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# Temperature distribution in a laminar plane wall jet due to variably heated wall

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**Abstract.** The heat transfer in a laminar incompressible plane wall jet due to a variably heated wall has been studied. It is assumed that the difference of temperatures between the wall and the issuing jet is inversely proportional to an arbitrary exponent of the distance from the slit. A similar solution of the energy equation is possible. The solutions, for arbitrary values of the Prandtl number and of the exponent are obtained. It is found that in some cases the heat transfer at the wall may become zero or negative.

**Keywords.** Jet flow; viscous incompressible flow; heat transfer.

## List of symbols

| | |
|---|---|
| $c_p$ | specific heat at constant pressure |
| $\mathrm{Nu}(x)$ | local nusselt number |
| $\mathrm{Re}_x$ | local Reynolds number |
| $T$ | temperature of the fluid in the boundary layer |
| $T_w(x)$ | temperature distribution (function of $x$) at the wall |
| $T_\infty$ | temperature of the fluid at rest |
| $u, v$ | velocity components along and normal to the plane wall respectively |
| $x, y$ | rectangular coordinates along and normal to the plane wall respectively |
| $\eta$ | similarity variable |
| $\theta$ | difference temperature in boundary layer $= (T - T_\infty)$ |
| $\chi$ | coefficient of thermal conductivity |
| $\nu$ | coefficient of kinematic viscosity |
| $\sigma$ | Prandtl number |
| $\mu$ | coefficient of viscosity |

## 1. Introduction

The velocity distribution in a laminar wall jet was investigated by Akatnow [1] and Glauert [5] and the corresponding temperature distributions with and without frictional heat were obtained by Tak and Bansal [8] and Bansal and Tak [3] respectively. The effects of compressibility on a laminar radial wall jet was studied numerically by Riley [6]. Bansal and Tak [4] considered the effects of compressibility on a plane wall jet.

In the present paper the heat transfer in a laminar incompressible plane wall jet due to a variably heated wall has been studied. It is assumed that the difference of temperatures between the wall and the issuing jet is inversely proportional to an arbitrary exponent $n$ of distance $x$ from the slit ($T_w - T_\infty \propto x^{-n}$). A similarity solution of the energy equation is possible and the resulting ordinary linear differential equation is reduced to a hypergeometric equation by a proper transformation of the similarity variable. The solutions, for arbitrary values of Prandtl number and the exponent $n$ are obtained. It is found that in some cases the heat transfer at the wall may become zero or even negative.

## 2. Formulation of the problem

Let an incompressible fluid be discharged through a narrow slit at a temperature $T_\infty$ in half space along a plane wall and mix with the same surrounding fluid which is at rest and has the same temperature $T_\infty$. Taking the origin in the slit and the coordinate axes $x$ and $y$ along and normal to the plane wall respectively: the velocity and thermal boundary layer equations, from Schlichting [7], are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

and

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{\nu}{\sigma}\frac{\partial^2 \theta}{\partial y^2}, \tag{3}$$

where

$$\theta = T - T_\infty, \quad \sigma = \frac{\mu c_p}{\chi} \text{ (Prandtl number)}, \tag{4}$$

and the other symbols have their usual meanings.
Let the temperature of the wall be $T_w(x)$ and is given by

$$T_w(x) = T_\infty + c\sqrt{\frac{E}{\nu}}\, x^{-n}, \tag{5}$$

where $c$ is a constant, $n$ can have an assigned value, and $E$ is the exterior momentum flux defined by (7).

*Boundary conditions*

$$y = 0 : u = 0,\ v = 0;\ \theta = c\sqrt{\frac{E}{\nu}}\,x^{-n},$$

$$y = \infty : u = 0; \qquad \theta = 0, \tag{6}$$

Further, $\int\limits_0^\infty u^2 \left( \int\limits_0^y u\, dy \right) dy = E$ (const.). (7)

## 3. Analysis

An exact solution of the velocity boundary layer may be found in the book by Bansal [2] and the results, which we shall require, are as follows:

$$u = \sqrt{\frac{E}{\nu x}}\, f'(\eta),$$

$$v = \frac{1}{4}\left(\frac{\nu E}{x^3}\right)^{1/4} \{3\eta f'(\eta) - f(\eta)\},$$

$$\eta = \left(\frac{E}{\nu^3}\right)^{1/4} y x^{-3/4} \text{ (similarity variable)}, \tag{8}$$

where a prime denotes differentiation with respect to $\eta$ and the function $f(\eta)$ satisfies the following equation :

$$4f''' + ff'' + 2f'^2 = 0, \tag{9}$$

with the boundary conditions

$$\eta = 0 : f = 0,\ f' = 0,$$

$$\eta = \infty :\ f' = 0, \tag{10}$$

and integral condition

$$\int\limits_0^\infty ff'^2\, d\eta = 1, \tag{11}$$

whose solution is given by

$$f(\eta) = f_\infty F(\eta), \tag{12}$$

such that $$\eta = \frac{2}{f_\infty}\left[ \ln \frac{1 + \sqrt{F} + F}{(1 - \sqrt{F})^2} + 2\sqrt{3} \operatorname{arc\,tg} \frac{\sqrt{3F}}{2 + \sqrt{F}} \right], \tag{13}$$

$$f_\infty = (40)^{1/4} = 2{\cdot}515. \tag{14}$$

To solve the energy equation, we take

$$\theta = c\sqrt{\frac{E}{\nu}}\,x^{-n}\, h(\eta). \tag{15}$$

Equation (3) then becomes

$$4h'' + \sigma (fh' + 4nf'h) = 0, \tag{16}$$

with the boundary conditions:

$$\eta = 0 : \ h = 1,$$

$$\eta = \infty : \ h = 0. \tag{17}$$

Without restricting the scope of the Prandtl number $\sigma$ and the exponent $n$, we apply the following transformation of the independent variable:

$$s = [F(\eta)]^{3/2}. \tag{18}$$

Equation (16) becomes

$$s(1-s)\frac{d^2h}{ds^2} + \left\{\frac{2}{3} - \left(\frac{2}{3} - \sigma + 1\right)s\right\}\frac{dh}{ds} + \frac{8n}{3}\sigma h = 0, \tag{19}$$

with the boundary conditions

$$s = 0 : h = 1,$$

$$s = 1 : h = 0. \tag{20}$$

Equation (19) is a hypergeometric equation, whose solution is given by

$$h = A_2F_1(\alpha, \beta \ ; \ \gamma \ ; \ s) + Bs^{1-\gamma} {}_2F_1 \ (\alpha - \gamma + 1, \ \beta - \gamma + 1 \ ; \ 2 - \gamma \ ; \ s), \tag{21}$$

where $\alpha + \beta = \frac{2}{3} - \sigma, \ \ \alpha\beta = -\frac{8}{3}n\sigma, \ \ \gamma = \frac{2}{3}.$ (22)

The constants $A$ and $B$ are determined with the help of boundary conditions (20) as

$$A = 1, B = -\frac{{}_2F_1(\alpha, \ \beta; \ \gamma; \ 1)}{{}_2F_1(\alpha - \gamma + 1, \beta - \gamma + 1; \ 2 - \gamma; \ 1)}, \tag{23}$$

finally, $h = {}_2F_1(\alpha, \beta; \ \gamma; \ s) - s^{1/3}\dfrac{{}_2F_1(\alpha, \beta; \ \gamma; \ 1)}{{}_2F_1(\alpha - \gamma + 1, \ \beta - \gamma + 1; \ 2 - \gamma; \ 1)}$

$$\times \, {}_2F_1(\alpha - \gamma + 1, \ \beta - \gamma + 1; \ 2 - \gamma; \ s). \tag{24}$$

Equation (24) represents a general solution from which, by substituting the desired value of $n$, the required solution can be obtained for arbitrary values of the Prandtl number $\sigma$, except for $n = (1 + 3\sigma)/8\sigma$ when the boundary conditions are not compatible with the solution (21).

## 4. Heat transfer

The local Nusselt number for the heat transfer at the wall, from (4), (5), (18) and (24) is given by

$$\mathrm{Nu}\,(x) = -\frac{\left(\frac{\partial T}{\partial y}\right)_{y=0}}{(T_w - T_\infty)}\, x = -\left(\frac{Ex}{\nu^3}\right)^{1/4} \frac{f_\infty B}{12},$$

$$= -\frac{f_\infty B}{12} (\mathrm{Re}_x)^{1/4}, \tag{25}$$

where $\mathrm{Re}_x$ is local Reynolds number and is defined by

$$\mathrm{Re}_x = \frac{Ex}{\nu^3}. \tag{26}$$

## 5. Numerical discussion

In figure 1, the function $h(\eta)$, which is proportional to the temperature difference $(T - T_\infty)$, is plotted against the similarity variable $\eta$ for various values of Prandtl

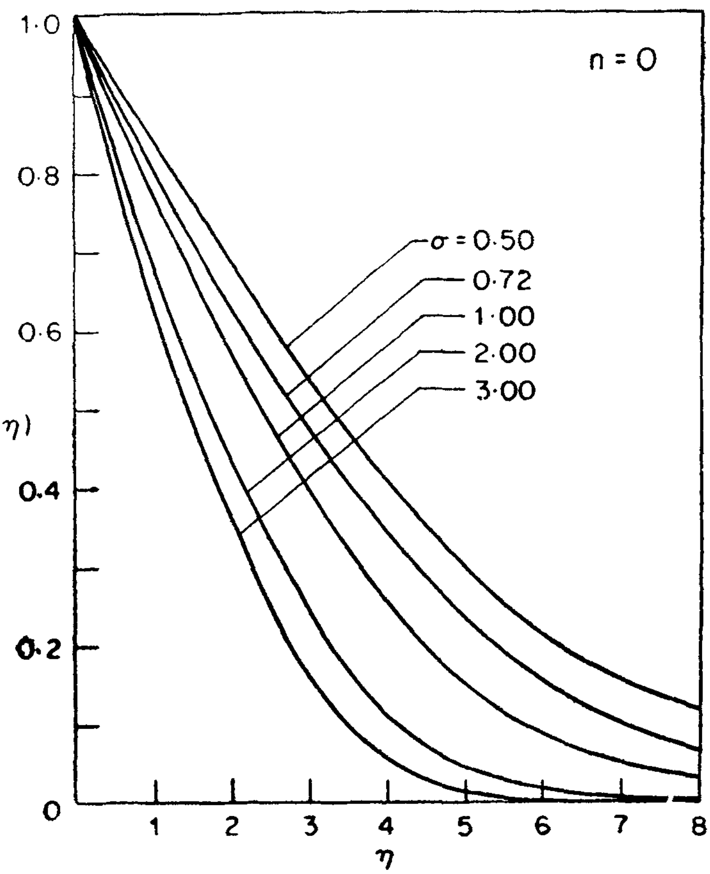


**Figure 1.** Temperature distribution plotted against the similarity variable $\eta$ in a plane wall jet (wall at constant temperature).

number $\sigma$ in the case of constant wall temperature ($n = 0$). It may be noted that thermal boundary layer decreases with the increase in the Prandtl number $\sigma$.

In figures 2 to 5, the function $h(\eta)$ is plotted against $\eta$ for the cases $n = \frac{1}{4}$ $\frac{3}{8}$, $\frac{1}{2}$ and 1 respectively for various values of the Prandtl number $\sigma$. It may be pointed out that the variation of the wall temperature has a strong effect on the temperature profiles and the profiles assume $S$ shape. Thermal boundary layers decrease with the increase in the Prandtl number as in the case of constant wall temperature. In the case of $n = \frac{1}{2}$ and 1, the solution of energy equation is not compatible with the boundary conditions if the Prandtl number is taken as unity and $\frac{1}{8}$ respectively.

For the heat transfer at the plate the ratio of the local Nusselt number and Reynolds number given by (25), for different values of $n$ and $\sigma$, is tabulated in table 1. It may be noted that in the case of constant wall temperature ($n = 0$), the heat transfer from the plate to fluid increases with the increase in the Prandtl number. When $n = \frac{1}{4}$, the transfer of heat is zero and the wall behaves as an adiabatic wall. Further, it may be pointed out that in certain cases for $n > 0$, the Nusselt number may become negative, i.e., even if the local wall temperature is greater than the fluid temperature, the transfer of heat takes place from fluid to plate. This phenomenon can be explained in the following way. A positive value of $n$ means a negative exponent of $x$ and therefore, the wall temperature decreases in flow direction, as a consequence, the fluid layers in the immediate

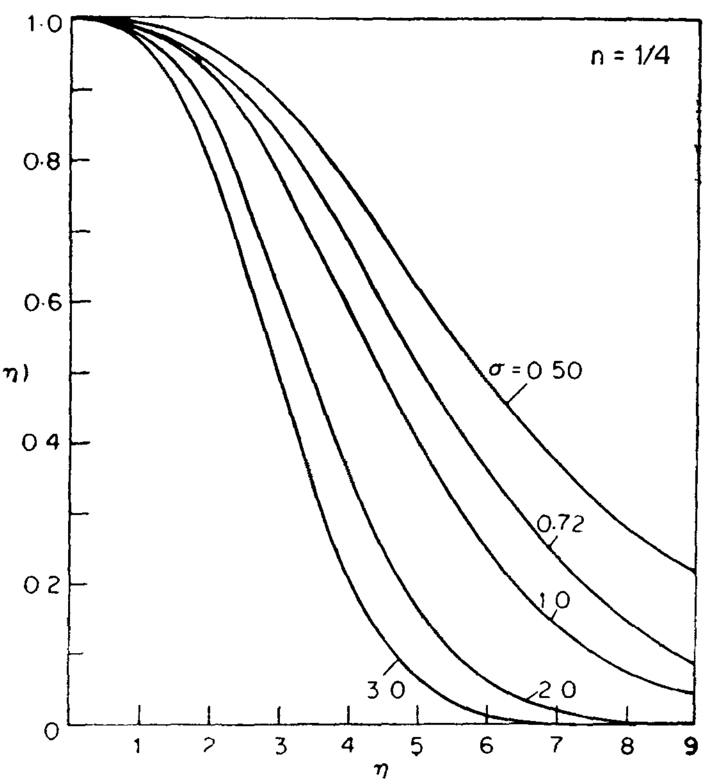


**Figure 2.** Temperature distribution plotted against the similarity variable $\eta$ in a plane wall jet (wall temperature varies as $x^{-1/4}$).

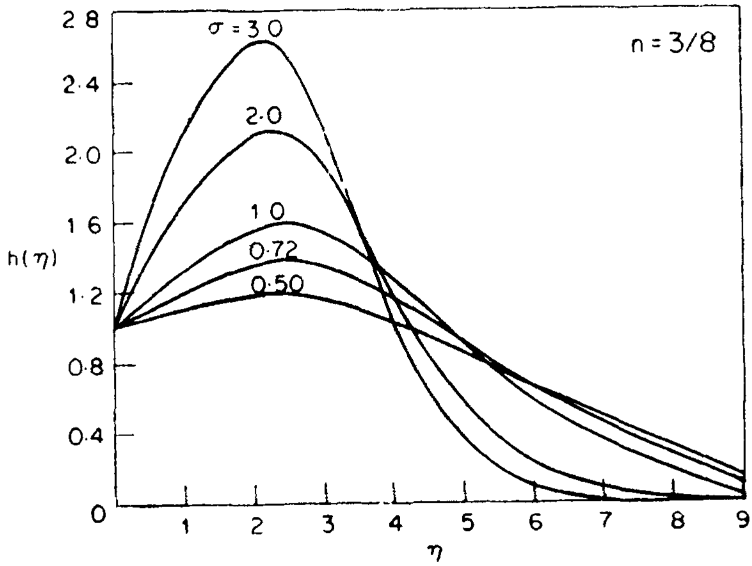


**Figure 3.** Temperature distribution plotted against the similarity variable $\eta$ in a plane wall jet (wall temperature varies as $x^{-3/8}$).

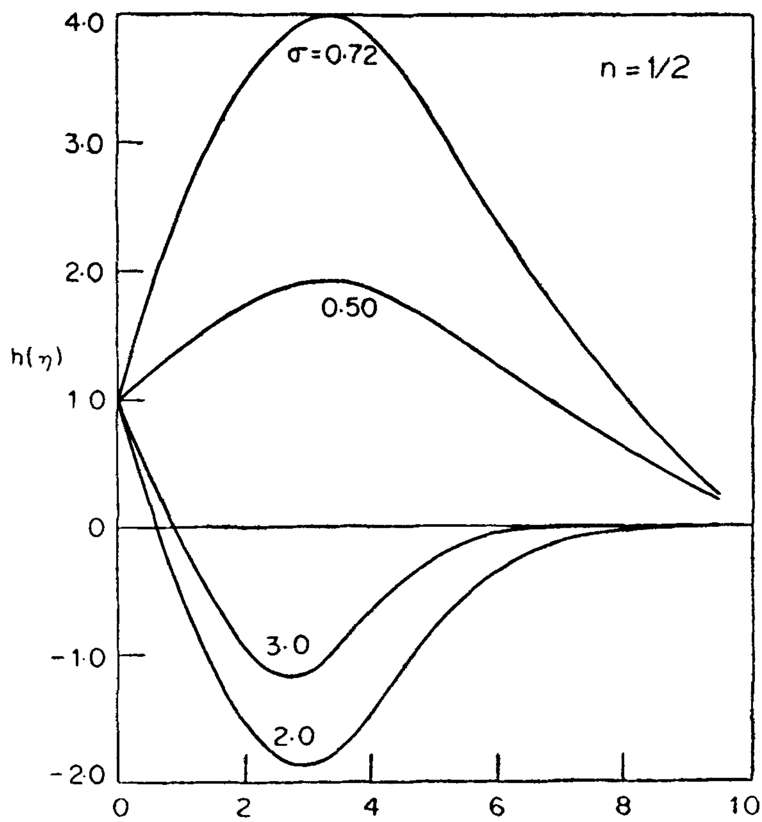


**Figure 4.** Temperature distribution plotted against the similarity variable $\eta$ in a plane wall jet (wall temperature varies as $x^{-1/2}$).

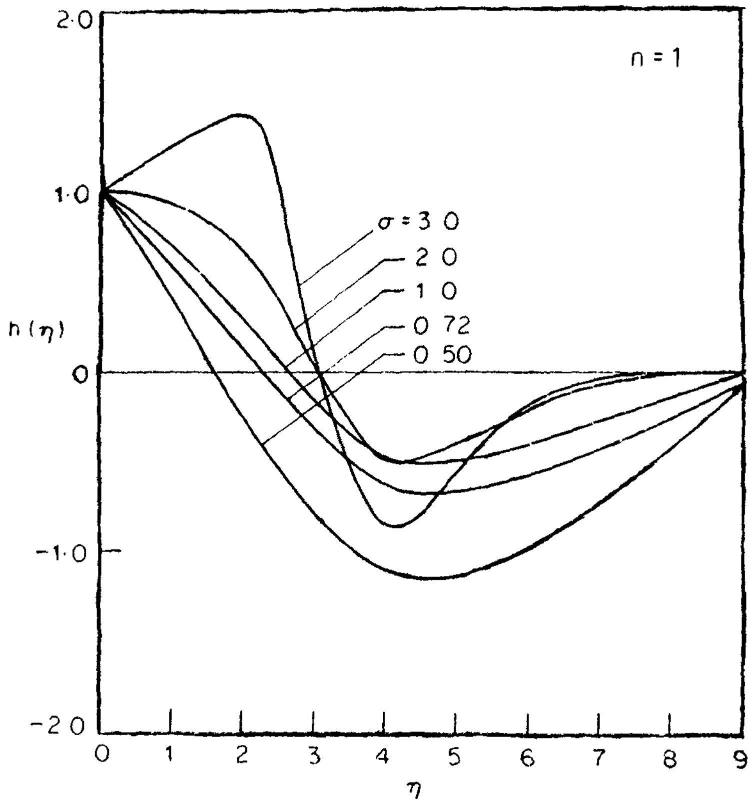


**Figure 5.** Temperature distribution plotted against the similarity variable $\eta$ in a plane wall jet. (wall temperature varies as $x^{-1}$).

**Table 1.** Nusselt number for arbitrary values of $n$ and $\sigma$ $\left[\frac{Nu_x}{(Re_x)^{1/4}} = -\frac{Bf_\infty}{12}\right]$

| $\sigma$ | $n = 0$ | $n = \frac{1}{4}$ | $n = \frac{3}{8}$ | $n = \frac{1}{2}$ | $n = 1$ |
|---|---|---|---|---|---|
| 0·50 | 0·153573 | 0 | −0·121694 | −0·440738 | 0·596850 |
| 0·72 | 0·180777 | 0 | −0·227214 | −1·352592 | 0·376508 |
| 0·80 | 0·189634 | 0 | −0·264551 | −2·199160 | 0·332559 |
| 1·00 | 0·209583 | 0 | −0·353088 | .. | 0·246216 |
| 2·00 | 0·279444 | 0 | −0·769496 | 1·349788 | −0·160709 |
| 3·00 | 0·326016 | 0 | −1·235733 | 1·045344 | −1·214241 |

neighbour of the wall at any location come from up stream regions where they were in contact with a hotter wall. They carry this temperature down stream and causes the fluid near the wall to be hotter than the wall itself, and it is this situation which finally causes a heat flow from the fluid to the wall.

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