}\left[\frac{5a^2}{\rho_1^2}\left(\frac{7z^2}{\rho_1^2}-3\right)-1\right], \tag{69}$$

where

$$\rho_1^2 = r^2+z^2.$$

If we neglect the micropolar effect, we get the results for the classical elasticity

$$u_\theta = \frac{r}{\mu\rho_1^3}\left[1+\frac{3a^2}{\rho_1^2}\left(1-\frac{5z^2}{\rho_1^2}\right)\right], \tag{70}$$

$$t_{z\theta} = -\frac{3rz}{\rho_1^5}\left[1+\frac{5a^2}{\rho_1^2}\left(3-\frac{7z^2}{\rho_1^2}\right)\right], \tag{71}$$

6. Numerical results and discussion

Couple stress, force stress, microrotation and displacement have been calculated in the plane $z = 1$ for various values of γ, $\alpha = 0{\cdot}01$, $\beta = 0{\cdot}03$, $\varepsilon = 0{\cdot}02$, $\mu = 0{\cdot}3$ and $a = 1$ for the range $0 \leqslant r \leqslant 4{\cdot}0$.

It is observed (figure 1) that for $\gamma = 0{\cdot}05$, $0{\cdot}1$, $0{\cdot}15$, m_{zz} decreases for $0 \leqslant r \leqslant 0{\cdot}8$, increases for $0{\cdot}8 \leqslant r \leqslant 2{\cdot}8$ and decreases as r increases further for $r \geqslant 2{\cdot}8$.

It is evident from figure 2 that for $\gamma = 0{\cdot}05, 0{\cdot}1, 0{\cdot}15$, ω_z increases monotonically in the intervals $0 \leqslant r \leqslant 0{\cdot}4$ and $0{\cdot}8 \leqslant r \leqslant 1{\cdot}6$, whereas it decreases monotonically for $0{\cdot}4 \leqslant r \leqslant 0{\cdot}8$ and $r \geqslant 1{\cdot}6$.

We notice that for $\gamma = 0{\cdot}05$, $t_{z\theta}$ increases monotonically for $0 \leqslant r \leqslant 0{\cdot}4$ and $1{\cdot}2 \leqslant r \leqslant 2{\cdot}8$ whereas it decreases monotonically for $0{\cdot}4 \leqslant r \leqslant 1{\cdot}2$ and $r \geqslant 2{\cdot}8$. For $\gamma = 0{\cdot}1$, $t_{z\theta}$ decreases monotonically for $0 \leqslant r \leqslant 0{\cdot}4$ and $r \geqslant 2{\cdot}0$ but increases monotonically for $0{\cdot}4 \leqslant r \leqslant 2{\cdot}0$. For $\gamma = 0{\cdot}15$, $t_{z\theta}$ decreases monotonically for $0 \leqslant r \leqslant 0{\cdot}4$ and $r \geqslant 1{\cdot}6$ but increases monotonically for $0{\cdot}4 \leqslant r \leqslant 1{\cdot}6$. It is also

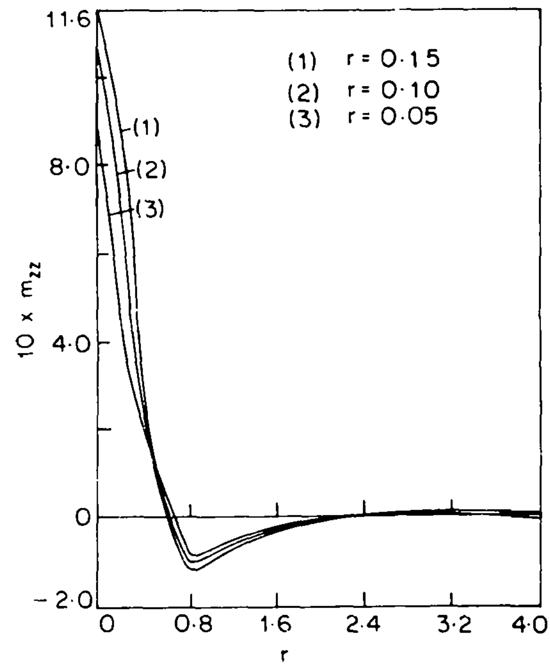

Figure 1. Couple stress m_{zz}.

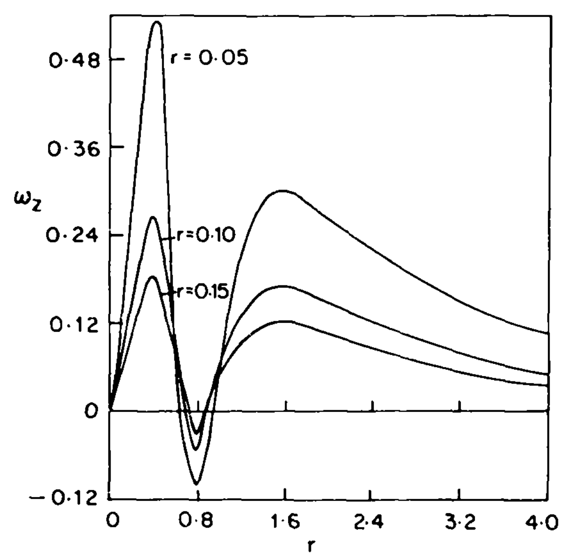

Figure 2. Microrotation ω_z.

observed that in the case of classical elasticity, $t_{z\theta}$ decreases monotonically for $0 \leqslant r \leqslant 0{\cdot}4$ and $r \geqslant 1{\cdot}6$ whereas it increases monotonically for $0{\cdot}4 \leqslant r \leqslant 1{\cdot}6$. These variations have been shown graphically in figure 3.

The variations of the displacement component u_θ have been shown graphically in figure 4. For $\gamma = 0{\cdot}05$, u_θ decreases monotonically for $0 \leqslant r \leqslant 0{\cdot}4$ and $r \geqslant 2{\cdot}4$

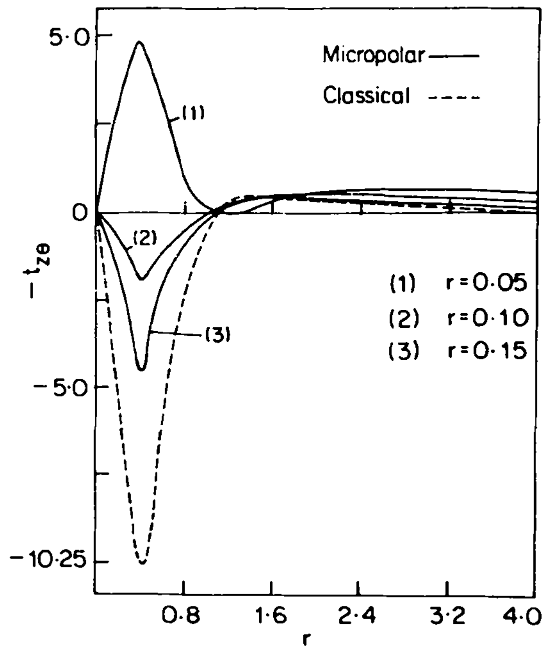

Figure 3. Force stress $t_{z\theta}$.

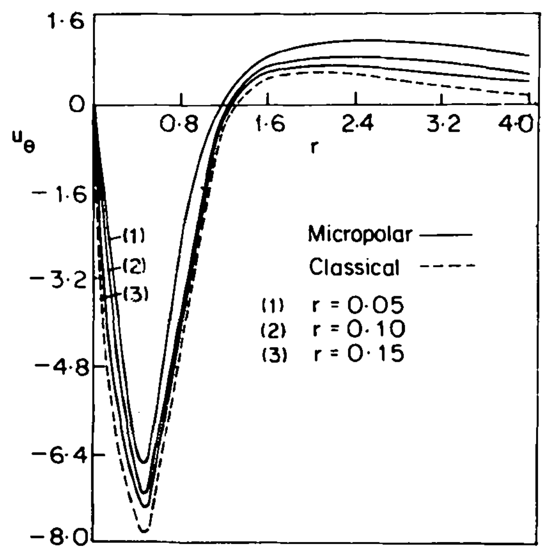

Figure 4. Displacement u_θ.

whereas it increases monotonically for $0{\cdot}4 \leqslant r \leqslant 2{\cdot}4$. For $\gamma = 0{\cdot}1$ and $0{\cdot}15$, u_θ decreases monotonically for $0 \leqslant r \leqslant 0{\cdot}4$ and $r \geqslant 2{\cdot}0$ whereas it increases monotonically for $0{\cdot}4 \leqslant r \leqslant 2{\cdot}0$. In the case of classical elasticity, u_θ decreases monotonically for $0 \leqslant r \leqslant 0{\cdot}4$, $r \geqslant 2{\cdot}0$ whereas it increases monotonically for $0{\cdot}4 \leqslant r \leqslant 2{\cdot}0$.

Thus, it is seen that force stress, couple stress, displacement and microrotation oscillate in certain range of r but all of them start decreasing beyond certain values of r.

Acknowledgement

The authors are thankful to Prof. Kehar Singh for help and suggestions.

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