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Wave propagation in piezoelectric medium of hexagonal symmetry

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Abstract. The wave propagation at large distances from a source of disturbance (isolated harmonic electric charge or body force of fixed frequency) in an infinite piezoelectric medium belonging to classes (6), (6 m m) or ceramic (α m) and (6 2 2) is discussed by means of asymptotic evaluation (at large distances) of Fourier integrals. Numerical results are given for the (6 m m) crystal class using the constants of cadmium selenide crystal.

Keywords. Wave surface; wave propagation; Fourier integrals; piezoelectric medium.

1. Introduction

The geometry of wave propagation in elastic medium has been investigated by Musgrave [4], Synge [6] and others. The asymptotic behaviour (at large distances) of solutions of linear partial differential equations with constant coefficients has been demonstrated by Lighthill [3]. He has also studied the geometry of wave propagation using the asymptotic evaluation of Fourier integrals in many dimensions for large arguments. Buchwald [2] has discussed the wave propagation at large distances in elastic anisotropic medium due to radiation from a source of body force. The ‘ Forcing Function ’ which is necessary to represent the radiation from a localized disturbance could be a body force or any other source which vanishes outside a finite region. Paldas [5] has studied the propagation of waves through piezoelectric α-quartz crystals when the source of disturbance is an isolated harmonic body force. He has studied wave surfaces due to propagation of longitudinal waves and obtained slowness surface following the analysis of Buchwald [2]. In the present paper the propagation of waves at large distances from a source of disturbance in an infinite piezoelectric medium of hexagonal symmetry belonging to classes (6), (6 m m) or ceramic (∞ m) and (6 2 2) is investigated. Two cases are analysed in which (a) the source of disturbance is an isolated harmonic electric charge of fixed frequency and (b) the source of disturbance is an isolated harmonic body force of fixed frequency. We have followed the analysis of Lighthill [3] to obtain the wave surface from the slowness surface and studied the wave amplitudes as well. Numerical results are presented for cadmium selenide crystal which belongs to (6 m m) class. It is interesting to note that the source of disturbance due to electric charge generates longitudinal waves in (6 m m) class and torsional waves in (6 2 2) class whereas the source of disturbance due to body force generates flexural waves in both (6 m m) class and (6 2 2) class.

2. Source of electric charge

We choose a rectangular coordinate system with the axis of symmetry of (6) crystal class as z-axis. The stresses T_{ij} and electric displacements D_i for crystal class (6) can be expressed in terms of displaceemnt components (u, v, w) and electric potential ϕ as

$$\begin{aligned}
T_{xx} &= c_{11}\, u_{,x} + c_{12} v_{,y} + c_{13} w_{,z} + e_{31}\phi_{,z} \\
T_{yy} &= c_{12} u_{,x} + c_{11} v_{,y} + c_{13} w_{,z} + e_{31}\phi_{,z} \\
T_{zz} &= c_{13}\,(u_{,x} + v_{,y}) + c_{33} w_{,z} + e_{33}\phi_{,z} \\
T_{yz} &= c_{44}\,(v_{,z} + w_{,y}) + e_{14}\phi_{,x} + e_{15}\phi_{,y} \\
T_{xz} &= c_{44}\,(u_{,z} + w_{,x}) + e_{15}\phi_{,x} - e_{14}\phi_{,y} \\
T_{xy} &= c_{66}\,(u_{,y} + v_{,x}) \\
D_x &= e_{14}\,(w_{,y} + v_{,z}) + e_{15}\,(w_{,x} + u_{,z}) - \epsilon_{11}\phi_{,x} \\
D_y &= e_{15}\,(w_{,y} + v_{,z}) - e_{14}\,(w_{,x} + u_{,z}) - \epsilon_{11}\phi_{,y} \\
D_z &= e_{31}\,(u_{,x} + v_{,y}) + e_{33}\, w_{,z} - \epsilon_{33}\phi_{,z}
\end{aligned} \tag{1}$$

where c_{ij}, e_{ij} and ϵ_{ij} are elastic, piezoelectric and dielectric constants respectively. Comma followed by subscript denotes partial differentiation with respect to the subscript variable. Also $c_{66} = (c_{11} - c_{12})/2$.

The equations of motion (in the absence of a body force) are

$$c_{11}\, u_{,xx} + c_{66}\, u_{,yy} + c_{44}\, u_{,zz} + (c_{12} + c_{66})\, v_{,xy} + (c_{13} + c_{44})\, w_{,xz} + (e_{15} + e_{31})\,\phi_{,xz} - e_{14}\phi_{,yz} = s u_{,tt} \tag{2}$$

$$(c_{12} + c_{66})\, u_{,xy} + c_{66}\, v_{,xx} + c_{11}\, v_{,yy} + c_{44}\, v_{,zz} + (c_{13} + c_{44})\, w_{,yz} + (e_{15} + e_{31})\,\phi_{,yz} + e_{14}\phi_{,xz} = s v_{,tt} \tag{3}$$

$$(c_{13} + c_{44})\,(u_{,xz} + v_{,yz}) + c_{44}\,(w_{,xx} + w_{,yy}) + c_{33} w_{,zz} + e_{15}\,(\phi_{,xx} + \phi_{,yy}) + e_{33}\phi_{,zz} = s w_{,tt} \tag{4}$$

where s is the density of the medium.

When electric charge is present, Gauss's equation

Div $\mathbf{D} = 4\pi\, q$ gives

$$(e_{15} + e_{31})\, u_{,xz} - e_{14}\, u_{,yz} + e_{14}\, v_{,xz} + (e_{15} + e_{31})\, v_{,yz} + e_{15}\,(w_{,xx} + w_{,yy}) + e_{33}\, w_{,zz} - \epsilon_{11}\,(\phi_{,xx} + \phi_{,yy}) - \epsilon_{33}\,\phi_{,zz} = 4\pi q \tag{5}$$

From (2) and (3) we get

$$a_4 \nabla_1^2 \Lambda + a_5 \Lambda_{,zz} + a_9 \nabla_1^2 \Sigma = \Lambda_{,tt} \tag{6}$$

$$a_1 \nabla_1^2 \triangle + a_5 \triangle_{,zz} + a_3 \nabla_1^2 \Gamma + a_8 \nabla_1^2 \Sigma = \triangle_{,tt}. \tag{7}$$

From (4) and (5) we obtain

$$a_3 \triangle_{,zz} + a_5 \nabla_1^2 \Gamma + a_2 \Gamma_{,zz} + a_6 \nabla_1^2 \Sigma + a_7 \Sigma_{,zz} = \Gamma_{,tt} . \tag{8}$$

$$a_9 \Lambda_{,zz} + a_8 \triangle_{,zz} + a_6 \nabla_1^2 \Gamma + a_7 \Gamma_{,zz} - a_{10} \nabla_1^2 \Sigma - a_{11} \Sigma_{,zz} = 4 \pi q_{,z} . \tag{9}$$

In eqs (6) to (9) we have taken

$$\begin{aligned} &\Lambda = (v_{,x} - u_{,y}); \quad \triangle = (u_{,x} + v_{,y}); \\ &\Gamma = w_{,z} \quad ; \quad \Sigma = \phi_{,z} \quad \text{and} \\ &s\, a_1 = c_{11}; \quad s\, a_2 = c_{33} \quad ; \quad s\, a_3 = c_{44} + c_{13}; \\ &s\, a_4 = c_{66}; \quad s\, a_5 = c_{44} \quad ; \quad s\, a_6 = e_{15} \quad ; \\ &s\, a_7 = e_{33}; \quad s\, a_8 = (e_{15} + e_{31}); \quad s\, a_9 = e_{14} \quad ; \\ &s\, a_{10} = \epsilon_{11}; \quad s\, a_{11} = \epsilon_{33} \end{aligned} \tag{10}$$

The displacements u, v, w and potential ϕ are completely determined by the variables Λ, $\triangle$, Γ and Σ.

We first consider the case (case 1) when the point source is an isolated harmonic electric charge of fixed frequency ω acting at the origin; that is,

$$q = q_0\, \delta(x)\, \delta(y)\, \delta(z) \exp(-i\, \omega\, t) \tag{11}$$

where q_0 is constant and $\delta(x)$ is Dirac's delta function.

We now express the variables Λ, $\triangle$, Γ, Σ and q in terms of Fourier integrals in the form

$$P = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \bar{P} \exp[i(\alpha x + \beta y + \gamma z - \omega t)]\, d\alpha\, d\beta\, d\gamma. \tag{12}$$

Substituting integrals of the above form for the variables in eqs (6) to (9) and considering the Fourier transform of the source term, we obtain

$$[a_4(\alpha^2+\beta^2) + a_5 \gamma^2 - \omega^2]\, \bar{\Lambda} + a_9 (\alpha^2 + \beta^2)\, \bar{\Sigma} = 0$$

$$[a_1(\alpha^2 + \beta^2) + a_5 \gamma^2 - \omega^2]\, \bar{\triangle} + a_3 (\alpha^2 + \beta^2)\, \bar{\Gamma} + a_8 (\alpha^2 + \beta^2)\, \bar{\Sigma} = 0$$

$$a_3 \gamma^2 \bar{\triangle} + [a_5 (\alpha^2 + \beta^2) + a_2 \gamma^2 - \omega^2] \bar{\Gamma}$$

$$+ [a_6 (\alpha^2 + \beta^2) + a_7 \gamma^2] \bar{\Sigma} = 0$$

$$a_9 \gamma^2 \bar{\Lambda} + a_8 \gamma^2 \bar{\triangle} + [a_6 (\alpha^2 + \beta^2) + a_7 \gamma^2] \bar{\Gamma}$$

$$- [a_{10} (\alpha^2 + \beta^2) + a_{11} \gamma^2] \bar{\Sigma} = - iq_0 \gamma/(2\pi^2) \tag{13}$$

3. Slowness surface

Solutions of eq. (13) are

$$\bar{\Lambda} = K_1/H; \quad \bar{\triangle} = K_2/H; \quad \bar{\Gamma} = K_3/H \text{ and } \bar{\Sigma} = K_4/H \tag{14}$$

where

$$H \equiv H(\alpha, \beta, \gamma, \omega) = \text{det.} |M_{ij}| \; (i, j = 1 \text{ to } 4)$$

$$M_{11} = a_4 (\alpha^2 + \beta^2) + a_5 \gamma^2 - \omega^2, M_{12} = M_{13} = 0$$

$$M_{14} = a_9 (\alpha^2 + \beta^2), M_{21} = 0$$

$$M_{22} = a_1 (\alpha^2 + \beta^2) + a_5 \gamma^2 - \omega^2, \; M_{23} = a_3 (\alpha^2 + \beta^2);$$

$$M_{24} = a_8 (\alpha^2 + \beta^2), M_{31} = 0$$

$$M_{32} = a_3 \gamma^2; M_{33} = a_5 (\alpha^2 + \beta^2) + a_2 \gamma^2 - \omega^2$$

$$M_{34} = a_6 (\alpha^2 + \beta^2) + a_7 \gamma^2; M_{41} = a_9 \gamma^2; \; M_{42} = a_8 \gamma^2$$

$$M_{43} = M_{34}; M_{44} = - [a_{10} (\alpha^2 + \beta^2) + a_{11} \gamma^2] \tag{15}$$

and $K_i \equiv K_i (\alpha, \beta, \gamma, \omega)$ is the determinant obtained from H, by replacing ith column with the column whose elements are respectively 0, 0, 0, $-i\, q_0\, \gamma/(2\pi^2)$.
From (14) and (12), we find

$$\Lambda = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (K_1/H) \exp \left[i (\alpha x + \beta y + \gamma z - \omega t)\right] d\alpha\, d\beta\, d\gamma. \tag{16}$$

Similar integrals hold for $\triangle$, Γ and Σ, the integrand containing (K_i/H) ($i = 2, 3, 4$ respectively) instead of K_1/H. The only singularities of the integrand are the roots of $H = 0$. It is known (Buchwald [2]) that for large arguments, important contributions in evaluating integrals (16) arise only from points on the real surface $H = 0$ in α, β, γ space (α-space). Further outward travelling waves from source will have large contribution to the integrals. $H = 0$ regarded as an equation of α, β, γ gives the slowness surface for Λ, $\triangle$, Γ and Σ for real values of α, β, γ. We find that it is a surface of revolution. The plane $\gamma = 0$ is a plane of symmetry and γ-axis is the axis of revolution. This surface can be represented as follows: Consider the section of $H = 0$ with plane $\beta = 0$ given by

$$H(\alpha, 0, \gamma) = 0. \tag{17}$$

Since H is homogeneous in α, β, γ and ω we can put $\gamma = k\alpha$ where k is a constant. The resulting quartic equation in α^2 can be solved. One of the four roots is found to be zero and the corresponding sheet of the surface collapses to a point viz. the origin. This corresponds to a wave with infinite velocity generated at the origin. Corresponding to each real positive root of the quartic, one real sheet of the surface is obtained. By giving arbitrary values of k at suitable intervals, we obtain a set of values $(\alpha/\omega, \gamma/\omega)$ corresponding to each non-zero positive root, which when plotted on the plane $\beta = 0$ gives a curve. Rotating this curve about γ-axis, a sheet of the slowness surface is obtained. Hence we find that in a piezoelectric medium of class (6), the slowness surface has in general three sheets.

In the case of piezoelectric medium of class (6 m m) only $a_9 = 0$. Corresponding to eqs (14) we have

$$\begin{aligned}
\bar{\Lambda} &= 0 \\
\bar{\triangle} &= [i\, q_0/(2\pi^2)]\, \gamma\, (\alpha^2 + \beta^2) \{(a_5 a_8 - a_3 a_6)(\alpha^2 + \beta^2) \\
&\quad + (a_2 a_8 - a_3 a_7)\, \gamma^2 - a_8\, \omega^2\} / H_1 \\
\bar{\Gamma} &= [i\, q_0/(2\pi^2)]\, \gamma\, \{[a_1 (\alpha^2 + \beta^2) + a_5\, \gamma^2 - \omega^2] \\
&\quad \times [a_6 (\alpha^2 + \beta^2) + a_7\, \gamma^2] - a_3 a_8 (\alpha^2 + \beta^2)\, \gamma^2\} / H_1 \\
\bar{\Sigma} &= [i\, q_0/(2\pi^2)]\, \gamma\, H_2/H_1
\end{aligned} \tag{18}$$

where H_1 is det $|M_{ij}|$, $(i, j = 2, 3, 4)$

and

$$\begin{aligned}
H_2 &= a_3^2 (\alpha^2 + \beta^2)\, \gamma^2 - [a_1 (\alpha^2 + \beta^2) + a_5\, \gamma^2 - \omega^2] \\
&\quad \times [a_5 (\alpha^2 + \beta^2) + a_2\, \gamma^2 - \omega^2].
\end{aligned} \tag{19}$$

We conclude that rotational wave Λ does not propagate in the medium and the waves of $\triangle$, Γ, and Σ are coupled, the slowness surface being $H_1 = 0$. We find that this surface has two real sheets corresponding to the two real positive roots of the quadratic equation in α^2 given by $H_1(\alpha, 0, \gamma) = 0$.

If we consider piezoelectric medium of class (6 2 2) we have $a_6 = a_7 = a_8 = 0$ and $a_9 \neq 0$. In this case, eq. (18) becomes

$$\begin{aligned}
\bar{\Lambda} &= [i\, q_0/(2\pi^2)]\, a_9\, \gamma\, (\alpha^2 + \beta^2)/H_3 \\
\bar{\triangle} &= \bar{\Gamma} = 0 \text{ and} \\
\bar{\Sigma} &= [i\, q_0/(2\pi^2)]\, \gamma\, H_4/H_3
\end{aligned} \tag{20}$$

where

$$H_3 = [a_4(\alpha^2+\beta^2)+a_5\gamma^2-\omega^2][a_{10}(\alpha^2+\beta^2)+a_{11}\gamma^2]$$
$$+ a_9^2(\alpha^2+\beta^2)\gamma^2. \tag{21}$$

and $H_4 = a_4(\alpha^2+\beta^2)+a_5\gamma^2-\omega^2$

We find that waves of dilation $\triangle$ and Γ do not propagate. Also Λ and Σ have the slowness surface $H_3 = 0$ which has only one sheet.

4. Wave surface

The wave surface is the envelope (at time $t = 1$) of plane waves originating from a source at the origin at time $t = 0$. A point (α, β, γ) of α-space corresponds to a plane wave passing through a point (x, y, z) of x-space provided the vector (x, y, z) is parallel to the normal at (α, β, γ) to the slowness surface and the parametric equation of the wave surface is

$$x = -H_{,\alpha}/H_{,\omega};\ y = -H_{,\beta}/H_{,\omega};\ z = -H_{,\gamma}/H_{,\omega}. \tag{22}$$

We obtain the wave surface corresponding to the slowness surface $H_1 = 0$. This is the surface of revolution obtained by rotating about the z-axis in plane $y = 0$, the curve

$$Rx = \alpha\omega[b_1\alpha^4 - b_2\alpha^2\gamma^2 - b_3\gamma^4 - b_4\alpha^2\omega^2 - b_5\gamma^2\omega^2 + a_{10}\omega^4] \tag{23}$$

$$Rz = \gamma\omega[\tfrac{1}{2}b_2\alpha^4 - 2b_3\alpha^2\gamma^2 + b_6\gamma^4 - b_5\alpha^2\omega^2 - b_7\gamma^2\omega^2 + a_{11}\omega^4]$$

$$R = \omega^2\{a_8^2\alpha^2\gamma^2 + (a_6\alpha^2 + a_7\gamma^2)^2$$
$$+ (a_{10}\alpha^2 + a_{11}\gamma^2)[(a_1+a_5)\alpha^2 + (a_2+a_5)\gamma^2 - 2\omega^2]\}$$

where $b_1 = 3a_1(a_5a_{10} + a_6^2)$

$$b_2 = 2[2a_6(a_3a_8 - a_7a_1) - a_5a_6^2$$
$$- a_5(a_5a_{10} + a_1a_{11} + a_8^2) - a_{10}(a_1a_2 - a_3^2)]$$

$$b_3 = a_7[2(a_3a_8 - a_5a_6) - a_1a_7] \tag{24}$$
$$- a_2(a_5a_{10} + a_1a_{11} + a_8^2) - a_{11}(a_5^2 - a_3^2)$$

$$b_4 = 2[a_{10}(a_1 + a_5) + a_6^2]$$

$$b_5 = 2a_6a_7 + a_5a_{11} + a_2a_{10} + a_5a_{10} + a_1a_{11} + a_8^2$$

$$b_6 = 3a_5(a_2a_{11} + a_7^2)$$

and $b_7 = 2[a_{11}(a_2 + a_5) + a_7^2]$

The method of Lighthill [3] gives us the asymptotic contributions (in 6 m m) medium for $\triangle$, Γ and Σ for large r as

$$\triangle \sim (2/r) \sum_{n=1}^{N} A_n q_0 i \lambda_n \gamma_n(\alpha_n^2 + \beta_n^2) \times$$

$$[(a_8a_5 - a_3a_6)(\alpha_n^2 + \beta_n^2) + (a_2a_8 - a_3a_7)\gamma_n^2 - a_8\omega^2]$$

$$\Gamma \sim (2/r) \sum_{n=1}^{N} A_n q_0 i\lambda_n \gamma_n \{ [a_1(\alpha_n^2 + \beta_n^2) + a_5 \gamma_n^2 - \omega^2]$$

$$\times [a_6(\alpha_n^2 + \beta_n^2) + a_7\gamma_n^2) - a_3a_8(\alpha_n^2 + \beta_n^2)\gamma_n^2\} \tag{25}$$

$$\Sigma \sim (2/r) \sum_{n=1}^{N} A_n q_0 i \lambda_n\gamma_n \{ a_3^2(\alpha_n^2 + \beta_n^2)\gamma_n^2 -$$

$$- [a_1(\alpha_n^2 + \beta_n^2) + a_5 \gamma_n^2 - \omega^2][a_6(\alpha_n^2 + \beta_n^2) + a_7\gamma_n^2] \}$$

where summation is taken for points $(\alpha_n, \beta_n, \gamma_n)$ on the slowness surface at which the normal is parallel to OP, the point $P(x, y, z)$ lying on the wave surface. A_n is phase constant. For an axisymmetric case, it is enough if we evaluate amplitude coefficient λ_n corresponding to $\beta=0$. Then λ_n^2 is obtained as

$$\lambda_n^2 = (H_{,\alpha}^2 + H_{,\gamma}^2)/[H_{,\beta\beta}(H_{,\alpha}^2 H_{,\gamma\gamma} - 2H_{,\alpha} H_{,\gamma} H_{,\alpha\gamma} + H_{,\gamma}^2 H_{,\alpha\alpha})] \tag{26}$$

For (6 2 2) medium the asymptotic contributions for Λ, Σ are

$$\Lambda \sim (2/r) \sum_{n=1}^{N} A_n q_0 i \lambda_n a_9 \gamma_n (\alpha_n^2+\beta_n^2)$$

and

$$\Sigma \sim (2/r) \sum_{n=1}^{N} A_n q_0 i \lambda_n \gamma_n [a_4(\alpha_n^2+\beta_n^2)+a_5 \gamma_n^2-\omega^2].$$

5. Source of body force

The source of disturbance could be a body force instead of an electric charge. We now consider (case 2) an isolated harmonic body force of fixed frequency acting at the origin along the x-axis, the electric charge q being absent. The body force is given by

$$X = X_0 \delta(x)\delta(y)\delta(z)\exp(-i\omega t) \tag{27}$$

$$Y = Z = 0.$$

Gauss's equation in this case is Div $\mathbf{D}=0$ and eqs (13) are modified as

$$[a_4(\alpha^2+\beta^2)+a_5\gamma^2-\omega^2]\bar{\Lambda}+a_9(\alpha^2+\beta^2)\bar{\Sigma}=-i\beta X_0/(8\pi^3)$$

$$[a_1(\alpha^2+\beta^2) + a_5\gamma^2-\omega^2]\bar{\triangle} + a_3(\alpha^2 + \beta^2)\bar{\Gamma}$$

$$+ a_8(\alpha^2 + \beta^2)\bar{\Sigma} = i\alpha X_0/(8\pi^3) \tag{13'}$$

$$a_3\gamma^2 \bar{\triangle} + [a_5(\alpha^2+\beta^2) + a_2\gamma^2 - \omega^2]\bar{\Gamma} + [a_6(\alpha^2+\beta^2) + a_7\gamma^2)\bar{\Sigma} = 0$$

$$a_9\gamma^2\bar{\Lambda} + a_8\gamma^2\bar{\triangle} + [a_6(\alpha^2+\beta^2) + a_7\gamma^2)\bar{\Gamma}$$

$$- [a_{10}(\alpha^2+\beta^2) + a_{11}\gamma^2)\bar{\Sigma} = 0.$$

Solving the above equations we obtain

$$\bar{\Lambda} = L_1/H;\ \bar{\triangle} = L_2/H;\ \bar{\Gamma} = L_3/H \text{ and } \bar{\Sigma} = L_4/H \tag{28}$$

where $L_i \equiv L_i(\alpha, \beta, \gamma, \omega)$ is the determinant obtained from H by replacing ith column with the column whose elements are respectively $-i\beta X_0/(8\pi^3)$, $i\alpha X_0/(8\pi^3)$, 0, 0.

If we consider the medium of class (6), the slowness surface is the same as in case 1. In (6 m m) piezoelectric medium we obtain

$$\bar{\Lambda} = - i\beta X_0 (8\pi^3 H_4)$$

$$\bar{\triangle} = - i\alpha X_0 \{[a_5(\alpha^2+\beta^2) + a_2\gamma^2 - \omega^2][a_{10}(\alpha^2+\beta^2) + a_{11}\gamma^2)]$$

$$+ [a_6(\alpha^2+\beta^2) + a_7\gamma^2]^2\}/(8\pi^3 H_1) \tag{29}$$

$$\bar{\Gamma} = i\alpha X_0\gamma^2[(a_3a_{10} + a_6a_8)(\alpha^2+\beta^2) + (a_3a_{11} + a_7a_8)\gamma^2]/(8\pi^3 H_1)$$

and

$$\bar{\Sigma} = i\alpha X_0\gamma^2[(a_3a_6 - a_5a_8)(\alpha^2+\beta^2) + (a_3a_7 - a_2a_8)\gamma^2 + a_8\omega^2]/(8\pi^3 H_1).$$

The slowness surface for Λ exists and is given by $H_4=0$. In this case the slowness surface is identical with that of transversely isotropic elastic medium. Here the slowness surface is a spheroid and the corresponding wave surface is also a spheroid. The slowness surface for $\triangle$, Γ and Σ is the same as in case 1.

The asymptotic contributions for large r can be obtained as in case 1.

Considering medium of class (6 2 2), we obtain

$$\bar{\Lambda} = i\beta X_0[a_{10}(\alpha^2+\beta^2) + a_{11}\gamma^2]/(8\pi^3 H_3)$$

$$\bar{\triangle} = i\alpha X_0[a_5(\alpha^2+\beta^2) + a_2\gamma^2 - \omega^2]/(8\pi^3 H_2)$$

$$\bar{\Gamma} = i\alpha X_0 a_3\gamma^2/(8\pi^3 H_2), \text{ and} \tag{30}$$

$$\bar{\Sigma} = i\beta X_0 a_9\gamma^2/(8\pi^3 H_3)$$

Λ and Σ have the same slowness surface as in case 1. The surface for $\triangle$ and Γ exists and is given by $H_2 = 0$. The asymptotic contributions for large r can be obtained as in case 1.

6. Numerical results

Since all the constants for a class (6) crystal are not available we shall consider (6 m m) piezoelectric medium in this section. We choose the surface $H_1 = 0$ for obtaining numerical results and use the constants of cadmium selenide crystal (Berlincourt *et al* [1])

$$\begin{aligned} c_{11} &= 7{\cdot}41 \times 10^{10}\ \mathrm{N/m^2} & e_{15} &= -0{\cdot}138\ \mathrm{coul/m^2} \\ c_{12} &= 4{\cdot}51 & e_{31} &= -0{\cdot}159 \\ c_{13} &= 3{\cdot}93 & e_{33} &= -0{\cdot}347 \\ c_{33} &= 8{\cdot}36 & s &= 5684\ \mathrm{kg/m^3} \\ c_{44} &= 1{\cdot}32 & \epsilon_{11} &= 84{\cdot}4 \times 10^{-12}\ \mathrm{F/m} \\ & & \epsilon_{33} &= 90{\cdot}3 \end{aligned} \tag{31}$$

We consider the section of the surface H_1 $(\alpha, \beta, \gamma)=0$ with the plane $\beta=0$. In $H_1(\alpha, 0, \gamma)=0$ we put $\gamma=k\alpha$ and evaluate the non-zero roots of the resulting quadratic equation in α^2 for different values of k^2. The corresponding values of γ^2 are obtained.

Table 1. Piezoelectric case

Coordinates of slowness surface with $\beta=0$ $\times 10^{-4}$ sec/met		Coordinates of wave surface with $y=0$ $\times 10^3$ met/sec		Amplitude coefficient $\times 10^{13}$ kg²/(Sec Coul)²
α/ω	γ/ω	x	z	λ
		First sheet		
0·00	2·59	0·00	3·86	6·6
1·09	2·44	1·04	3·63	5·2
1·23	2·40	1·21	3·54	4·9
1·36	2·35	1·38	3·45	4·7
1·46	2·31	1·52	3·37	4·5
1·67	2·20	1·83	3·16	4·2
1·98	1·98	2·34	2·70	3·9
2·04	1·93	2·44	2·61	3·8
2·10	1·87	2·54	2·50	3·8
2·16	1·81	2·65	2·37	3·8
2·66	0·84	3·46	0·93	4·3
2·77	0·00	3·61	0·00	4·6
		Second sheet		
0·00	6·56	0·00	1·52	0·2
2·37	5·30	1·34	1·29	0·8
2·60	5·06	1·40	1·26	1·1
2·79	4·84	1·42	1·25	1·8
2·95	4·66	1·43	1·24	∞
3·77	3·77	1·32	1·34	1·7
3·87	3·67	1·30	1·36	1·8
3·98	3·56	1·28	1·38	1·9
4·04	3·50	1·27	1·39	2·0
4·55	3·05	1·24	1·43	∞
5·80	1·83	1·38	1·09	0·8
6·51	0·00	1·54	0·00	0·5

Table 2. Elastic case

Coordinates of slowness surface with $\beta=0$ all $\times 10^{-4}$ sec/met		Coordinates of wave surface with $y=0$ all $\times 10^{3}$ met/sec		Amplitude coefficient all $\times 10^{-8}$ sec^2/met^2
α/ω	γ/ω	x	z	λ
		First sheet		
0·00	2·61	0·00	3·85	6·4
1·21	2·42	1·23	3·53	5·3
1·40	2·34	1·47	3·39	5·0
1·46	2·31	1·55	3·34	4·9
1·59	2·25	1·73	3·22	4·7
1·76	2·15	1·98	3·03	4·5
1·98	1·98	2·34	2·70	4·2
2·16	1·81	2·64	2·38	4·0
2·23	1·73	2·76	2·23	4·0
2·44	1·40	3·12	1·69	4·0
2·66	0·84	3·46	0·93	4·2
2·77	0·00	3·61	0·00	4·4
		Second sheet		
0·00	6·56	0·00	1·52	1·5
2·59	5·17	1·34	1·26	5·0
2·89	4·84	1·40	1·23	8·7
2·99	4·73	1·41	1·22	11·5
3·18	4·50	1·42	1·22	∞
3·43	4·21	1·41	1·23	15·2
3·80	3·80	1·35	1·28	26·9
4·01	3·59	1·32	1·31	24·1
4·28	3·32	1·28	1·36	15·4
4·80	2·84	1·26	1·39	∞
5·81	1·84	1·37	1·12	4·0
6·56	0·00	1·52	0·00	2·3

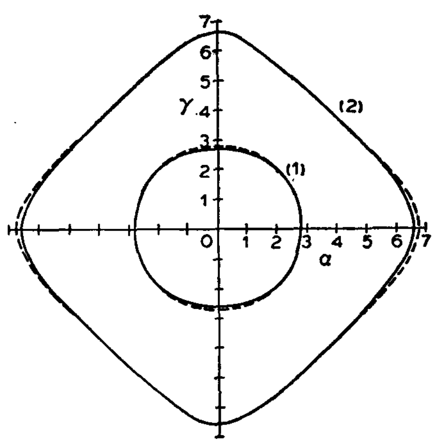

Figure 1. Slowness surfaces obtained by rotating the above curves about the γ-axis. (1) First sheet; (2) Second sheet.

x and z are found with the help of eqs (23). The amplitude coefficient λ_n is obtained from eq. (26). The results for the two sheets in the case of piezoelectric medium are given in table 1. The corresponding results, treating the medium as elastic, are given in table 2. The numerical results are also represented graphically in figures 1 and 2.

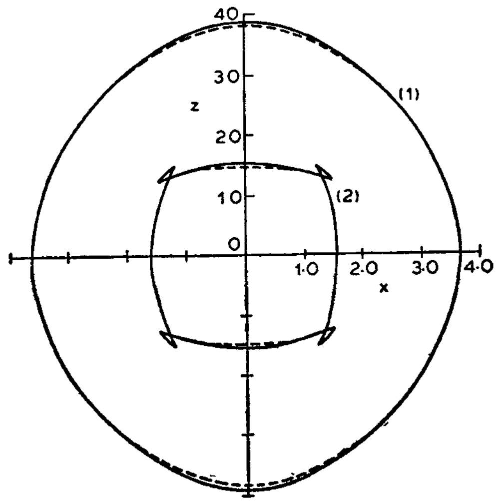

Figure 2. Wave surfaces obtained by rotating the above curves about the z-axis. (1) First sheet; (2) Second sheet.

The portions of curves where results for elastic medium are different from piezoelectric medium are shown by dotted line.

7. Conclusions

Case 1: Source of electric charge: Rotational wave Λ is not propagated in (6 m m) piezoelectric medium whereas $\triangle$, Γ and Σ waves possess a slowness surface of two sheets. The waves $\triangle$ and Γ are not propagated in (6 2 2) piezoelectric medium whereas Λ and Σ waves have a slowness surface of only one sheet. We put $\gamma = 0$ in (18) and (20) and find that $\bar{\Lambda}$, $\bar{\triangle}$, $\bar{\Gamma}$ and $\bar{\Sigma}$ are zero in both media. Hence there is no wave propagation in the plane $z = 0$.

Case 2: Source of body force: Λ wave has a slowness surface in (6 m m) piezoelectric medium and $\triangle$, Γ and Σ waves have the same slowness surface as in case 1. The waves $\triangle$ and Γ have a slowness surface in (6 2 2) piezoelectric medium. We note that in both the cases where the slowness surface does not exist for the source of electric charge, the surface obtained for the source of body force is the same as in elastic (6 m m) and (6 2 2) media. Λ and Σ in (6 2 2) medium have the same slowness surface as in case 1. We put $\alpha=\beta=0$ in equations (29) and (31) and find that $\bar{\Lambda}$, $\bar{\triangle}$, $\bar{\Gamma}$ and $\bar{\Sigma}$ are all zero. Therefore there is no wave propagation along the z-axis for both the media. The numerical results obtained for cadmium selenide (6 m m) medium enable us to observe the following. The slowness and wave surfaces are very nearly the same when the medium is considered piezoelectric or purely elastic. However, the amplitude coefficients vary considerably in the two media. Hence the electromechanical interaction does not affect much the slowness and wave surfaces but has considerable influence on the amplitude coefficient. It may be pointed out that the change will be appreciable if the medium of piezoelectric ceramic is considered instead of that of piezoelectric crystal since the electromechanical coupling factor for ceramic (∞ m) is large compared to a crystal. It may be mentioned that the piezoelectric constants noted in the table given in [5] are wrong. Hence numerical results given in [5] cannot be relied upon.

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