J. Astrophys. Astr. (2021) 42:12
https://doi.org/10.1007/s12036-021-09732-4

LETTER

Is the binding energy of galaxies related to their core black hole mass?

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MS received 28 February 2020; accepted 4 January 2021

Abstract. Most of the large galaxies host a supermassive black hole, but their origin is still not well understood. In this paper, we look at a possible connection between the gravitational binding energies of large galaxies, etc., and the masses of their central black holes. Using this relation (between the gravitational binding energy of the host structure and the black hole energy), we argue why globular clusters are unlikely to harbour large black holes and why dwarf galaxies, if they have to host black holes, should have observed mass-to-light ratios of $\sim$100.

Keywords. Binding energy—black holes—dwarf galaxies.

It is by now well established that most large galaxies (spiral, elliptical, etc.) host a supermassive black hole (SMBH) in their centre (Kaper *et al.* 2001). Again active galactic nuclei, quasars, etc., are powered by the gravitational energy of matter accreting onto these central SMBHs (Melia 2007). These black hole (BH) masses are typically of several millions of solar masses and can be as large as a few billion solar masses, such as in the case of M87. The origin of these SMBHs at the galactic cores is still an enigma. Again the BH masses have been related to the galactic bulge mass, suggesting a common origin (Zel'dovich & Novikov 1971; Shapiro & Teukolsky 1983).

Various studies have pointed out the correlation between the masses of the SMBHs at the centre of galaxies and the different characteristics of their host galaxies (Kormendy & Richstone 1995), such as the spheroid luminosity and spheroid mass (Magorrian *et al.* 1998), the spheroid velocity dispersion (Gultekin *et al.* 2009), and the kinetic energy of random motions (Feoli & Mancini 2009). The Kormendy relation connects the estimated mass of the BH to the K-band luminosity and the velocity dispersion of the bulge of the host galaxy measured outside the BH sphere of influence (Kormendy 1977).

However, the underlying physics of these various observations is not yet fully understood. It is thought that feedback processes are responsible for such correlations (Silk & Rees 1998). Also, there is no understanding as to whether these relations evolve with the evolution of the galaxy (and the central BH) or are set at the time of their formation (Di Matteo *et al.* 2005; Volonteri & Natarajan 2009). Again, many other large stellar conglomerations such as globular clusters or dwarf galaxies also do not seem to host massive BHs in their core regions.

Here we propose a possible connection between the gravitational binding energies of large galaxies, etc., and the masses of their central BHs. The idea is similar to what happens in the core collapse of massive stars. The gravitational binding energy released in stellar collapse is carried away mainly by neutrinos (as in the case of SN1987A). The total energy carried away is just the gravitational binding energy of the remnant neutron star (Burrows 1990).

In the case of SN1987A, the estimated 3×10^{53} erg carried away by neutrinos just corresponds to the binding energy of a 1.4 $M_{\odot}$ neutron star (Bahcall 1989). More massive stars would release more binding energy, and this would correspond to the formation of a BH. The gravitational binding energy released in the formation of a BH would correspond to its rest energy, i.e., to the mass of the BH (for the same reason, a supernova would not give rise to a white dwarf as a remnant, as the binding energy of the white dwarf is two or more orders smaller than the energy released in the explosion).

Published online: 30 April 2021

By analogy, we argue that whatever processes led to the formation of the BHs at the galaxy cores was connected with the formation of the galaxy and should thus be related to the gravitational binding energy of the final structure which forms.

We note that in the case of galactic structures, the binding energy of the parent galaxy is comparable with that of the BH hosted by the galaxy. The binding energy of a typical galaxy such as the Milky Way is given by

$$(BE)_{\text{spiral}} = \frac{GM_{\text{spiral}}^2}{R_{\text{spiral}}} \approx 10^{61}\,\text{erg}$$
$$\left(M_{\text{spiral}} \approx 10^{12}\,M_\odot,\ R_{\text{spiral}} \approx 10\,\text{kpc}\right). \tag{1}$$

The galaxy harbours a $\sim 10^6\,M_\odot$ BH. The energy (mass) of the BH is as follows:

$$(BE)_{\text{BH}} = M_{\text{BH}}c^2 = \frac{GM_{\text{BH}}^2}{R_{\text{BH}}} \approx 10^{61}\,\text{erg}. \tag{2}$$

The equality of Equations (1) and (2) is suggestive of what happens in the release of gravitational binding energy in stellar collapses leading to the formation of a remnant compact object.

Again for large elliptical galaxies, mass is few orders higher, that is $M_{\text{elliptical}} \approx 10^{14}\,M_\odot$, so the binding energy is as follows:

$$(BE)_{\text{elliptical}} \approx 10^{63}\,\text{erg}. \tag{3}$$

Thus, galaxies, such as M87 harbour billion solar mass BH, whose binding energy is as follows:

$$(BE)_{\text{BH}} \approx 10^{63}\,\text{erg}. \tag{4}$$

Therefore, we have, in the case of galaxies, the binding energy of the galaxy being comparable with the BH energy:

$$M_{\text{BH}}c^2 = \frac{\text{G}M_{\text{galaxy}}^2}{R}. \tag{5}$$

Hence, the fraction of BH mass to the galactic mass is given by

$$\frac{M_{\text{BH}}}{M_{\text{galaxy}}} = \frac{GM_{\text{galaxy}}}{Rc^2} \approx 10^{-5} - 10^{-6}. \tag{6}$$

This implies that smaller galaxies of $10^8\,M_\odot$ could harbour BHs of $100\,M_\odot$.

This is in agreement with the observations of host galaxy mass and its central BH mass, as shown in Table 1 (Bandara *et al.* 2009).

The result from Equation (6) could also provide a reason as to why globular clusters do not host a BH. The binding energy of the cluster is as follows:

$$(BE)_{\text{cluster}} = \frac{GM_{\text{cluster}}^2}{R_{\text{cluster}}} \approx 10^{51}\,\text{erg}, \tag{7}$$

which is lesser than the binding energy of even neutron stars.

In the case of some globular clusters such as the Mayall II (G1) in the Andromeda galaxy, there is evidence of a $2 \times 10^4\,M_\odot$ BH. There are also studies (Meylan *et al.* 2001) that have suggested that the globular cluster G1 could in fact be the core of a dwarf elliptical galaxy. The BH in G1 could have been formed at an earlier epoch when the cluster was more closely packed or due to the merger of few clusters. X-ray and radio emissions from Mayall II appear to be consistent with an intermediate-mass BH. The energy of the $10^3\,M_\odot$ BH is $E = M_{\text{BH}}c^2 \approx 10^{57}$ erg. In order for the cluster to have a binding energy of this order, its size should be

$$R = \frac{GM_{\text{cluster}}^2}{M_{\text{BH}}c^2} \approx 10^{-4}\,\text{pc}, \tag{8}$$

where $M_{\text{cluster}} \sim 10^6\,M_\odot$.

As the potential decreases, the kinetic energy of the cluster increases, hence spreading it out over time.

One of the possible models for the formation of an IMBH is the collapse of a cluster of stars (Miller & Hamilton 2002). The collapsed core can accrete matter ejected during the formation. If a collection of a thousand 10 solar mass stars in a volume of a parsec cube collapses, ejecting 30% of its mass, then the total mass of the ambient gas, $nm_P \approx 6 \times 10^{33}$ kg. If $\sim 30\%$ of the mass of each star is ejected, then the accretion rate given by $\dot{m} = 16\pi n m_P G^2 M^2/c^3 \approx 4 \times 10^7$ kg s^{-1}. For a constant density, the dynamical friction (Chandrasekhar 1942) is given as follows:

$$F_{\text{d}} = C\frac{G^2M^2\rho}{v_m^2}, \tag{9}$$

where G is the gravitational constant, M the mass of the moving object, ρ the density, and v_m the velocity of the object in the frame in which the surrounding matter was initially at rest. C is not a constant but depends on how v_m compares with the velocity dispersion of the surrounding matter.

The dynamical friction comes into effect in the evolution of the cluster due to the interaction between the ambient gas and dust, and the central Intermediate Mass Black Hole (IMBH). The initial BH is subjected

Table 1. Correlation between SMBH mass (M_{BH}) and mass of its host galaxy (M_{galaxy}).

Galaxy system	$M_{BH}(M_\odot)$	$M_{galaxy}(M_\odot)$	M_{BH}/M_{galaxy}
SDSS J0029−0055	2.65×10^8	1.04×10^{13}	2.5×10^{-5}
SDSS J0252+0039	6.48×10^7	1.27×10^{13}	5.1×10^{-6}
SDSS J0330−0020	1.88×10^8	1.51×10^{13}	1.2×10^{-5}
SDSS J0728+3835	1.90×10^8	1.73×10^{13}	1.1×10^{-5}
SDSS J0737+3216	1.28×10^9	2.41×10^{13}	5.3×10^{-5}
SDSS J0822+2652	4.67×10^8	2.00×10^{13}	2.3×10^{-5}
SDSS J0955+0101	1.14×10^8	1.21×10^{13}	9.4×10^{-6}
SDSS J0959+0410	2.25×10^8	1.08×10^{13}	2.1×10^{-5}
SDSS J1142+1001	2.81×10^8	1.68×10^{13}	1.7×10^{-5}
SDSS J1403+0006	1.74×10^8	1.18×10^{13}	1.5×10^{-5}
SDSS J1451−0239	2.21×10^8	1.17×10^{13}	1.9×10^{-5}
SDSS J1538+5817	1.06×10^8	1.17×10^{13}	9.1×10^{-6}
SDSS J2238−0754	1.47×10^8	1.44×10^{13}	1.0×10^{-5}
SDSS J2303+1422	3.83×10^8	2.57×10^{13}	1.5×10^{-5}
SDSS J2341+0000	1.44×10^8	1.87×10^{13}	7.7×10^{-6}

to a gravitational force due to the star cluster, which is given by

$$\nabla^2\phi(r) = 4\pi G\rho(r) = 4\pi Gmn(r), \tag{10}$$

where $n(r)$ is the density distribution of stars and m the typical mass of the star.

For a constant density, $\rho(r) = \rho_0$, the potential is as follows:

$$\phi(r) = -2\pi G\rho_0\left(R^2 - \frac{1}{3}r^2\right), \tag{11}$$

where R is the radius of the cluster.

The gravitational force on the BH is given by

$$F_g = -M_{BH}\nabla\phi(r) = -\frac{4}{3}\pi G\rho_0 M_{BH} r. \tag{12}$$

The particle inside a homogeneous gravitational system performs simple harmonic motion.

The equation of motion of the BH in the star cluster is given by

$$M_{BH}\frac{d^2r}{dt^2} + kr + \gamma\frac{dr}{dt} = 0, \tag{13}$$

where $k = (4/3)\pi G\rho_0 M_{BH}$, $\gamma = -3\pi G^2 mn(r)M_{BH}^2/(\sqrt{2}\sigma)^3$ and σ the velocity dispersion of stars in the cluster. The BH undergoes a damped oscillation in the star cluster, with a damping time corresponding to $t_{damp} = M_{BH}/\gamma$.

In the case of the M82 system , which harbours an IMBH of mass in the range of $500\,M_\odot$ (Pasham *et al.* 2014), the BH is not found at the centre but displaced by about 1 kilo-parsec from the centre. The frequency corresponding to the oscillation of the BH, $\omega = \sqrt{k/M_{BH}} \approx 10^{-13}\,s^{-1}$.

For the M82 system, the number density of the stars in the cluster, $\approx 10^4\,pc^{-3}$, with a typical mass of $1\,M_\odot$ has a velocity dispersion, $\sigma \approx 30\,km\,s^{-1}$. We obtain the damping time for the system as $t_{damp} \approx 4 \times 10^{12}$ s, which is of the order of 10^5 years. With the effects of dynamical friction, the relaxation time for the system to form the IMBH is $t_{form} \approx 10^7$ years. For a denser core, i.e., $\approx 10^3/(0.01\,pc)^3$, the time taken to form the IMBH will be $\approx 10^6$ years (Sivaram & Arun 2014).

Similarly, certain dwarf galaxies could also harbour $10^3\,M_\odot$ BH, even though their binding energy (with visible mass) suggests that they should not. The binding energy of the dwarf galaxy is as follows:

$$(BE)_{dwarf} = \frac{GM_{dwarf}^2}{R_{dwarf}} \approx 10^{55}\,erg. \tag{14}$$

The energy of the $10^3\,M_\odot$ BH is of the order of 10^{57} erg. This would suggest that there is a substantial amount of dark matter to make up for the binding energy of the dwarf galaxy (Faber & Gallagher 1979; Kolb & Turner 1990):

$$\frac{M}{L} \approx 100. \tag{15}$$

Recent studies (Reines *et al.* 2020) with Very Large Array (VLA) have found 13 massive BHs in dwarf galaxies that are less than a billion light-years from

Earth. Their study indicates that roughly half of the massive BHs in dwarf galaxies are not in the centres of those galaxies. This probably indicates that the galaxies likely have merged with others earlier in their history, as elaborated in this work, thus maintaining the validity of our argument.

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