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Little rip phenomena from coupled dark energy with quadratic equation of state with time-dependent parameters

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Abstract. The purpose of this article was to examine the behavior of the Little rip (LR) and Pseudo rip (PR) models with two interacting ideal fluids related to dark energy and dark matter with the quadratic equation of state with time-dependent parameters $\omega(t)$ and $\Lambda(t)$ in flat Friedmann–Lemaitre–Robertson–Walker cosmological model. In this article, the gravitational equations of motion for dark matter have been solved. The equation of the state parameter $\omega(t) \rightarrow -1$ has been discovered. Also, discovered $\Lambda(t) \rightarrow \infty$ as $t \rightarrow \infty$, it shows that the future behavior of our universe depends on specific model parameters $\omega(t)$ and $\Lambda(t)$ for the coupled dark energy. In this formalism, the properties of the early universe are pointed out.

Keywords. Cosmology models—Dark energy—Dark matter—Little rip (LR)—Pseudo rip (PR)—Quadratic equation of state (EOS).

1. Introduction

One of the most unforeseen disclosures about our understanding of the universe is that it is not dominated by the ordinary baryonic matter, but instead, by a form of nonluminous matter called dark matter and is about five times more abundant than baryonic matter (Ade *et al.* 2014). The method for explaining the observed expansion is to introduce a dark energy fluid with negative pressure and negative entropy for the universe that derives the positively accelerated phase of the universe expansion (Riess *et al.* 1998; Perlmutter *et al.* 1999; Sahni & Starobinsky 2000; Peebles & Ratra 2003; Li *et al.* 2011). According to present observational data, dark energy currently accounts for about 73% of the total mass/energy of the universe and only 27% of a combination of dark matter and baryonic matter (Kowalski *et al.* 2008).

Very little has been explored about dark energy and its properties that determine the fate of our universe. In addition to that, there is no clear evidence that they interact with each other or even they are interlinked while it is usually believed that they weakly interact with ordinary matter. However, there is a enough possibility to develop a generalized model of quintessence field that the background and the dark energy develop independently, but have nonminimal coupling between both dark components (Amendola 2000; Chimento *et al.* 2000; Zimdahl *et al.* 2001; Amendola & Tochini 2002; Chimento *et al.* 2003a,b; Gonzalez *et al.* 2006; Farooq *et al.* 2011). Since the nature of the dark matter is not completely discovered, we have the liberty to consider additional interactions between the dark components without bothering about the facts observed so far. Nevertheless, solar system tests impose some restrictions on the nonminimal coupling between dark matter and dark energy (Will 1933). Currently, no specific coupling between the dark sectors has been known based on fundamental theories. Therefore, suggested coupling models will necessarily be phenomenological (Amendola & Tocchini-Valentini 2001; Boehmer *et al.* 2008), though some models seem to have more physical justification than others (Gonzalez *et al.* 2006; Boehmer *et al.* 2015; Gleyzes *et al.* 2015; D'Amico *et al.* 2016; Pan *et al.* 2020a,b).

Here, new interaction can be phenomenologically introduced in several ways (Koyama *et al.* 2009) in the investigation, which follows similar approaches

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(He & Wang 2008; Gavela *et al.* 2009, 2010; He *et al.* 2009a,b, 2011; Jackson *et al.* 2009; Salvatelli *et al.* 2013; Costa *et al.* 2014; Abdalla *et al.* 2017; Valentino *et al.* 2017; Yang *et al.* 2018, 2019). We phenomenologically parameterized the coupling between dark matter and dark energy through an energy transfer from one sector to the other.

In the coupled phantom/fluid model, dark energy and dark matter are usually described by assuming an ideal fluid with an unusual equation of state (EOS). In the future, the dark energy universe may have interesting implications (Nojiri *et al.* 2005). It tends to be described by an EOS dark energy parameter ω. It is the proportion of the pressure to the density $\omega = p_x/\rho_x < 0$, where ρ_x is the dark energy and p_x is the dark pressure. Its EOS parameter ω is < -1. The condition $\omega < -1$ corresponds to a dark energy density that monotonically increases with time t and scale factor R.

Brevik *et al.* (2004), Nojiri & Odintsov (2005, 2006) and Capozziello *et al.* (2006) presented the dark fluid models with an inhomogeneous EOS. There exist various cosmological scenarios for the evolution of the universe with the big rip (Caldwell *et al.* 2003; Nojiri & Odintsov 2003, 2004), the little rip (LR) (Brevik *et al.* 2011; Frampton *et al.* 2011, 2012a; Astashenok *et al.* 2012a,b,c; Nojiri *et al.* 2012; Makarenko *et al.* 2012), the pseudo rip (PR) Frampton *et al.* (2012b) and the quasi rip (Wei *et al.* 2012). Shelote & Khadekar (2018) have suggested that both LR and PR are nonsingular and also studied the behavior of LR and PR for dark energy in the flat Friedmann–Lemaitre–Robertson–Walker (FLRW) cosmological model. Khadekar *et al.* (2015) and Vinutha *et al.* (2019) are the authors who have analyzed the LR and PR models behavior in the flat Kaluza–Klein cosmological model.

In this article, the effect of the coupling between dark energy and dark matter of special form through the time-dependent parameters $\omega(t)$ and $\Lambda(t)$ has been investigated. In this investigation, we have considered the account of the impact of the interaction rate between dark energy and dark matter on the evolution of the LR and PR universe. Equation of state has an important role for LR as well as PR phenomena in the framework of coupled dark energy models.

With regard to general relativity, the impacts of a quadratic EOS have been examined by Nojiri & Odintsov (2005), Ananda & Bruni (2006) and Capozziello *et al.* (2006) to describe homogeneous and inhomogeneous cosmological models. This quadratic EOS of the type $p_x = p_0 + \alpha\rho_x + \beta\rho_x^2$, where p_0, α, and β are parameters, is only the Taylor expansion of arbitrary barotropic EOS, $p_x(\rho_x)$. Different EOS has been discussed by Nojiri & Odintsov (2005) and Capozziello *et al.* (2006) for dark energy universe and demonstrated that the quadratic EOS may describe dark energy or unified dark matter.

Rahman *et al.* (2009), Feroze & Siddiqui (2011), Chavanis (2013a,b), Maharaj & Takisa (2013), Sharma & Ratanpal (2013), Malaver (2014) and Takisa *et al.* (2014) are several researcher's works with a quadratic EOS.

Shelote & Khadekar (2018) studied the dark energy model with quadratic EOS with time-dependent parameters by considering the below form:

$$p_x = [1 + \omega(t)]\rho_x^2 + \Lambda(t), \tag{1}$$

in which LR and PR behaviors were encountered. Here p_x and ρ_x are the pressure and the energy density for dark energy, respectively.

Vinutha *et al.* (2019) have examined the flat FLRW type Kaluza–Klein model within sight of ideal fluid with a quadratic EOS with time-dependent parameters. Raushan *et al.* (2020) have studied the dynamical systems analysis of the FLRW model of the universe with a quadratic EOS and bulk viscosity in the structure of general relativity.

Bervik *et al.* (2013) have studied the impact from the interaction between dark energy and dark matter of time parameters in the inhomogeneous EOS $p_x = \omega(t)\rho_x + \Lambda(t)$ and $p_m = \widetilde{\omega}(t)\rho_m$ for dark energy and dark matter, respectively, upon the occurrence of LR and PR models.

In this work, we have examined the particular dark energy models with time-dependent parameters $\omega(t)$ and $\Lambda(t)$ in the quadratic EOS and inhomogeneous EOS for dark matter in which LR and PR conducts have been experienced.

2. Model and field equations

Consider the flat FLRW cosmological model of the following structure

$$\mathrm{d}s^2 = \mathrm{d}t^2 - R^2(t)[\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)], \tag{2}$$

where $R(t)$ is a scale factor.

Einstein's field equations take the standard structure as

$$G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R = -8\pi G T_{ij}. \tag{3}$$

The energy–momentum tensor of the fluid contained in this universe is given by

$$T_{ij} = (\rho_x + \rho_m + p_x + p_m)\mu_i\mu_j - (p_x + p_m)g_{ij},$$

where p_x, ρ_x and p_m, ρ_m are the pressure, the energy density of dark energy and the pressure, the energy density of dark matter, respectively.

Here, we neglect baryons and radiation since at the late time they are not much significant Aghanim *et al.* (2018a,b).

The Einstein field Equation (3) for the FLRW model Equation (2) takes the form

$$H^2 = \frac{1}{3}\chi^2(\rho_x + \rho_m), \tag{4}$$

$$\dot{H} = -\frac{1}{2}\chi^2(p_x + \rho_x + p_m + \rho_m), \tag{5}$$

where $\chi^2 = 8\pi G$ with G representing the Newton's gravitational constant, H is the Hubble parameter, and dot $(\cdot)$ for differentiation with respect to the proper time t.

For dark energy and dark matter, the conservation equation takes the form

$$\dot{\rho}_x + 3H(p_x + \rho_x) = -Q \tag{6}$$

and

$$\dot{\rho}_m + 3H(p_m + \rho_m) = Q, \tag{7}$$

where Q is the interaction rate between dark energy and dark matter.

If $Q > 0$, the energy transfer is from dark energy to dark matter, which means dark energy decays into dark matter, whereas if $Q<0$, the energy flux has the opposite direction and dark matter turns into dark energy.

Consider that the universe is filled with two interacting perfect fluids, fulfilling the quadratic EOS with time-dependent parameters $\omega(t)$ and $\Lambda(t)$ as mentioned in Equation (1) for dark energy.

Also, in the accompanying EOS parameter between the pressure of dark matter and the energy density of the dark matter, a component is taken to comply with another inhomogeneous EOS (Nojiri *et al.* 2005)

$$p_m = \widetilde{\omega}(t)\rho_m. \tag{8}$$

After substituting the value of P_m from Equation (8) into Equation (7), it results into

$$\dot{\rho}_m + 3H(1 + \widetilde{\omega})\rho_m = Q. \tag{9}$$

By using Equations (1) and (4) in Equation (6), the gravitational equation of motion for the dark energy takes the form as below:

$$\frac{6H\dot{H}}{\chi^2} - \dot{\rho}_m + 3H([(1 + \omega(t))\rho_x + 1]\rho_x + \Lambda(t)) = -Q. \tag{10}$$

We will consider the development of the universe beginning at some time $t = 0$, which will at first be left undefined, except that it refers to an initial instance in the very early universe. Its precise meaning will be dependent on which model we consider. We have considered the LR and PR models.

We will now explore LR and PR cosmological models in which interaction between dark energy and dark matter components is described in terms of the quadratic EOS with time-dependent parameters $\omega(t)$ and $\Lambda(t)$.

2.1 *LR model*

LR cosmology is characterized by an energy density, increasing with time t but in an asymptotic sense so that an infinite time is required to reach the singularity. It corresponds to an EOS parameter $\omega < -1$, but $\omega \to -1$ asymptotically. This is a simple variant of the future singularity approach.

Consider the two different cases of LR model in four-dimensional cosmology with a given Hubble parameter H:

Case (a): $H(t) = H_0 e^{\lambda t}$ (Frampton *et al.* 2012a).
Case (b): $H(t) = (t^n/\tau^n) - 1$.

In both cases as $t \to \infty$, $H \to \infty$.

Case (a): $H(t) = H_0 e^{\lambda t}, \quad \lambda > 0, \quad H_0 > 0.$

In Case (a), if $t = 0$, we get $H = H_0$, so that H_0 turns into the present-time Hubble parameter.

To solve Equation (9), assume that the thermodynamic parameter $\widetilde{\omega}(t)$ for dark matter has the accompanying structure as below (Bervik *et al.* 2013):

$$\widetilde{\omega}(t) = -1 + e^{-\lambda t}. \tag{11}$$

Consider that interaction rate $Q(t)$ between dark energy and dark matter depends on quadratic on time, in the following form:

$$Q = Q_1 t^2 + Q_2, \tag{12}$$

where Q_1 and Q_2 are constants.

This choice (Equation 12) is based on the physically natural assumption that the coupling increases when the corresponding rate of change of the scale factor increases (it means time increases). Also, Brevik *et al.* (2013) considered $Q(t)$ dependents linearly on time and choices are motivated by modified gravity theory (Nojiri & Odintsov 2007, 2011).

By substituting the value of $\widetilde{\omega}(t)$ from Equation (11) and the value of Q from Equation (12) in

Equation (9), energy density of dark matter can be written as

$$\rho_m = \frac{1}{3H_0}\left[t^2 Q_1 - \frac{2tQ_1}{3H_0} + \frac{2Q_1}{9H_0^2} + Q_2\right] + c\mathrm{e}^{-3H_0 t}, \quad (13)$$

where c is the constant of integration and H_0 is the present-time Hubble parameter.

In Equation (13), during evolution of the universe (i.e., $t > 0$) for the value $Q_1 > 0$ and $Q_2 > 0$, ρ_m will always be positive.

At $t = 0$, Equation (13) becomes

$$\rho_m(0) = \frac{1}{3H_0}\left[\frac{2Q_1}{9H_0^2} + Q_2\right] + c.$$

In the above equation, for the value of $c = -(1/3H_0)((2Q_1/9H_0^2) + Q_2)$, ρ_m is zero. It indicates that in the universe at initial time $t = 0$, dark matter starts to exist.

When $t \to \infty$, $\rho_m \to \infty$.

Consider the value of $\omega(t)$ for dark energy in the form (Brevik *et al.* 2013)

$$\omega(t) = -1 - \frac{1}{3\chi^2 H^2}. \quad (14)$$

In Equation (14), for $t<0$ and $t > 0$, results in $\omega < -1$.

In the limit $t \to \pm\infty$, gives $\omega(t) \to -1$.

Subsequently when $t \to -\infty$, the accelerating expansion happens, which may relate to the inflation in the early universe.

By puting the value of $\omega(t)$ from Equation (14) in Equation (10), we obtain

$$\Lambda(t) = \frac{1}{\chi^2}\left[3H^2\left(\frac{1}{\chi^4} - 1\right) - 2\lambda H\right] - \rho_m\left[\frac{2}{\chi^4} + \widetilde{\omega} - \frac{\rho_m}{3\chi^2 H^2}\right]. \quad (15)$$

When $t \to \infty$ we obtain $\Lambda(t) \to \infty$.

At initial time $t = 0$, from Equations (14) and (15) we get

$$\omega(0) = -1 - \frac{1}{3\chi^2 H_0^2} \quad \text{and}$$

$$\Lambda(0) = \frac{1}{\chi^2}\left[3H_0^2\left(\frac{1}{\chi^4} - 1\right) - 2\lambda H_0\right]. \quad (16)$$

From Equation (10), for the value of $\Lambda = \Lambda(t)$, we obtain $\omega(t)$ as

$$\omega(t) = \frac{-1}{\rho_m^2[(3H^2/\chi^2\rho_m) - 1]^2} \times \left[\frac{(2\lambda + 3)H}{\chi^2} + \widetilde{\omega}\rho_m + \Lambda(t)\right] - 1. \quad (17)$$

From Equation (17), we observed that, for any value of $\Lambda(t)$ (like $\Lambda(t)$ as an increasing function of t or decreasing function of t or constant value), we get $\omega(t) \to -1$. This equation corresponds to $\omega < -1$ at any given time. As a result, LR cosmology has been formulated in terms of the EOS parameters $\Lambda(t)$ and $\omega(t)$ for the coupled dark energy.

Case (b): $H(t) = (t^n/\tau^n) - 1$, where τ is constant and $n > 0$.

In Case (b), if $t \to \infty$, $H(t) \to \infty$ and for $t = 0$, $H(t) \to -1$.

Here, we considered Case (b) as a LR model because on solving Case (b) for the value of scale factor $R(t)$, it gives

$$R(t) = \exp\left[\frac{t^{n+1}}{\tau^n(n+1)} - t + c\right].$$

The above value of $R(t)$ is of the form $R(t) = \mathrm{e}^{f(t)}$, where $f(t) = (t^{n+1}/(\tau^n(n+1))) - t + c$ is a nonsingular function, which is satisfied as long as $\ddot{f} > 0$.

According to Frampton *et al.* (2011), all LR models are described by an equation of the form $R(t) = \mathrm{e}^{f(t)}$, with nonsingular f satisfying equation $\ddot{f} > 0$ and physically in the LR, the scale factor $R(t)$ and the density is never infinite at a finite time.

For the thermodynamic dark matter $\widetilde{\omega}$ in the inhomogeneous EOS Equation (8), consider the following form:

$$\widetilde{\omega}(t) = -1 + t^n. \quad (18)$$

In Equation (18), $\widetilde{\omega}(t)$ is always -1 at an initial value of t, i.e., $t = 0$, thereafter with increasing t, $\widetilde{\omega}(t)$ increases.

Consider the value of $Q(t)$ in the following form:

$$Q(t) = \left(\mathrm{e}^{3t^{n+1}/(n+1)}\right)\frac{3t^{2n}}{\tau^n}. \quad (19)$$

By using the value of $\widetilde{\omega}(t)$ from Equation (18) and $Q(t)$ from Equation (19) to solve Equation (9) for dark matter, we obtain

$$\rho_m(t) = \mathrm{e}^{3t^{n+1}/(n+1)} + c_1 \exp\left[\frac{3t^{(n+1)}}{n+1} - \frac{3t^{2n+1}}{\tau^n(2n+1)}\right], \quad (20)$$

where c_1 is constant of integration.

At $t = 0$, Equation (20) simply yields

$$\rho_m(0) = 1 + c_1. \quad (21)$$

By considering the value of constant of integration $c_1 = -1$ in Equation (21), it gives $\rho_m = 0$.

In the far future, $t \to \infty$, we additionally observed that the dark matter $\rho_m(\infty) \to \infty$.

To get the value of $\Lambda(t)$, put the value of parameter $\omega(t)$ from Equation (14), $Q(t)$ from Equation (19) and $\rho_m(t)$ from Equation (20), in Equation (10), gives

$$\Lambda(t) = \frac{1}{\chi^2}\left[3H^2\left(\frac{1}{\chi^4} - 1\right) - \frac{2nt^{n-1}}{\tau^n}\right] - \rho_m\left[\frac{2}{\chi^4} + \widetilde{\omega} - \frac{\rho_m}{3\chi^2 H^2}\right]. \tag{22}$$

From Equations (14) and (22), for a Case (b) at $t = 0$, gives

$$\omega(0) = -1 - \frac{1}{3\chi^2}$$

and

$$\Lambda(0) = \frac{1}{\chi^2}\left[3\left(\frac{1}{\chi^4} - 1\right)\right] - (1 + c_1)\left[\frac{2}{\chi^4} - 1 - \frac{1 + c_1}{3\chi^2}\right]. \tag{23}$$

For $H(t) = (t^n/\tau^n) - 1$ and the value of $\Lambda = \Lambda(t)$ in Equation (10), gives the value of $\omega(t)$ as

$$\omega(t) = \frac{-1}{\rho_m^2[(3H^2/\chi^2\rho_m) - 1]^2}[A + \widetilde{\omega}\rho_m + \Lambda(t)] - 1, \tag{24}$$

where

$$A = \frac{2nt^{n-1}}{\chi^2\tau^n} + \frac{3}{\chi^2}\left(\left(\frac{t^n}{\tau^n}\right) - 1\right)^2.$$

Up to this point, we have in this manner built two instances of LR cosmology, regarding time-dependent parameters in the EOS, by considering an interaction rate $Q(t)$ between dark energy and dark matter.

It should be noted that for both LR model's Cases (a) and (b) at the point $t \to \infty$, it is observed that $\rho_m \to \infty$. At initial time $t = 0$, for Case (a) at $c = -((2Q_1/9H_0^2) + Q_2)$ and for Case (b) at $c_1 = -1$, energy density for dark matter is zero. This implies, in the universe there might be a time in this model when there is no dark matter in it or it begins to appear.

2.2 *PR model*

In this section, a model has been investigated, in which the Hubble parameter moves toward a constant in the far future. That implies, the universe approaches asymptotically to a de-Sitter space.

Let us consider that PR model with different behavior of Hubble parameter has the following form:

$$H = H_0 - H_1 e^{-\lambda t^{(n+1)}}, \tag{25}$$

where H_0, H_1, λ, and n are positive constants, $H_0 > H_1$ when $t > 0$.

For the solution of Equation (9) for dark matter for the PR model, consider the parameter $\widetilde{\omega}(t)$ has the structure as in Equation (18).

Let us consider $Q(t)$ as

$$Q(t) = 3H_0 t^n\left[\exp\left(\frac{-3H_1}{\lambda(n+1)}\exp(-\lambda t^{(n+1)})\right)\right]. \tag{26}$$

With respect to above equation, we found that the solution of Equation (9) for dark matter is

$$\rho_m = e^{((-3H_1/\lambda(n+1))e^{-\lambda t^{(n+1)}})}[1 + c_2 e^{-3H_0/(n+1)t^{n+1}}], \tag{27}$$

where c_2 is constant of integration.

From Equation (27), the dark matter energy density

At $t \to 0$, $\rho_m \to e^{-3H_1/(\lambda(n+1))}(1 + c_2)$.

At $t \to \infty$, $\rho_m \to 1$.

At $t \to -\infty$, $\rho_m \to 0$.

For the PR model, from the above expression, we observed that at a past time, there is no dark matter. It will start to appear in the universe with increasing time.

Energy density for dark matter has value ranging from 0 to 1 for the elapsed time from $t \to -\infty$ to $t \to \infty$.

If the thermodynamic parameter $\omega(t)$ for dark energy has the form as in Equation (14), we obtain from Equation (10) the following expression for the cosmological constant:

$$\Lambda(t) = \frac{1}{\chi^2}\left[3H^2\left(\frac{1}{\chi^4} - 1\right) - 2(n+1)H_1\lambda e^{-\lambda t^{(n+1)}} t^n\right] - \rho_m\left[\frac{2}{\chi^4} - \frac{\rho_m}{3\chi^2 H^2} + \widetilde{\omega}\right]. \tag{28}$$

At $t = 0$, for PR model from Equations (14) and (28) gives

$$\omega(0) = -1 - \frac{1}{3\chi^2(H_0 - H_1)^2}$$

and

$$\Lambda(0) = \frac{1}{\chi^2}\left[3(H_0 - H_1)^2\left(\frac{1}{\chi^4} - 1\right)\right] - e^{(-3H_1/\chi(n+1))} \times [1 + c_2]\left[\frac{2}{\chi^4} - \frac{e^{(-3H_1/\lambda(n+1))}[1 + c_2]}{3\chi^2(H_0 - H_1)^2} + \widetilde{\omega}\right]. \tag{29}$$

For PR model Equation (25), for the value of $\Lambda = \Lambda(t)$ in Equation (10), we get the value of $\omega(t)$ as follows:

$$\omega(t) = \frac{-1}{\rho_m^2[(3H^2/\chi^2\rho_m) - 1]^2}[B + \widetilde{\omega}\rho_m + \Lambda(t)] - 1, \quad (30)$$

where

$$B = \frac{2H_1\lambda(n+1)t^n e^{-\lambda t^{n+1}}}{\chi^2} + \frac{3H^2}{\chi^2}.$$

From the above equation, we observed that for any value of $\Lambda(t)$, we get $\omega(t) \to -1$ for the PR model.

Thus, the influence of the coupling between dark energy and dark matter on the evolution of the LR as well as the PR model has been investigated by the time-dependent parameters of the quadratic EOS.

3. Conclusion

The physics of dark matter and dark energy has remained challenging even after a series of astronomical missions.

In this paper, we have presented models with two interacting ideal fluids corresponding to dark matter and dark energy with quadratic EOS with time-dependent parameters $\omega(t)$ and $\Lambda(t)$, in which LR and PR behaviors, described in flat FLRW model, have been encountered in the far future. It shows that LR and PR cosmology changes exponentially with parameter $\Lambda(t)$.

For the coupled dark energy, we have obtained expression for dark energy, the dark matter density, and time-dependent parameters $\omega(t)$ and $\Lambda(t)$. In this expression, the descriptions of the LR and PR universes in terms of the quadratic EOS parameters were given.

For LR model Case (a), LR model Case (b), and PR model (Equation 25), the gravitational equations of motion for dark matter were solved as Equations (13), (20), and (27), respectively. For LR model, Equations (13) and (20) at $t = 0$ simply yield

$$\rho_m(0) = (1/3H_0)((2Q_1/9H_0^2) + Q_2) + c$$

and

$$\rho_m(0) = 1 + c_1.$$

It indicates that, at the initial time $t = 0$, dark matter is constant in the universe. For the result

$$\rho_m(0) = (1/3H_0)((2Q_1/9H_0^2) + Q_2) + c,$$

if we take the value of constant of integration

$$c = -(1/3H_0)((2Q_1/9H_0^2) + Q_2),$$

we obtain $\rho_m(0) = 0$. In short, at $t \to 0$, it is seen that p and ρ exist, whereas ρ_m and p_m tend to zero and this implies there might be a period in this model when there is no dark matter in it.

Dissimilar to a model containing simply pure dark energy, the presence of an interaction rate between dark energy and dark matter in the gravitational equations prompts changes in the EOS parameters (Bervik *et al.* 2013).

Similarly for LR model Case (b) and PR model, if we consider the value of integration constants c_1 and c_2 as -1 in Equations (20) and (27), at the initial value of t, i.e., $t = 0$, we found that energy density for dark matter is also zero. For PR model at $t \to \infty$, we found $\rho_m \to 1$ and at $t \to -\infty$, we found $\rho_m \to 0$. From this expression, we observed that in the past time, there is no dark matter and it will start to appear in the universe with increasing time. So, we can say, energy density for dark matter has a value between 0 and 1, which means it is negligible for the elapsed time from $t \to -\infty$ to $t \to \infty$.

From Equations (17), (24), and (30), we observed that for any value of $\Lambda(t)$, like if we take the value of $\Lambda(t)$ as an increasing function of t or decreasing function of t or constant value, we always get the value of $\omega(t) \to -1$ for both cases of LR model as well as PR model. It shows that the effect of coupling between dark energy and dark matter is responsible for the accelerated expansion of the universe.

Additionally, discovered behavior of LR and PR for coupled dark energy with quadratic EOS with time-dependent parameters $\omega(t)$ and $\Lambda(t)$.

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