Proc. Indian Acad. Sci. (Math. Sci.), Vol. 91, Number 2, July 1982, pp. 139-154.

Scattering of impulsive elastic waves by a fluid cylinder

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MS received 7 December 1981

Abstract. We consider the scattering of impulsive sv waves by a fluid circular cylinder. The cylinder is embedded in an unbounded isotropic homogeneous elastic medium and it is filled with some acoustic fluid. The line source, generating the incident pulse is situated outside the cylinder parallel to its axis. We investigate the problem by the method of dual integral transformation as developed by Friedlander. The resulting integrals are evaluated approximately to obtain the short time estimate of the motion near the wave-front in the illuminated region of the elastic medium. We also interpret the approximate solution in terms of geometrical optics.

Keywords. Elastic waves; scattering; dual integral transformation; geometrical optics.

1. Introduction

The scattering and diffraction of two-dimensional elastic waves with a cylindrical obstacle in an unbounded medium has been considered in recent years. Gilbert and Knopoff (1959) discussed the scattering of impulsive elastic waves by a rigid circular cylinder situated in a homogeneous isotropic elastic medium. Gilbert (1960) considered the scattering of impulsive elastic waves by a smooth convex cylinder and obtained the formal solution of the problem using the technique of dual integral transformation developed by Friedlander (1954). An approximate evaluation of the solution was then obtained corresponding to the short-time behaviour of the scattered field near the wave-front both in the shadow as well as in the illuminated zone. Jha (1974) used the same technique to investigate the problem of diffraction of compressional waves by a fluid cylinder in a homogeneous medium. Rajhans and Mishra (1980) also considered the diffraction of impulsive elastic waves by a fluid cylinder in a homogeneous medium using the above technique.

In this paper, we investigate and discuss the scattering of the impulsive sv pulses by a circular cylinder filled with inviscid fluid material. The obstacle is supposed to be situated in an unbounded homogeneous isotropic elastic medium and the incident pulse is generated by a line source situated in the surrounding elastic medium at a finite distance parallel to the axis of the cylinder. It is well-known that line stone formations usually exhibit a shear-wave arrival stronger than the compressional waves. Therefore in such cases shear waves are more prominent than the compressional waves (White 1965). We suppose that the velocities of P and SV waves outside the cylinder are α and β respectively and that of P waves inside the cylinder is α_0. To be specific, we assume $\alpha > \alpha_0 > \beta$. This assumption of the velocity distribution corresponds to the actual velocity distribution of elastic waves inside the earth and to the location of the source in the mantle and the outer core as the obstacle (Bullen 1963). The present investigation therefore throws some light on the scattering of SV waves by the core of the earth. We also suppose the density of the medium outside the cylinder is ρ and that inside the cylinder is ρ' where $\rho > \rho'$.

2. Formulation of the problem

Let the axis of the cylinder be taken as the z-axis and let a co-ordinate system (r, θ) be located in the (x, y) plane with $\theta = 0$, $r = r_0\ (> a)$ corresponding to the location of the line source which is parallel to the axis of the cylinder. The equation of the cylinder is $r = a$.

We define the elastic velocity potentials Φ_0, Φ and ψ corresponding to the wave equations inside and outside the cylinder respectively. Since z-axis is taken along the axis of the cylinder, the state of the media is fully determined if the velocity potentials Φ_0, Φ and ψ are obtained as a function of r, θ and t. It is thus required to find out the velocity potentials Φ_0, Φ and ψ as a function of r, θ and t which satisfy the wave equations

$$\frac{1}{\beta^2}\frac{\partial^2\psi}{\partial t^2} - \nabla^2\psi = \frac{2\pi}{r}\,\delta(r - r_0)\,\delta(t)\,\delta(\theta), \quad (r \geqslant a), \tag{1}$$

$$\frac{1}{\alpha^2}\frac{\partial^2\Phi}{\partial t^2} - \nabla^2\Phi = 0, \quad (r \geqslant a), \tag{2}$$

$$\frac{1}{\alpha_0^2}\frac{\partial^2\Phi_0}{\partial t^2} - \nabla^2\Phi_0 = 0, \quad (r \leqslant a), \tag{3}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2},$$

the initial conditions

$$\begin{aligned} &\psi = \partial\psi/\partial t = 0, \text{ when } t = 0 \text{ except at } r = r_0,\ \theta = 0,\\ &\Phi = \partial\Phi/\partial t = 0, \text{ when } t = 0,\\ &\Phi_0 = \partial\Phi_0/\partial t = 0, \text{ when } t = 0, \end{aligned} \tag{4}$$

and the boundary conditions

$$\begin{aligned} &[T_{r\theta}]_{r=a+0} = 0,\\ &[T_{rr}]_{r=a+0} = [T_{rr}]_{r=a-0},\\ &[u_r]_{r=a+0} = [u_r]_{r=a-0}, \end{aligned} \tag{5}$$

where

$$T_{r\theta} = \nu\left\{\frac{1}{r}\frac{\partial u_r}{\partial\theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right\},$$

$$T_{rr} = \lambda\frac{1}{a^2}\frac{\partial^2\Phi}{\partial t^2} + 2\nu\frac{\partial u_r}{\partial r},$$

$$u_r = \frac{\partial\Phi}{\partial r} + \frac{1}{r}\frac{\partial\psi}{\partial\theta},$$

$$u_\theta = \frac{1}{r}\frac{\partial\Phi}{\partial\theta} - \frac{\partial\psi}{\partial r},$$

Here λ and ν are Lame's parameters and the symbol δ stands for Dirac delta function.

3. The formal solution

Let us define the Laplace transform $\bar{\psi}\,(r,\theta,s)$ of $\psi\,(r,\theta,t)$ by

$$\bar{\psi}\,(r,\theta,s) = \int_0^\infty \psi\,(r,\theta,t)\exp(-st)\,\mathrm{d}t, \tag{6}$$

where s is the transform variable. Again we denote the Fourier transform ψ^* of $\bar{\psi}$ by

$$\psi^*\,(r,\mu,s) = \frac{1}{(2\pi)^{1/2}}\int_{-\infty}^{\infty}\bar{\psi}\,(r,\theta,s)\,\exp\,(-i\mu\theta)\,\mathrm{d}\theta \tag{7}$$

Applying these transformations to (1), (2) and (3) we get

$$\frac{\mathrm{d}^2\psi^*}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}\psi^*}{\mathrm{d}r} - \left(\frac{s^2}{\beta^2} + \frac{\mu^2}{r^2}\right)\psi^* = -\frac{(2\pi)^{1/2}}{r}\delta\,(r - r_0), \quad (r \geqslant a) \tag{8}$$

$$\frac{\mathrm{d}^2\Phi^*}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}\Phi^*}{\mathrm{d}r} - \left(\frac{s^2}{a^2} + \frac{\mu^2}{r^2}\right)\Phi^* = 0, \quad (r \geqslant a), \tag{9}$$

and

$$\frac{\mathrm{d}^2\Phi_0^*}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}\Phi_0^*}{\mathrm{d}r} - \left(\frac{s^2}{a_0^2} + \frac{\mu^2}{r^2}\right)\Phi_0^* = 0, \ (r \leqslant a). \tag{10}$$

Also the Laplace-Fourier transformed boundary conditions are given by

$$[T_{r\theta}^*]_{r\to a+0} = 0,$$

$$[T_{rr}^*]_{r\to a+0} = [T_{rr}^*]_{r\to a-0},$$

and

$$[u_r^*]_{r\to a+0} = [u_r^*]_{r\to a-0}. \tag{11}$$

It may be assumed that ψ^* is continuous at $r = r_0$.

Then (8) is equivalent to

$$\frac{\mathrm{d}^2\psi^*}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}\psi^*}{\mathrm{d}r} - \left(\frac{s^2}{\beta^2} + \frac{\mu^2}{r^2}\right)\psi^* = 0, \quad (r \geqslant a), \tag{12}$$

and

$$[\psi^*]_{r_0-0}^{r_0+0} = 0, \; [\mathrm{d}\psi^*/\mathrm{d}r]_{r_0-0}^{r_0+0} = -(2\pi)^{1/2}/r_0. \tag{13}$$

After some mathematical calculation and Fourier inversion, we find that the Laplace transforms of the solution are given by

$$\bar{\psi}(r,\theta,s) = \int_{-\infty}^{\infty} I_{|\mu|}\left(\frac{sr}{\beta}\right) K_{|\mu|}\left(\frac{sr_0}{\beta}\right) \exp(i\mu\theta)\, \mathrm{d}\mu$$

$$+ \int_{-\infty}^{\infty} K_\mu\left(\frac{sr}{\beta}\right) K_\mu\left(\frac{sr_0}{\beta}\right) \frac{L}{M} \exp(i\mu\theta)\, \mathrm{d}\mu, \quad (r_0 \geqslant r \geqslant a) \tag{14}$$

$$\bar{\Phi}(r,\theta,s) = \int_{-\infty}^{\infty} K_\mu\left(\frac{sr}{a}\right) K_\mu\left(\frac{sr_0}{\beta}\right) \frac{N}{M} \exp(i\mu\theta)\, d\mu \quad (r \geqslant a) \tag{15}$$

$$\bar{\Phi}_0(r,\theta,s) = \int_{-\infty}^{\infty} \frac{2\rho\mu^2\beta^2/a^{3}\cdot L + N\cdot P}{M\cdot Q} I_{|\mu|}\left(\frac{sr}{a_0}\right)$$

$$\times K_\mu\left(\frac{sr_0}{\beta}\right) \exp(i\mu\theta)\, \mathrm{d}\mu, \quad (r \leqslant a) \tag{16}$$

where

$$L = \frac{2\rho s^4}{a\beta a_0} I'_\mu(sa/a_0)\, I'_\mu(sa/\beta)\, K_\mu(sa/a) - \frac{4\rho s^3\beta}{a^2\, a a_0}(I'_\mu)(sa/a_0)$$

$$\times I'_\mu(sa/\beta)\, K'_\mu(sa/a) - \frac{2\rho' s^4}{a\, a\beta} I_\mu(sa/a_0)\, I'_\mu(sa/\beta)\, K'_\mu(sa/a)$$

$$- \frac{\rho s\beta^2}{a_0}\left(\frac{s^2}{\beta^2} + \frac{2\mu^2}{a^2}\right) I'_\mu(sa/a_0)\, I_\mu(sa/\beta)\, K_\mu(sa/a)$$

$$+ \frac{2\rho s^4}{a a a_0} I'_\mu(sa/a_0)\, I_\mu(sa/\beta)\, K'_\mu(sa/a) + \frac{\rho' s^5}{a\beta^2} I_\mu(sa/a_0)$$

$$\times I_\mu(sa/\beta)\, K'_\mu(sa/a) + \frac{4\rho\mu^2 s^2\beta}{a^2\, a a_0} I'_\mu(sa/a_0)\, I'_\mu(sa/\beta)\, K'_\mu(sa/a)$$

$$+ \frac{4\rho s\mu^2\beta^3}{a^4 a_0} I'_\mu(sa/a_0)\, I_\mu(sa/\beta)\, K_\mu(sa/a)$$

$$+ \frac{2\rho'\mu^2 s^2}{a^2} I_\mu(sa/a_0)\, I_\mu(sa/\beta)\, K_\mu(sa/a),$$

$$M = \frac{4\rho s^3\beta}{a^2\, a a_0} I'_\mu(sa/a_0)\, K'_\mu(sa/a)\, K'_\mu(sa/\beta) - \frac{4\rho\beta\mu^2 s^2}{a^2\, a_0}$$

$$\times I'_\mu(sa/a_0)\, K'_\mu(sa/a)\, K'_\mu(sa/\beta) - \frac{4\rho s\mu^2\beta^2}{a^4 a_0} I'_\mu(sa/a_0)$$

$$\times K_\mu(sa/\alpha) K_\mu(sa/\beta) - \frac{2\rho'\mu^2 s^2}{a^3} I_\mu(sa/\alpha_0) K_\mu(sa/\alpha) K_\mu(sa/\beta)$$

$$- \frac{2\rho s^4}{a\beta\alpha_0} I'_\mu(sa/\alpha_0) K_\mu(sa/\alpha) K'_\mu(sa/\beta) + \frac{2\rho' s^4}{a\,\alpha\beta} I_\mu(sa/\alpha_0)$$

$$\times K'_\mu(sa/\alpha) K'_\mu(sa/\beta) + \frac{\rho s\beta^2}{\alpha_0}\left(\frac{s^2}{\beta^2} + \frac{2\mu^2}{a^2}\right)^2 I'_\mu(sa/\alpha_0)$$

$$\times K_\mu(sa/\alpha) K_\mu(sa/\beta) - \frac{2\rho s^4}{a\,\alpha\alpha_0} I'_\mu(sa/\alpha_0) K'_\mu(sa/\alpha) K_\mu(sa/\beta)$$

$$- \frac{\rho' s^5}{a\beta^2} I_\mu(sa/\alpha_0) K'_\mu(sa/\alpha) K_\mu\, sa/\beta),$$

$$N = \frac{4i\mu\rho s\beta^2}{a^4\alpha_0} I'_\mu(sa/\alpha_0) + \frac{2i\mu\rho' s^2}{a^3} I_\mu(sa/\alpha_0)$$

$$- \frac{2i\mu\rho s\beta^2}{a^4\alpha_0}\left(\frac{s^2}{\beta^2} + \frac{2\mu^2}{a^2}\right) I'_\mu(sa/\alpha_0),$$

$$P = \frac{i\mu\rho\beta^2}{a}\left(\frac{s^2}{\beta^2} + \frac{2\mu^2}{a^2}\right) I_\mu\left(\frac{sa}{\beta}\right) K_\mu\left(\frac{sa}{\alpha}\right)$$

$$- \frac{2i\mu\rho\beta s^2}{a\alpha} I'_\mu\left(\frac{sa}{\beta}\right) K'_\mu\left(\frac{sa}{\alpha}\right)),$$

and

$$Q = \frac{i\mu\rho' s^2}{a} I_\mu\left(\frac{sa}{\beta}\right) I_\mu\left(\frac{sa}{\alpha_0}\right) + \frac{2i\mu\rho s\beta^2}{a^2\alpha_0} I_\mu\left(\frac{sa}{\beta}\right)$$

$$\times I'_\mu\left(\frac{sa}{\alpha_0}\right) - \frac{2i\mu\rho s^2\beta}{a\alpha_0} I'_\mu\left(\frac{sa}{\beta}\right) I'_\mu\left(\frac{sa}{\alpha_0}\right). \tag{17}$$

It is evident that (14), (15) and (16) give the integral representation of Laplace transform of the formal solution. The time solution can be obtained on performing Laplace inversion. But it is difficult to evaluate the integrals (14), (15) and (16) as they stand. However, if one is interested in short-time behaviour of the pulses, these integrals can be evaluated approximately for large, positive and real s.

4. Incident, reflected and refracted pulses

We first give a brief description of the geometry of the problem. Initially the incident sv pulse striking the outer surface of the cylinder gives rise to reflected S, reflected P and refracted P pulses according to the laws of ordinary geometrical optics (figure 1). When the rays strike the outer surface of the cylinder at the critical angle, the reflected P rays become tangential to the surface and as such they move along the surface. These surface waves at each point of their path give rise to lateral sv waves at critical angle in the outer medium and P waves along the tangent to the surface (Keller 1958). The former are denoted by $S(P)S$ and the latter by $S(P)P$ (Gilbert and Knopoff 1959). In the case of grazing incidence, the disturbances move along the surface and at each point of their path they shed diffracted sv waves in the outer medium tangential to the surface. These are denoted by $S(S)S$ (Bullen 1963).

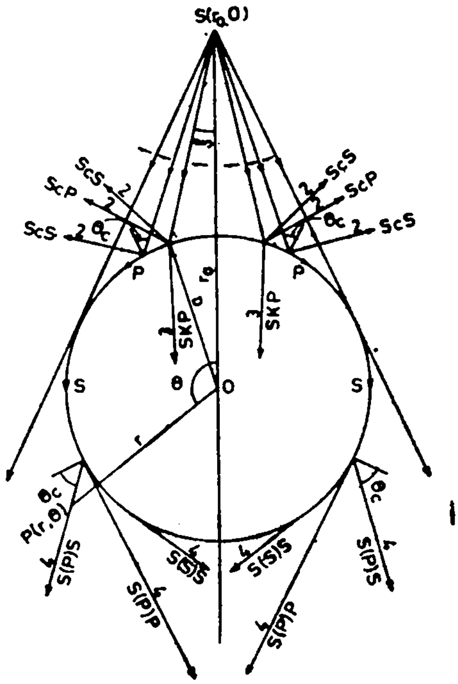

Figure 1. Incident SV pulse striking the outer surface of the cylinder. 1. Incident ray, 2. Reflected ray, 3. Refracted ray and 4. Diffracted ray.

Now we proceed to obtain the solution in the illuminated region of the elastic medium. For this purpose, we use the method of saddle point integration (Jeffreys and Jeffreys 1956). Therefore, we use the asymptotic approximations for modified Bessel functions occurring in the integrals (14), (15) and (16). We assume s to be large, real and positive. We know that for such s, the principal contributions to the integrals in (14), (15) and (16) arise from large values of $|\mu|$. Therefore using the various approximations for modified Bessel functions as given by (Mishra 1964a, b) in (14), (15) and (16), we find that

$$\bar{\psi}(r,\theta,s) \sim \int_{-\infty}^{\infty} f_1(\mu)\exp\{g_1(\mu)\}\,d\mu + \int_{-\infty}^{\infty} f_2(\mu)\exp\{g_2(\mu)\}\,d\mu, \quad (r_0 \geqslant r \geqslant a) \tag{18}$$

$$\bar{\Phi}(r,\theta,s) \sim \int_{-\infty}^{\infty} f_3(\mu)\exp\{g_3(\mu)\}\,d\mu, \quad (r \geqslant a) \tag{19}$$

$$\bar{\Phi}_0(r,\theta,s) \sim \int_{-\infty}^{\infty} f_4(\mu)\exp\{g_4(\mu)\}\,d\mu, \quad (r \leqslant a) \tag{20}$$

where

$$f_1(\mu) = \frac{\beta}{2}(\mu^2\beta^2 + s^2r^2)^{-1/4}(\mu^2\beta^2 + s^2r_0^2)^{-1/4},$$

$$g_1(\mu) = i\mu\theta + \left(\mu^2 + \frac{s^2r^2}{\beta^2}\right)^{1/2} - \mu\sinh^{-1}\left(\frac{\mu\beta}{sr}\right) - \left(\mu^2 + \frac{s^2r_0^2}{\beta^2}\right)^{1/2}$$
$$+ \mu\sinh^{-1}\left(\frac{\mu\beta}{sr_0}\right),$$

$$
\begin{aligned}
f_2(\mu) = {} & \frac{\beta}{2}(\mu^2\beta^2 + s^2 r^2)^{-1/4}(\mu^2\beta^2 + s^2 r_0^2)^{-1/4} \\
& \times \Bigg[4\rho\mu^2\beta^4\left(1 + \frac{s^2 a^2}{\mu^2 \alpha^2}\right)^{1/2}\left(1 + \frac{s^2 a^2}{\mu^2 \alpha_0^2}\right)^{1/2}\left(1 + \frac{s^2 a^2}{\mu^2 \beta^2}\right)^{1/2} \\
& - 4\rho\mu^4\beta^4\left(1 + \frac{s^2 a^2}{\mu^2 \alpha^2}\right)^{1/2}\left(1 + \frac{s^2 a^2}{\mu^2 \alpha_0^2}\right)^{1/2}\left(1 + \frac{s^2 a^2}{\mu^2 \beta^2}\right)^{1/2} \\
& + 2\mu\rho s^2 a^2 \beta^2\left(1 + \frac{s^2 a^2}{\mu^2 \alpha_0^2}\right)^{1/2}\left(1 + \frac{s^2 a^2}{\mu^2 \beta^2}\right)^{1/2} \\
& + 2\mu\rho' s^2 a^2 \beta^2\left(1 + \frac{s^2 a^2}{\mu^2 \alpha^2}\right)^{1/2}\left(1 + \frac{s^2 a^2}{\mu^2 \beta^2}\right)^{1/2} \\
& - \rho\beta^4 a^4\left(\frac{s^2}{\beta^2} + \frac{2\mu^2}{a^2}\right)^2\left(1 + \frac{s^2 a^2}{\mu^2 \alpha_0^2}\right)^{1/2} \\
& - 2\mu\rho s^2 a^2 \beta^2\left(1 + \frac{s^2 a^2}{\mu^2 \alpha^2}\right)^{1/2}\left(1 + \frac{s^2 a^2}{\mu^2 \alpha_0^2}\right)^{1/2} \\
& + 4\rho\mu^2\beta^4\left(1 + \frac{s^2 a^2}{\mu^2 \alpha_0^2}\right)^{1/2} - \rho' s^4 a^4\left(1 + \frac{s^2 a^2}{\mu^2 \alpha^2}\right)^{1/2} \\
& + 2\mu\rho' s^2 a^2 \beta^2 \Bigg]
\end{aligned}
$$

$$
\begin{aligned}
& \Bigg[4\rho\mu^2\beta^4\left(1 + \frac{s^2 a^2}{\mu^2 \alpha^2}\right)^{1/2}\left(1 + \frac{s^2 a^2}{\mu^2 \alpha_0^2}\right)^{1/2}\left(1 + \frac{s^2 a^2}{\mu^2 \beta^2}\right)^{1/2} \\
& - 4\rho\mu^4\beta^4\left(1 + \frac{s^2 a^2}{\mu^2 \alpha^2}\right)^{1/2}\left(1 + \frac{s^2 a^2}{\mu^2 \alpha_0^2}\right)^{1/2}\left(1 + \frac{s^2 a^2}{\mu^2 \beta^2}\right)^{1/2} \\
& + 2\mu\rho s^2 a^2 \beta^2\left(1 + \frac{s^2 a^2}{\mu^2 \alpha_0^2}\right)^{1/2}\left(1 + \frac{s^2 a^2}{\mu^2 \beta^2}\right)^{1/2} \\
& + 2\mu\rho' s^2 a^2 \beta^2\left(1 + \frac{s^2 a^2}{\mu^2 \alpha^2}\right)^{1/2}\left(1 + \frac{s^2 a^2}{\mu^2 \beta^2}\right)^{1/2} \\
& + \rho\beta^4 a^4\left(\frac{s^2}{\beta^2} + \frac{2\mu^2}{a^2}\right)^2\left(1 + \frac{s^2 a^2}{\mu^2 \alpha_0^2}\right)^{1/2} \\
& + 2\mu\rho s^2 a^2 \beta^2\left(1 + \frac{s^2 a^2}{\mu^2 \alpha^2}\right)^{1/2}\left(1 + \frac{s^2 a^2}{\mu^2 \alpha_0^2}\right)^{1/2} \\
& - 4\rho\mu^2\beta^4\left(1 + \frac{s^2 a^2}{\mu^2 \alpha_0^2}\right)^{1/2} + \rho' s^4 a^4\left(1 + \frac{s^2 a^2}{\mu^2 \alpha^2}\right)^{1/2} \\
& - 2\mu\rho' s^2 a^2 \beta^2 \Bigg]
\end{aligned}
$$

$$g_2(\mu) = i\mu\theta + 2\left(\mu^2 + \frac{s^2 a^2}{\beta^2}\right)^{1/2} - 2\mu \sinh^{-1}\left(\frac{\mu\beta}{sa}\right)$$

$$- \left(\mu^2 + \frac{s^2 r^2}{\beta^2}\right)^{1/2} + \mu \sinh^{-1}\left(\frac{\mu\beta}{sr}\right) - \left(\mu^2 + \frac{s^2 r_0^2}{\beta^2}\right)^{1/2}$$

$$+ \mu \sinh^{-1}\left(\frac{\mu\beta}{sr_0}\right),$$

$$f_3(\mu) = \frac{2i\beta^2\left(1 + \frac{s^2 a^2}{\mu^2 \alpha^2}\right)^{1/4}\left(1 + \frac{s^2 a^2}{\mu^2 \beta^2}\right)^{1/4}}{\left(1 + \frac{s^2 r^2}{\mu^2 \alpha^2}\right)^{1/4}\left(1 + \frac{s^2 r_0^2}{\mu^2 \beta^2}\right)^{1/4}}$$

$$\times \frac{\left\{(2\mu\rho\beta^2\left(1 + \frac{s^2 a^2}{\mu^2 \alpha_0^2}\right)^{1/2} + \rho' s^2 a^2 - \mu\rho\beta^2 a^2\left(\frac{s^2}{\beta^2} + \frac{2\mu^2}{a^2}\right)\left(1 + \frac{s^2 a^2}{\mu^2 \alpha_0^2}\right)^{1/2}\right\}}{\begin{aligned}&\left\{4\rho\mu^2\beta^4\left(1 + \frac{s^2 a^2}{\mu^2 \alpha^2}\right)^{1/2}\left(1 + \frac{s^2 a^2}{\mu^2 \alpha_0^2}\right)^{1/2}\left(1 + \frac{s^2 a^2}{\mu^2 \beta^2}\right)^{1/2}\right.\\ &- 4\rho\mu^4\beta^4\left(1 + \frac{s^2 a^2}{\mu^2 \alpha^2}\right)^{1/2}\left(1 + \frac{s^2 a^2}{\mu^2 \alpha_0^2}\right)^{1/2}\left(1 + \frac{s^2 a^2}{\mu^2 \beta^2}\right)^{1/2}\\ &+ 2\mu\rho s^2 a^2 \beta^2\left(1 + \frac{s^2 a^2}{\mu^2 \alpha_0^2}\right)^{1/2}\left(1 + \frac{s^2 a^2}{\mu^2 \beta^2}\right)^{1/2}\\ &+ 2\mu\rho' s^2 a^2 \beta^2\left(1 + \frac{s^2 a^2}{\mu^2 \alpha^2}\right)^{1/2}\left(1 + \frac{s^2 a^2}{\mu^2 \beta^2}\right)^{1/2}\\ &+ \rho\beta^4 a^4\left(\frac{s^2}{\beta^2} + \frac{2\mu^2}{a^2}\right)^2\left(1 + \frac{s^2 a^2}{\mu^2 \alpha_0^2}\right)^{1/2}\\ &+ 2\mu\rho s^2 a^2 \beta^2\left(1 + \frac{s^2 a^2}{\mu^2 \alpha^2}\right)^{1/2}\left(1 + \frac{s^2 a^2}{\mu^2 \alpha_0^2}\right)^{1/2}\\ &- 4\rho\mu^2\beta^4\left(1 + \frac{s^2 a^2}{\mu^2 \alpha_0^2}\right)^{1/2} + \rho' s^4 a^4\left(1 + \frac{s^2 a^2}{\mu^2 \alpha^2}\right)^{1/2}\\ &\left.- 2\mu\rho' s^2 a^2\beta^2\right\},\end{aligned}}$$

$$g_3(\mu) = i\mu\theta + \left(\mu^2 + \frac{s^2 a^2}{\alpha^2}\right)^{1/2} - \mu \sinh^{-1}\left(\frac{\mu\alpha}{sa}\right)$$

$$+ \left(\mu^2 + \frac{s^2 a^2}{\beta^2}\right)^{1/2} - \mu \sinh^{-1}\left(\frac{\mu\beta}{sa}\right) - \left(\mu^2 + \frac{s^2 r^2}{\alpha^2}\right)^{1/2}$$

$$+ \mu \sinh^{-1}\left(\frac{\mu\alpha}{sr}\right) - \left(\mu^2 + \frac{s^2 r_0^2}{\beta^2}\right)^{1/2} + \mu \sinh^{-1}\left(\frac{\mu\beta}{sr_0}\right),$$

$$f_4(\mu) = -2i\beta\alpha_0 a^2 \frac{\left(1+\frac{s^2a^2}{\mu^2\beta^2}\right)^{1/4}\left(1+\frac{s^2a^2}{\mu^2\alpha_0^2}\right)^{1/4}}{\left(1+\frac{s^2r^2}{\mu^2\alpha_0^2}\right)^{1/4}\left(1+\frac{s^2r_0^2}{\mu^2\beta^2}\right)^{1/4}}$$

$$\times\{2\mu\alpha\rho^2\beta^2s^2(\mu^2\beta^2+s^2a^2)^{1/2}(\mu^2\alpha_0^2+s^2a^2)^{1/2}$$

$$-2\mu\rho^2s^2\beta^3(\mu^2\alpha^2+s^2a^2)^{1/2}(\mu^2\alpha_0^2+s^2a^2)^{1/2}$$

$$-\rho\rho'\mu\beta\alpha_0a^2s^4(\mu^2\alpha^2+s^2a^2)^{1/2}-2\mu\alpha\rho^2s^2\beta^3$$

$$\times(\mu^2\alpha_0^2+s^2a^2)^{1/2}-\rho\rho'\mu\alpha\alpha_0\beta a^2s^4+2\mu\rho^2\beta^2s^2$$

$$\times(\mu^2\alpha^2+s^2a^2)^{1/2}(\mu^2\beta^2+s^2a^2)^{1/2}(\mu^2\alpha_0^2+s^2a^2)^{1/2}\}$$

$$\{4\rho\beta^3(\mu^2\alpha^2+s^2a^2)^{1/2}(\mu^2\beta^2+s^2a^2)^{1/2}(\mu^2\alpha_0^2+s^2a^2)^{1/2}$$

$$-4\rho\mu^2\beta^3(\mu^2\alpha^2+s^2a^2)^{1/2}(\mu^2\beta^2+s^2a^2)^{1/2}(\mu^2\alpha_0^2+s^2a^2)^{1/2}$$

$$-4\rho\alpha\mu^2\beta^4(\mu^2\alpha_0^2+s^2a^2)^{1/2}-2\rho'\alpha\alpha_0\mu^2s^2a^2\beta^2$$

$$+2\rho\alpha\beta s^2a^2(\mu^2\beta^2+s^2a^2)^{1/2}(\mu^2\alpha_0^2+s^2a^2)^{1/2}$$

$$+2\rho'\alpha_0\beta s^2a^2(\mu^2\alpha^2+s^2a^2)^{1/2}(\mu^2\beta^2+s^2a^2)^{1/2}$$

$$+\rho\alpha a^4\beta^4\left(\frac{s^2}{\beta^2}+\frac{2\mu^2}{a^2}\right)^2(\mu^2\alpha_0^2+s\,a^2)^{1/2}$$

$$+\rho'\alpha_0s^4a^4(\mu^2\alpha^2+s^2a^2)^{1/2}+2\rho\,s^2a^2\beta^2(\mu^2\alpha^2+s^2a^2)^{1/2}$$

$$\{(\mu^2\alpha_0^2+s^2a^2)^{1/2}\}\times\{\rho'\alpha_0s^2a^2+2\rho\beta^2(\mu^2\alpha_0^2+s^2a^2)^{1/2}$$

$$-2\rho\beta(\mu^2\beta^2+s^2a^2)^{1/2}(\mu^2\alpha_0^2+s^2a^2)^{1/2}\},$$

and

$$g_4(\mu) = i\mu\theta+\left(\mu^2+\frac{s^2a^2}{\beta^2}\right)^{1/2}-\mu\sinh^{-1}\left(\frac{\mu\beta}{sa}\right)-\left(\mu^2+\frac{s^2a^2}{\alpha_0^2}\right)^{1/2}$$

$$+\mu\sinh^{-1}\left(\frac{\mu\alpha_0}{sa}\right)+\left(\mu^2+\frac{s^2r^2}{\alpha_0^2}\right)^{1/2}-\mu\sinh^{-1}\left(\frac{\mu\alpha_0}{sr}\right)$$

$$-\left(\mu^2+\frac{s^2r_0^2}{\beta^2}\right)^{1/2}+\mu\sinh^{-1}\left(\frac{\mu\beta}{sr_0}\right). \tag{21}$$

Evaluating the integrals in (18), (19) and (20), we find that the saddle points of all the four integrals are at $i\lambda s$ where

$$\lambda = \frac{r_0\sin\xi}{\beta}.$$

The first integral in (18) has a saddle point μ_0 where μ_0 is the solution of

$$i\theta+\sinh^{-1}\left(\frac{\mu_0\beta}{sr_0}\right)-\sinh^{-1}\left(\frac{\mu_0\beta}{sr}\right)=0 \tag{22}$$

We can solve this equation geometrically. Let S be the source and $P_1(r,\theta)$ a point in the illuminated region (figure 2)

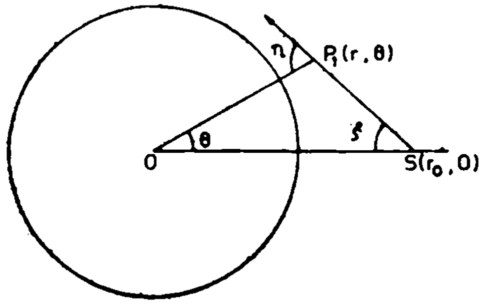

Figure 2. Geometrical interpretation of the saddle point for the incident pulse.

Let

$$\angle OSP_1 = \zeta \text{ and } \pi - \angle OP_1S = \eta$$

then

$$\theta = \eta - \xi, \ \sin\eta = \frac{r_0 \sin\xi}{r} \tag{23}$$

Now $\sinh^{-1}\left(\frac{\mu_0 \beta}{s r_0}\right) = i\xi$ defines the solution of (22) because from (23) we have

$$\sinh^{-1}\left(\frac{\mu_0 \beta}{s r}\right) = i\eta \tag{24}$$

and therefore (22) reduces to the first equation of (23). Thus we see that the saddle point μ_0 of the first integral in (18) can be associated with the incident ray.

At the saddle point μ_0 where

$$\mu_0 = i \frac{r_0 \sin\xi}{\beta} s,$$

we have

$$f_1(\mu_0) = \beta / \{2(r_0 \cos\xi)^{1/2} (r^2 - r_0^2 \sin^2\xi)^{1/4} s$$

$$g_1(\mu_0) = -\frac{s}{\beta} R_1.$$

$$g_1''(\mu_0) = -\beta R_1 / \{r_0 \cos\xi (r^2 - r_0^2 \sin^2\xi)^{1/2} s\} \tag{25}$$

where $R_1 = r_0 \cos\xi - r\cos\eta$ is the distance between the source and the receiver on the incident ray path. We find that the line of steepest descent through the saddle point is parallel to the real axis in the μ-plane since $g_1''(\mu_0)$ is negative. Therefore performing saddle point integration and using (25) we obtain (Jeffreys and Jeffreys 1956)

$$\int_{-\infty}^{\infty} f_1(\mu) \exp\{g_1(\mu)\}\, d\mu \sim \left(\frac{\pi\beta}{2R_1 s}\right)^{1/2} \exp - (sR_1)/\beta. \tag{26}$$

The saddle point μ_0 of the second integral in (18) is the solution of

$$i\theta - 2\sinh^{-1}\left(\frac{\mu_0\beta}{sa}\right) + \sinh^{-1}\left(\frac{\mu_0\beta}{sr_0}\right) + \sinh^{-1}\left(\frac{\mu_0\beta}{sr}\right) = 0. \tag{27}$$

We will interpret this geometrically. Let $S(r_0, 0)$ be the source and $P_2(r, \theta)$ be a point in the illuminated zone (figure 3).

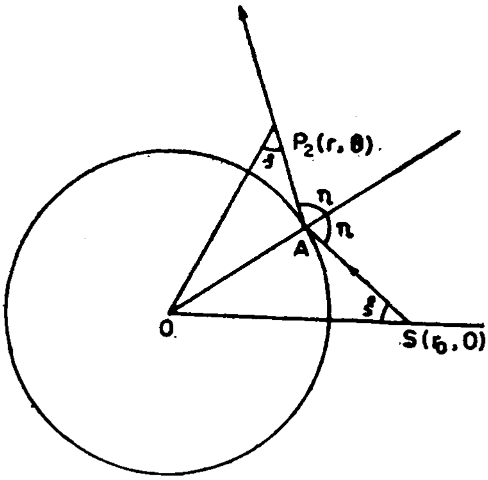

Figure 3. Geometrical interpretation of the saddle point for the reflected 'S' pulse.

Let $\angle OSA = \xi$, $\pi - \angle OAP_2 = \eta = \pi - \angle OAS$
and $\angle OP_2A = \zeta$ (28)
then

$$\theta = 2\eta - \xi - \zeta \quad \sin\eta = \frac{r_0 \sin\xi}{a}, \quad \sin\zeta = \frac{a}{r}\sin\eta \tag{29}$$

Hence we can solve (27) by setting

$$\sinh^{-1}\frac{\mu_0\beta}{sr_0} = i\xi, \quad \sinh^{-1}\frac{\mu_0\beta}{sa} = i\eta, \quad \sinh^{-1}\frac{\mu_0\beta}{sr} = i\zeta. \tag{30}$$

Therefore the saddle point μ_0 of the second integral in (18) can be associated with the reflected S ray. At the saddle point we have

$$f_2(\mu_0) = \frac{\beta}{2s(rr_0\cos\xi\cos\zeta)^{1/2}} \times \frac{\{4p\beta\sin^2\eta\cos\eta(1 - n^2\sin^2\eta)^{1/2}(1 - n_1^2\sin^2\eta)^{1/2} - \rho a(1 - 2\sin^2\eta)^2(1 - n_1^2\sin^2\eta)^{1/2} - \rho' a_0(1 - n^2\sin^2\eta)^{1/2}\}}{\{4p\beta\sin^2\eta\cos\eta(1 - n^2\sin^2\eta)^{1/2}(1 - n_1^2\sin^2\eta)^{1/2} + \rho a(1 - 2\sin^2\eta)^2(1 - n_1^2\sin^2\eta)^{1/2} + \rho' a_0(1 - n^2\sin^2\eta)^{1/2}\}},$$

$$g_2(\mu_0) = -\left(\frac{R_2 + R_3}{\beta}\right)s,$$

$$g_2''(\mu_0) = -\frac{\beta(rR_2\cos\zeta + r_0R_3\cos\xi)}{sarr_0\cos\xi\cos\eta\cos\zeta} \tag{31}$$

where $n = \alpha/\beta$,

$n_1 = \alpha_0/\beta$,

$R_2 = r_0 \cos\xi - a\cos\eta = SA$

$R_3 = r\cos\zeta - a\cos\eta = AP_3$.

Performing saddle point integration and using (31) we obtain

$$\int_{-\infty}^{\infty} f_3(\mu)\exp\{g_3(\mu)\}\,d\mu \sim$$

$$\frac{\{4p\beta\sin^2\eta\cos\eta(1-n^2\sin^2\eta)^{1/2}(1-n_1^2\sin^2\eta)^{1/2} - p\alpha(1-2\sin^2\eta)^2(1-n_1^2\sin^2\eta)^{1/2} - p'\alpha_0(1-n^2\sin^2\eta)^{1/2}\}}{\{4p\beta\sin^2\eta\cos\eta(1-n^2\sin^2\eta)^{1/2}(1-n_1^2\sin^2\eta)^{1/2} + p\alpha(1-2\sin^2\eta)^2(1-n_1^2\sin^2\eta)^{1/2} + p'\alpha_0(1-n^2\sin^2\eta^{1/2}\}},$$

$$\times\left\{\frac{\pi\alpha\beta\cos\eta}{2s(rR_2\cos\zeta + r_0R_3\cos\xi)}\right\}^{1/2}\exp-\left(\frac{R_2+R_3}{\beta}\right)s. \qquad (32)$$

Now it remains to evaluate the integrals in (19) and (20). As before we see that the saddle point μ_0 of the integral in (19) is the solution of

$$i\theta + \sinh^{-1}\left(\frac{\mu_0\alpha}{sr}\right) + \sinh^{-1}\left(\frac{\mu_0\beta}{sr_0}\right) - \sinh^{-1}\left(\frac{\mu_0.\alpha}{sa}\right) - \sinh^{-1}\left(\frac{(\mu_0\beta}{sa}\right) = 0. \qquad (33)$$

This again can be solved geometrically (figure 4). Let $S(r_0, 0)$ be the source, SA the incident ray and AP_3 the reflected P ray. Let the coordinates of the point P_3 be (r, θ). Let

$$\angle OSA = \xi, \pi - \angle OAS = \eta, \pi - \angle OAP_3 = \delta', \quad \angle OP_3A = \zeta' \qquad (34)$$

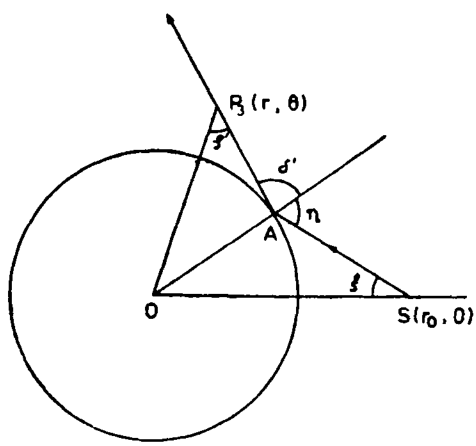

Figure 4. Geometrical interpretation of the saddle point for the reflected 'P' pulse.

then

$$\theta = \eta + \delta' - \xi - \zeta', \quad r_0 \sin \xi = a \sin \eta, r \sin \zeta' = a \sin \delta',$$

$$\sin \eta = \beta/\alpha \sin \delta'. \tag{35}$$

Thus the saddle point μ_0 of (19) is related to the reflected P pulse. Hence performing saddle point integrating as before one obtains

$$\int_{-\infty}^{\infty} f_3(\mu) \exp \{g_3(\mu)\} \, d\mu$$

$$\sim \frac{\{4\rho\alpha \sin \eta \cos \eta \, (1 - n_1{}^2 \sin^2 \eta)^{1/2} \; (1 - 2 \sin^2 \eta)\}}{\begin{aligned}\{4\rho\beta \sin^2 \eta \cos \eta \, (1 - n^2 \sin^2 \eta)^{1/2} (1 - n_1^2 \sin^2 \eta)^{1/2} \\ + \rho\alpha (1 - 2\sin^2 \eta)^2 (1 - n_1^2 \sin^2 \eta)^{1/2} + \rho' \alpha_0 (1 - n^2 \sin^2 \eta)^{1/2}\}\end{aligned}}$$

$$\times \left\{ \frac{\pi a \, \beta^2 \cos^2 \delta'}{2s\,(\beta R_3 \, r \cos \delta' \cos \zeta' + \alpha \, R_4 r_0 \cos \xi \cos \eta)} \right\}^{1/2}$$

$$\times \exp - \left(\frac{R_3}{\beta} + \frac{R_4}{\alpha} \right) s \tag{36}$$

where

$$R_3 = r_0 \cos \xi - a \cos \eta$$

$$R_4 = r \cos \zeta' - a \cos \delta'$$

The saddle point μ_0 of the integral in (20) is the solution of

$$i\theta + \sinh^{-1}\left(\frac{\mu_0 \beta}{s r_0}\right) + \sinh^{-1}\left(\frac{\mu_0 \alpha_0}{s a}\right)$$

$$- \sinh^{-1}\left(\frac{\mu_0 \alpha_0}{s r}\right) - \sinh^{-1}\left(\frac{\mu_0 \beta}{s a}\right) = 0. \tag{37}$$

Let $S(r_0, 0)$ be the source, SA the incident ray and AP_4 the refracted P ray Let the coordinates of P_4 be (r, θ) (figure 5). Let

$$\angle OSA = \xi, \; \pi - \angle OAS = \eta, \; \pi - \angle OP_4A = \zeta' \; \angle OAP_4 = \delta' \tag{38}$$

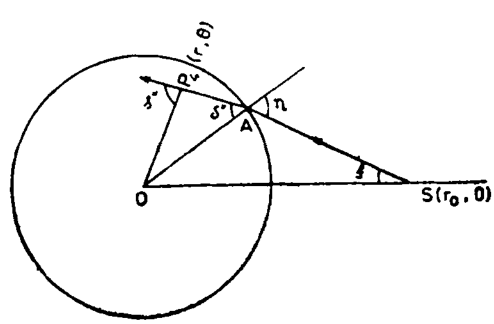

Figure 5. Geometrical interpretation of the saddle point for the refracted 'P' pulse

then

$$\theta = \eta + \zeta'' - \xi - \delta'', \ \sin\eta = \frac{r_0 \sin\xi}{a}, \sin\delta'' = \alpha_0/\beta \sin\eta,$$

$$\sin\zeta'' = a/r \sin\delta''. \tag{39}$$

Therefore (37) can be solved by putting

$$\sinh^{-1}\left(\frac{\mu_0\beta}{sr_0}\right) = i\xi, \ \sinh^{-1}\left(\frac{\mu_0\alpha_0}{sa}\right) = i\delta''$$

$$\sinh^{-1}\left(\frac{\mu_0\alpha_0}{sr}\right) = i\zeta'', \ \sinh^{-1}\left(\frac{\mu_0\beta}{sa}\right) = i\eta. \tag{40}$$

Hence the saddle point μ_0 of (20) is related to the refracted P pulse. Therefore if we approximate (20) by the method of steepest descents, we obtain.

$$\int_{-\infty}^{\infty} f_4(\mu) \exp\{g_4(\mu)\}\, d\mu \sim$$

$$-\frac{\{4\rho\alpha_0 \sin\eta\cos\eta\,(1 - n^2\sin^2\eta)^{1/2}\}}{\{4\rho\beta\sin^2\eta\cos\eta\,(1 - n^2\sin^2\eta)^{1/2}(1 - n_1^2\sin^2\eta)^{1/2} + \rho\alpha(1 - 2\sin^2\eta)^2(1 - n_1^2\sin^2\eta)^{1/2} + \rho'\alpha_0(1 - n^2\sin^2\eta)^{1/2}\}}$$

$$\times \left\{\frac{\pi\alpha\beta^2\cos^2\delta''}{2s\,(\beta R_2 r\cos\zeta''\cos\delta'' + \alpha_0 R_5 r_0\cos\xi\cos\eta)}\right\}^{1/2}$$

$$\times \exp -\left(\frac{R_2}{\beta} + \frac{R_5}{\alpha_0}\right)s \tag{41}$$

where

$$R_5 = a\cos\delta'' - r\cos\zeta'' = AP_4.$$

Substituting the results (26) and (32) in (18) we get

$$\bar{\psi}(r,\theta,s) \sim \left(\frac{\pi\beta}{2R_1 s}\right)^{1/2} \exp -(st_1)$$

$$+\frac{\{4\rho\beta\sin^2\eta\cos\eta\,(1 - n^2\sin^2\eta)^{1/2}(1 - n_1^2\sin^2\eta)^{1/2} - \rho\alpha(1 - 2\sin^2\eta)^2(1 - n_1^2\sin^2\eta)^{1/2} - \rho'\alpha_0(1 - n^2\sin^2\eta)^{1/2}\}}{\{4\rho\beta\sin^2\eta\cos\eta\,(1 - n^2\sin^2\eta)^{1/2}(1 - n_1^2\sin^2\eta)^{1/2} + \rho\alpha(1 - 2\sin^2\eta)^2(1 - n_1^2\sin^2\eta)^{1/2} + \rho'\alpha_0(1 - n^2\sin^2\eta)^{1/2}\}}$$

$$\times \left\{\frac{\pi\alpha\beta\cos\eta}{2s\,(r R_2\cos\zeta + r_0 R_3\cos\xi)}\right\}^{1/2} \exp -(st_2),$$

$$(r_0 \geqslant r \geqslant a) \tag{42}$$

where t_1 and t_2 are respectively the arrival times of the incident and reflected S pulses at the point (r,θ).

If we use (36) and (41) in (19) and (20) respectively, we get

$$\bar{\Phi}(r,\theta,s) \sim \frac{4\rho\alpha\sin\eta\cos\eta\,(1 - n_1^2\sin^2\eta)^{1/2}(1 - 2\sin^2\eta)}{\{4\rho\beta\sin^2\eta\cos\eta\,(1 - n^2\sin^2\eta)^{1/2}(1 - n_1^2\sin^2\eta)^{1/2} + \rho\alpha(1 - 2\sin^2\eta)^2(1 - n_1^2\sin^2\eta)^{1/2} + \rho'\alpha_0(1 - n^2\sin^2\eta)^{1/2}\}}$$

$$\times \left\{\frac{\pi a\beta^2 \cos^2 \delta'}{2s(\beta R_2 r \cos\zeta' \cos\delta' + aR_4 r_0 \cos\xi \cos\eta)}\right\}^{1/2}$$

$$\times \exp - (st_3), (r \geqslant a) \tag{43}$$

$$\bar{\Phi}_0(r,\theta,s) \sim -\frac{4\rho a_0 \sin\eta\cos\eta(1-n^2\sin^2\eta)^{1/2}}{\{4\rho\beta\sin^2\eta\cos\eta(1-n^2\sin^2\eta)^{1/2}(1-n_1^2\sin^2\eta)^{1/2}}$$

$$+ \rho a(1-2\sin^2\eta)^2(1-n_1{}^2\sin^2\eta)^{1/2} + \rho' a_0(1-n^2\sin^2\eta)^{1/2}\}$$

$$\times \left\{\frac{\pi a\beta^2\cos^2\delta''}{2s(\beta R_2 r\cos\zeta''\cos\delta'' + a_0R_5 r_0\cos\xi\cos\eta)}\right\}^{1/2}$$

$$\times \exp - (st_4), \qquad (r \leqslant a) \tag{44}$$

where t_3 and t_4 are respectively the arrival times of the reflected and refracted P pulses at the point (r, θ).

Now, we can obtain the short-time approximations for the solution by performing Laplace inversion. We use the known result of Laplace inversion (Churchill (1958)

$$\frac{1}{2\pi i}\int_{\epsilon-i\infty}^{\epsilon+i\infty} s^{-\mu}\exp\{(t-T)s\}\,ds = \frac{(t-T)^{\mu-1}}{\Gamma(\mu)}H(t-T) \tag{45}$$

where $\mu > 0$ and Γ stands for Gamma function.
Thus we have

$$\psi(r,\theta,t) \sim \frac{1}{\{2t_1(t-t_1)\}^{1/2}}H(t-t_1)$$

$$\{4\rho\beta\sin^2\eta\cos\eta(1-n^2\sin^2\eta)^{1/2}(1-n_1{}^2\sin^2\eta)^{1/2}$$

$$+\frac{-\rho a(1-2\sin^2\eta)^2(1-n_1^2\sin^2\eta)^{1/2} - \rho' a_0(1-n^2\sin^2\eta)^{1/2}\}}{\{4\rho\beta\sin^2\eta\cos\eta(1-n^2\sin^2\eta)^{1/2}(1-n_1^2\sin\eta)^{1/2}}$$

$$+ \rho a(1-2\sin^2\eta)^2(1-n_1^2\sin^2\eta)^{1/2} + \rho' a_0(1-n^2\sin^2\eta)^{1/2}\}$$

$$\times \left\{\frac{a\beta\cos\eta}{2(rR_2\cos\zeta + r_0R_3\cos\xi)}\right\}^{1/2}\frac{H(t-t_2)}{(t-t_2)^{1/2}}, \qquad (r_0 \geqslant r \geqslant a) \tag{46}$$

$$\Phi(r,\theta,t) \sim \frac{4\rho a\sin\eta\cos\eta(1-n_1{}^2\sin^2\eta)^{1/2}(1-2\sin^2\eta)}{\{4\rho\beta\sin^2\eta\cos\eta(1-n^2\sin^2\eta)^{1/2}(1-n_1^2\sin^2\eta)^{1/2}}$$

$$+ \rho a(1-2\sin^2\eta)^2(1-n_1^2\sin^2\eta)^{1/2} + \rho' a_0(1-n^2\sin^2\eta)^{1/2}\}$$

$$\times \left\{\frac{a\beta^2\cos^2\delta'}{2(\beta R_2 r\cos\zeta'\cos\delta' + aR_4 r_0\cos\xi\cos\eta)}\right\}^{1/2}$$

$$\times \frac{H(t-t_3)}{(t-t_3)^{1/2}}, \qquad (r \geqslant a) \tag{47}$$

$$\Phi_0(r,\theta,t) \sim -\frac{4\rho a_0\sin\eta\cos\eta(1-n^2\sin^2\eta)^{1/2}}{\{4\rho\beta\sin^2\eta\cos\eta(1-n^2\sin^2\eta)^{1/2}(1-n_1^2\sin^2\eta)^{1/2}}$$

$$+ \rho a(1-2\sin^2\eta)^2(1-n_1^2\sin^2\eta)^{1/2} + \rho' a_0(1-n^2\sin^2\eta)^{1/2}\}$$

$$\times \left\{\frac{a\beta^2 \cos^2 \delta''}{2\,(\beta R_2 r \cos \zeta'' \cos \delta'' + a_0 R_5 r_0 \cos \xi \cos \eta)}\right\}^{1/2}$$

$$\times \frac{H(t-t_4)}{(t-t_4)^{1/2}}, \qquad (r \leqslant a) \tag{48}$$

We can physically interpret the solutions (46), (47) and (48) in the following way (Friedlander 1958). The first term in (46) represents the incident pulse. The second term in (46) represents the reflected sv pulse. Its first factor is the reflection coefficient for a plane boundary (Brekhovshikh 1960). The second and the third factors may be interpreted as the divergence and source factors respectively. The approximation for the reflected P pulse is given by (47). Its first factor is the reflection coefficient for a plane boundary (Brekhovshikh 1960). The second may be interpreted as the divergence factor due to the curved surface and the third as the source term. Similarly (48) represents the refracted P pulse. The first factor is the refraction coefficient for a plane boundary (Brekhovshikh 1960) and the second and third factors may be interpreted as the divergence and source terms respectively.

5. Conclusion

We have obtained the solution and interpreted them in terms of geometrical optics as the incident, reflected and refracted pulses. We note that the arrival time of the reflected pulses is the same whether the obstacle is a rigid, weak or fluid cylinder. However, we have not obtained the pulses associated with multiple reflection and refraction as it becomes very lengthy. We expect to discuss this case separately.

Acknowledgement

The authors are grateful to the authorities of the Indian School of Mines, Dhanbad, for the financial support and other facilities provided during the preparation of this paper.

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