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Combined free and forced convection for laminar flow in vertical triangular channel with internal heat generation

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Abstract. The problem of combined, natural and forced convection for the laminar flow in a vertical channel of equilateral triangular cross-section is discussed. Internal heat generation is assumed to be uniform. The coupled equations in velocity and temperature are solved using equitriangular transform. Expressions for pressure drop parameter and Nusselt number are obtained and their behaviour for different values of Rayleigh number and heat generation function, is studied.

Keywords. Convection; heat generation function; pressure-drop parameter; equitriangular transform.

1. Introduction

There are many practical situations in which forced and free convections are of equal magnitude and neither can be ignored. The problem of combined free and forced convection for laminar flow has been extensively studied in vertical channels with and without internal heat generation. Hallman [4] studied the vertical circular tubes with internal heat generation, while Han [5] studied it in rectangular channels. Gupta [2, 3] investigated the unsteady laminar convective flow under a pressure gradient along a vertical pipe. The combined convection in horizontal tubes has also attracted attention. Nagendra [7] studied the interaction of free and forced convections in the transition regime. Patankar *et al* [8] discussed the effects of non-uniform heating on laminar combined convection, while El-Hawary [1] experimentally investigated the effects of combined free and forced convection on the stability of flow in a horizontal tube. The combined convection has been studied experimentally in inclined circular tubes also [9].

In the present investigation, the combined natural and forced convection has been analytically studied for fully established laminar flow in a vertical channel of equilateral triangular cross-section. Internal heat generation is also considered and assumed to be uniform. The coupled equations in velocity and temperature are solved using equitriangular transform (established on an incomplete set of eigen functions in an equilateral triangular region) due to Lu and Miller [6]. Expressions for pressure drop parameter and Nusselt number are obtained and their behaviour for different values of Rayleigh number and heat generation function is studied.

2. Formulation of the problem

The flow velocity is assumed as fully established and the heat input from the boundary of the lateral triangular channel to the fluid is constant in the flow direction. The wall temperature is uniform in the transverse plane. The temperatures of the walls and the fluid are linear along the flow direction. It is further assumed that the fluid properties are constant unless the body force is considered. Heat dissipation due to viscosity effect is assumed to be negligible. Heat sources, providing similar distributions in all transverse planes, are also present in the fluid. The fluid flow is along the positive direction of Z-axis.

Under these conditions, the equations of motion and temperature difference are

$$\nabla^2 u = 1/\mu\,[\partial p/\partial z - F_B], \tag{1}$$

and

$$k\nabla^2 T = \rho u C_p\,\partial T/\partial Z - Q, \tag{2}$$

where u, μ, ρ, k, T, C_p, F_B, p and Q represent the axial velocity, coefficient of viscosity, density, thermal conductivity of the fluid, temperature difference between the fluid and the wall, specific heat at constant pressure, body force in positive Z direction, pressure and heat generation rate respectively.

The flow channel configuration and the co-ordinate axes are shown in figure 1.

The equations of motion and temperature, in dimensionless form, reduce to

$$\nabla^2 U + R_A\theta = -L \tag{3}$$

and

$$\nabla^2\theta - U = F, \tag{4}$$

assuming that the fluid obeys the linear law of state and no phase change of the fluid occurs. Here U, θ, R_A, F and L represent non-dimensional velocity, temperature difference between the fluid and the boundary, Rayleigh number, heat generation

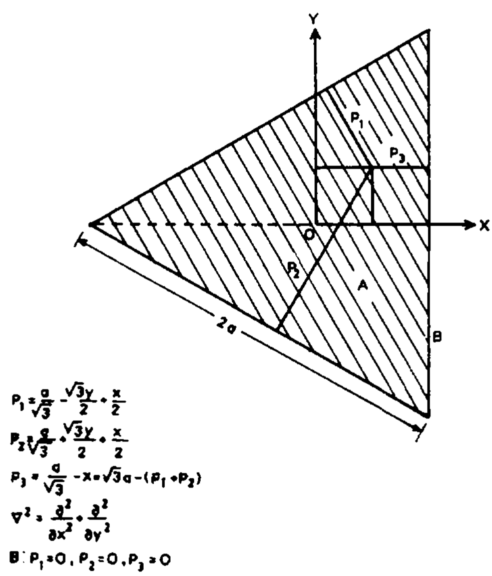

Figure 1. An equilateral triangular region.

function and pressure drop parameter respectively. The boundary conditions for the present problem are $U = 0 = \theta$, on the boundary B. (*i.e.* $p_1 = 0, p_2 = 0$ and $p_3 = 0$) (see figure 1).

2.1. *Method of solution*

Combining the coupled equations in velocity and temperature difference using a parameter λ, we obtain

$$\nabla^2 U + R_A\theta + \lambda(\nabla^2\theta - U - F) = -L, \tag{5}$$

or

$$\nabla^2 (U + \lambda\theta) + (R_A\theta - \lambda U) = \lambda F - L. \tag{6}$$

Choosing λ such that $1/\lambda = -\lambda/R_A$ *i.e.* $\lambda = i\sqrt{R_A}$, the equation reduces to

$$(\nabla^2 + k^2)W = -L + i\sqrt{R_A}\, F, \tag{7}$$

where $W = U + i\sqrt{R_A}\,\theta$ and $k^2 = -i\sqrt{R_A}$ and the boundary conditions reduce to

$$W = 0 \text{ on the boundary B.} \tag{8}$$

Applying equitriangular transform (established on an incomplete set of eigen functions $\psi_n = \sin\lambda_n p_1 + \sin\lambda_n p_2 + \sin\lambda_n p_3$ with eigenvalues $\lambda_n = 2n\pi/\sqrt{3}a$; $n = 1, 2, 3, \ldots$) (Lu and Miller [6]), we obtain

$$-\lambda_n^2 \tilde{W} + k^2\tilde{W} = (3\sqrt{3}a^2/n\pi)(-L + i\sqrt{R_A}F), \tag{9}$$

and $$\tilde{W} = 0 \text{ on the boundary B,} \tag{10}$$

where $\tilde{W}$ represents equitriangular transform of W. Now, using the inversion formula for this transform, we get

$$W = \sum_{n=1}^{\infty} (2/n\pi)(\sin\lambda_n p_1 + \sin\lambda_n p_2 + \sin\lambda_n p_3)(-L + i\sqrt{R_A}F)/(k^2 - \lambda_n^2). \tag{11}$$

Equating the real and imaginary parts, we get the expressions for 2 velocity and temperature difference as

$$U = \sum_{n=1}^{\infty} \frac{2}{n\pi}\frac{L\lambda_n^2 - R_A F}{\lambda_n^4 + R_A}(\sin\lambda_n p_1 + \sin\lambda_n p_2 + \sin\lambda_n p_3) \tag{12}$$

and

$$\theta = \sum_{n=1}^{\infty} -\frac{2}{n\pi}\frac{F\lambda_n^2 + L}{\lambda_n^4 + R_A}(\sin\lambda_n p_1 + \sin\lambda_n p_2 + \sin\lambda_n p_3). \tag{13}$$

3. Results and discussions

Figure 2 depicts the velocity distribution along the axis of x for different values of Rayleigh number (R_A) and heat generation function (F). Because of the symmetry, this distribution is the same along all the medians of a triangular section of the channel. The

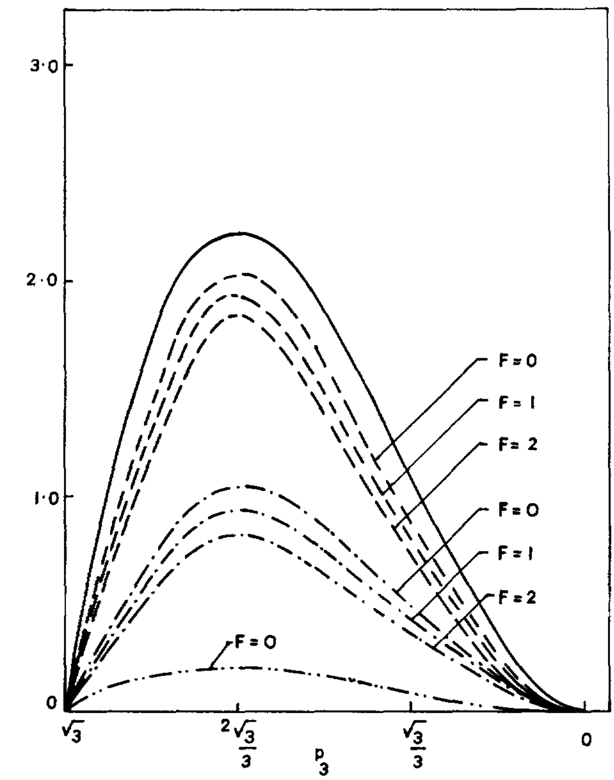

Figure 2. Velocity distribution along the axis of x for $R_A = 0$ (———), $R_A = \pi^4$ (- - -), $R_A = 10\pi^4$ (–·–·–) and $R_A = 100\pi^4$ (–··–··–··).

velocity of the fluid is seen to be maximum at the centroid. Increase in the Rayleigh number results in the decrease in the velocity. It is clear from the expression that for $R_A = 0$, F has no effect on the velocity. For very large values of R_A, the velocity is insignificantly affected by F. However, for the moderate values of R_A (π^4 and $10\pi^4$), the dependence on F is significant and increase in the positive values of F decreases the velocity at every point of the section.

Figure 3 shows the temperature difference between the fluid and the boundary, along the axis of x, for different values of R_A and F. It is also same along the three medians of a triangular section of the channel. For all values of R_A and F, the temperature difference is optimum at the centroid. For $F = 0$ (no heat generation in the fluid), the increase in Rayleigh number results in decrease in the temperature difference. For a particular R_A value, increase in F (heat source) gives increase in the temperature difference. A particular negative value of F (heat sink) makes the temperature of the fluid throughout the channel equal to that of the boundary. However, this negative value of F is different for different values of R_A.

4. Pressure-drop parameter

The pressure-drop parameter L is defined in terms of heat generation function F and

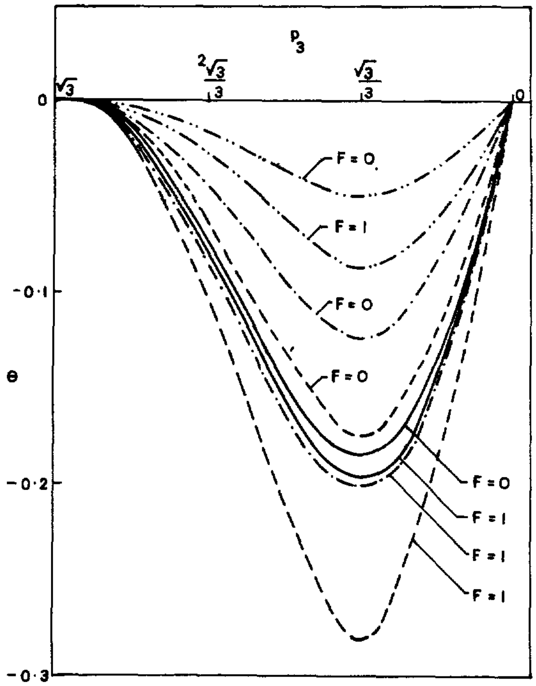

Figure 3. Temperature difference along the axis of x for $R_A = 0$ (———), $R_A = \pi^4$ (– – –), $R_A = 10\pi^4$ (–·–·–·) and $R_A = 100\pi^4$ (–··–··–··).

Rayleigh number R_A as follows:

Consider the equation of continuity,

$$\int_A U \mathrm{d}x\mathrm{d}y = \int_A \mathrm{d}x\mathrm{d}y, \tag{14}$$

where A is the cross-sectional area. Substituting the value of U from (12) and solving (14), we get,

$$L = \frac{1 + (6\pi^2) F R_A \sum_{n=1}^{\infty} 1/n^2[R_A + (16n^4\pi^4/9a^4)]}{(8/a^2) \sum_{n=1}^{\infty} [1/R_A + (16n^4\pi^4/9a^4)]}, \tag{15}$$

where $2a$ is the side of the equilateral triangle.

Equation (15) shows that the pressure drop parameter is a linearly increasing function of the heat generating function F. For $a = 1$, the following expressions for the pressure drop parameter L have been obtained from (15), for different positive values of R_A.

$$L = 20{\cdot}001 \text{ for } R_A = 0,$$

$$L = 30{\cdot}072 + 6{\cdot}755F \text{ for } R_A = \pi^4, \tag{16}$$

$$L = 100{\cdot}419 + 56{\cdot}354F \text{ for } R_A = 10\pi^4,$$

$$L = 483{\cdot}215 + 363{\cdot}778F \text{ for } R_A = 100\pi^4.$$

From (16), it is obvious that for the positive values of Rayleigh number, heat source/sink is responsible for the increase/decrease in the pressure-drop parameter. A particular negative value of F (*i.e.* heat sink) makes the pressure-drop parameter zero.

4.1. *Nusselt number*

In case of uniform heat generation, the Nusselt number is given by

$$\mathrm{Nu} = (F-1)/(-4\theta_{mx}), \tag{17}$$

where θ_{mx} represents the dimensionless mixed mean temperature difference between the fluid and the wall in the same transverse plane and is expressed as

$$\theta_{mx} = \int_A \theta U \,\mathrm{d}x\mathrm{d}y \Big/ \int_A \mathrm{d}x\mathrm{d}y. \tag{18}$$

Substituting the values of U and θ and solving (18), we get

$$\theta_{mx} = -(2/\pi^2)\sum_{n=1}^{\infty} 1/n^2 \frac{(L+F.4n^2\pi^2/3a^2)(L.4n^2\pi^2/3a^2 - FR_A)}{(R_A + 16n^4\pi^4/9a^4)^2}. \tag{19}$$

For $a = 1$, the Nusselt number values are obtained for different values of heat generation function F and Rayleigh number R_A and are given in table 1.

When $F = 1$, the Nusselt number is zero and this physically means that there is no heat transfer from the walls of the channel. This represents the condition of insulated walls. Equation (13) indicates that the temperature gradient on the boundaries is not identically zero. For $F = 1$, the temperature gradient changes sign along the boundary and hence heat flows in the reverse direction also, so that the net heat transfer from the wall is zero.

From table 1, it is clear that for $F > 1$, the Nusselt number is positive *i.e.* heat flows away from the walls, while for $F < 1$, the Nusselt number is negative which means that the heat is absorbed by the fluid through the boundary.

4.2. *Rayleigh number*

In the present study, a zero Rayleigh number corresponds to either of the two cases. The first corresponds to incompressible fluid in which the free convection effects cannot occur, while the second deals with zero axial temperature gradient where free convection effects occur if the heat generation function F is non-zero. Since F and θ are infinitely large in the second case, the zero Rayleigh number corresponds to the forced convection problem only.

Table 1. Nusselt number values.

R_A	Nu for $F = 0$	$F = 0{\cdot}5$	$F = 1{\cdot}5$	$F = 2$
0	$-2{\cdot}333$	$-0{\cdot}875$	$0{\cdot}583$	$1{\cdot}000$
π^4	$-2{\cdot}507$	$-0{\cdot}952$	$0{\cdot}640$	$1{\cdot}106$
$10\pi^4$	$-3{\cdot}716$	$-1{\cdot}443$	$1{\cdot}061$	$1{\cdot}925$
$100\pi^4$	$-7{\cdot}171$	$-2{\cdot}923$	$2{\cdot}448$	$4{\cdot}918$

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