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A test of Alzain's modified inertia model for MOND using galaxy cluster observations

SOWMYA SEERAM and SHANTANU DESAI*

Department of Physics, Indian Institute of Technology, Hyderabad 502285, India.
E-mail: shntn05@gmail.com

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Abstract. Recently, Alzain proposed a modified inertial formalism of modified Newtonian dynamics (MOND), wherein Galiliean invariance is violated and new Lorentz transformation properties from an inertial to an accelerated frames are posited. Milgrom's acceleration constant is then found to be invariant under the aegis of these new space-time coordinate transformations. In this model, a modified equation of hydrostatic equilibrium is proposed, which can be applied to relaxed galaxy clusters. However, when we apply this equation to a Chandra sample of 12 clusters, we find that the total dynamical mass obtained using Alzain's model is much smaller than the baryonic mass for almost all the clusters, thereby implying that this original model is not viable. A variant of this model with a value of a_0 about two times smaller than that which fits galaxy data, is necessary in order to reconcile the missing mass problem in clusters, without the need for dark matter.

Keywords. MOND—galaxy clusters—dark matter.

1. Introduction

One of the key edifice of the standard ΛCDM hot big bang model of the Universe (Ratra & Vogeley 2008) is the existence of 25% cold dark matter. This cold dark matter component is posited to be non-baryonic and decouples from the remaining components of the universe, while moving at non-relativistic velocities (Jungman *et al.* 1996; Bertone *et al.* 2005). Although, this ΛCDM model agrees remarkably well with observational astrophysical data at large-scales (Ade 2016), it is still disturbing that there is no laboratory evidence for any cold dark matter candidate, despite over three decades of searches through underground dark matter laboratory searches (Merritt 2017). The LHC has also failed to find evidence for supersymmetry or any other theories beyond standard model, which predict these cold dark matter candidates (Canepa 2019).

Therefore, one possible alternative to the above standard picture, which obviates the need for dark matter, is the modified Newtonian dynamics (MOND) paradigm (Milgrom 1983). One can think of MOND as a theoretical extension of Newtonian dynamics in the low acceleration limit, i.e., for acceleration below some constant value $a_0 \approx 10^{-10}$ m s^{-2}, where a_0 is referred to as Milgrom's constant. Milgrom established a smooth transition between the dark matter dominated and the Newtonian regimes, i.e., $g \gg a_0$ and $g \ll a_0$ using the formula

$$\mu\left(\frac{g}{a_0}\right)g = g_N, \tag{1}$$

where the interpolation function $\mu(x)$ is given by, $\mu(x) \rightarrow 1$ for $x \gg 1$ and $\mu(x) \rightarrow x$ for $x \ll 1$. Many examples of such MOND interpolation functions can be found in Famaey & McGaugh (2012). The MOND paradigm relates the observed gravitational acceleration g and the Newtonian acceleration calculated from only the baryonic mass ($g_{\rm bar}$) by radial acceleration relation (RAR), i.e., for acceleration above a_0, $g_{\rm obs} \approx g_{\rm bar}$ and for acceleration below the value, $g_{\rm obs} \propto g_{\rm bar}^{1/2}$ (Famaey & McGaugh 2012).

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This RAR has recently been shown to fit the rotation curves of SPARC galaxy samples remarkably well with negligible scatter of around 0.12 dex (McGaugh *et al.* 2016). MOND also has a number of other successes on galactic scales, as it can trivially explain many empirical observations such as Tully–Fisher relation, mass-acceleration discrepancy relation, Freeman's Law, Renzo's rule, etc., (Famaey & McGaugh 2012). These successes cannot be trivially predicted by the standard ΛCDM model. The latest scorecard comparing the successes of MOND *vs.* ΛCDM on galactic scales are reviewed in McGaugh (2020).

However, it has been known since a long time, that MOND does not work well for galaxy clusters (The & White 1988; Aguirre *et al.* 2001; Sanders 2003; Pointecouteau & Silk 2005; Angus *et al.* 2008; Natarajan & Zhao 2008). For relaxed galaxy clusters, MOND has been tested using two broad methods. In the first method, the dynamical MOND masses for clusters have been calculated using a modified equation of hydrostatic equilibrium and found to be much larger than the baryonic mass, consisting of hot diffuse gas and stars (The & White 1988; Sanders 2003; Pointecouteau & Silk 2005; Angus *et al.* 2008). Using this method, the dynamical MOND mass was found to be larger than the observed baryonic mass (The & White 1988; Sanders 2003; Pointecouteau & Silk 2005; Angus *et al.* 2008). MOND has also been tested using the usual (Newtonian) equation of hydrostatic equilibrium, but plugging the MONDian formula for the acceleration, in order to derive the resulting temperature profile (Aguirre *et al.* 2001). This estimated temperature profile was found to be much smaller than the observed temperature profiles, thus providing a strong challenge for MOND (Aguirre *et al.* 2001). Therefore, the conclusion from both these methods is that, even within the MOND paradigm, galaxy clusters have a larger dynamical mass, which cannot be accounted for using baryons. MOND also fails with lensing data from merging clusters such as the Bullet Cluster (Natarajan & Zhao 2008) and with the shape of the matter power spectrum (Dodelson 2011). Many relativistic theories of MOND are also in tension due to the coincident gravitational wave and electromagnetic observations from GW170817 (Boran *et al.* 2018).

Yet despite these setbacks, there are a large number of attempts to reconcile the successes of MOND on galactic scales with the failures at larger scales. Theoretically, MOND can be interpreted either as a modification of Newtonian gravity or as modified inertia and there continue to be new attempts to formulate MOND using both these pictures. In this work, we test a recently proposed interpretation of MOND as modified inertia (Alzain 2017) using a sample of 12 related galaxy clusters studied by Vikhlinin *et al.* (2005, 2006).

2. Alzain's model for MOND

Recently, Alzain proposed a model of MOND (Alzain 2017) based on modified inertia, which we briefly describe (please refer Alzain (2017) for more details). The starting point was based on a reformulation of MOND by Milgrom as a modification of Newton's second of motion (Milgrom 1994; Famaey & McGaugh 2012):

$$F = mg\mu\left(\frac{g}{a_0}\right), \tag{2}$$

where F is the total force on the particle, m the inertial mass, g the total acceleration and a_0 the Milgrom's constant. Using any of the commonly used MOND interpolation functions (Famaey & McGaugh 2012), Equation (2) can be written in the low-acceleration limit as

$$g = g_N + \sqrt{a_0 g_N}, \tag{3}$$

where g_N is the acceleration due to Newtonian gravity.

In Alzain's model, locality and Galilean invariance are violated in the low-velocity limit. Instead of the usual Lorentz transformations, one gets the following non-linear space-time transformations between an inertial frame (x, t) and an accelerated frame (x', t') moving with acceleration g_D relative to the inertial frame:

$$x' = \alpha\left(x - \frac{1}{2}g_D t^2\right), \tag{4}$$

$$t'^2 = \alpha\left(t^2 - 2\frac{g_D}{a_\dagger^2}x\right), \tag{5}$$

where α is an arbitrary constant and is given by

$$\alpha = \frac{1}{\sqrt{1 - g_D^2/a_\dagger^2}}, \tag{6}$$

and $a_\dagger$ is the universal constant proportional to Milgrom's constant ($a_\dagger \equiv \eta a_0$). Alzain then showed that $a_\dagger$ is invariant under the above sets of non-linear Lorentz transformations in Equations (4) and (5), i.e.,

$$a_\dagger = \frac{d^2x'}{dt'^2} = \frac{d^2x}{dt^2}.$$

Therefore, $a_\dagger$ plays the role of a Lorentz scalar in special relativistic space-times.

In the above model the zeroth (time) component of the force four-vector (F^0) can be written as:

$$F^0 = \frac{ma_\dagger}{\sqrt{1 - g_D^2/a_\dagger^2}} = ma_\dagger + \frac{1}{2}m\frac{g_D^2}{a_\dagger} + \cdots, \quad (7)$$

where the last term is obtained from a Taylor expansion, neglecting higher-order terms.

From Milgrom's modified inertia formulation Equation (3), the excess acceleration compared to the Newtonian acceleration can be written as

$$a_D = g - g_N = \sqrt{a_0 g_N}. \quad (8)$$

Alzain then argued that a_D in Equation (8) is equal to g_D in Equation (7) and the constant of proportionality η used to define $a_\dagger = \frac{1}{2}$. The total force acting on the particle is then equal to F^0 from Equation (7) and reproduces Milgrom's formula in the low acceleration limit.

Furthermore, to come up with a viable model for galaxy clusters, Alzain also incorporated weak equivalence principle in addition to the above space-time transformations. To incorporate such a universality of free-fall, Alzain used the following *ansatz* to replace Milgroms modified inertia formula from Equation (2)

$$\mu\left(\frac{g}{a_0}\right)g = g_N + \eta a_0, \quad (9)$$

where the interpolation function is chosen to resemble the μ-function of Milgroms formula, i.e, $\mu(x) \rightarrow 1$ for $x \gg \eta$ and $\mu(x) \rightarrow x$ for $x \ll \eta$ and the value of $\eta = \frac{1}{2}$ as before. Then the observed acceleration in high acceleration systems $g \gg \frac{1}{2}a_0$ is

$$g = g_N + \frac{1}{2}a_0 \quad (10)$$

and that in low acceleration systems $g \ll \frac{1}{2}a_0$ is

$$g = \sqrt{a_0 g_N + \frac{1}{2}a_0^2}, \quad (11)$$

which is expected to become relevant within large clusters of galaxies. Considering the dynamical mass for clusters in hydrostatic equilibrium and Equation (11) we get,

$$\sqrt{a_0 G M_{\rm dyn} + \frac{1}{2}a_0^2 r^2} = -\frac{k_b T}{\mu m_p}\left(\frac{d\ln\rho}{d\ln r} + \frac{d\ln T}{d\ln r}\right), \quad (12)$$

where $M_{\rm dyn}$ is the dynamical mass within Alzain's theory, T the observed temperature profile and ρ the gas density. Alzain asserted that this is a more robust relation between mass and temperature for clusters than $M \propto T^2$ previously predicted by MOND (Aguirre *et al.* 2001).

To test this model, we note that the r.h.s. of Equation (12) is a simple function of the total Newtonian dynamical mass (M_N) for clusters in hydrostatic equilibrium (Allen *et al.* 2011). This total dynamical mass under the aegis of Newtonian gravity is dominated by dark matter, and also includes about 10% contribution from hot gas and stars. Therefore, Equation (12) can be re-written as

$$\sqrt{a_0 G M_{\rm dyn} + \frac{1}{2}a_0^2 r^2} = \frac{GM_N(r)}{r}. \quad (13)$$

In this work, we try to verify this empirical formulation using a sample of galaxy clusters. An acid test of any modified gravity theory, which dispenses with the need for dark matter is that the total dynamical mass in that theory must be equal to the total observed baryonic mass, given by the sum of gas and star mass (Rahvar & Mashhoon 2014). Therefore, for Alzain's model to be viable, $M_{\rm dyn}$ given by Equation (13) must be equal to the observed baryonic mass in each cluster. A test of this model using a cluster sample observed with Chandra is described in the next section.

3. Analysis and results

Vikhlinin *et al.* (2005, 2006) derived gas density and temperature profiles for 13 different clusters using archival and pointed Chandra and ROSAT X-ray observations with exposure times of $\mathcal{O}$ (100) ks. For our analysis, we use 12 of these clusters, *viz.*, A133, A262, A383, A478, A907, A1413, A1795, A1991, A2029, A2390, RXJ1159 and MKW4 for which mass estimates are available. The redshifts of these clusters range approximately upto $z = 0.2$. These measurements extended up to very large radii of about R_{500} for some of the clusters. The temperatures span the range between 1 and 10 keV and masses from

$(0.5-10) \times 10^{14} M_\odot$. Based on the gas density and temperature profiles, one can test modified gravity theories (Rahvar & Mashhoon 2014; Gupta & Desai 2019) including MOND-based versions (Edmonds *et al.* 2018). We have also previously used this sample for a variety of tests, such as testing effects of relativistic corrections to Newtonian hydrostatic equation, modified gravity theories, constancy of halo surface density (Gupta & Desai 2019, 2020; Gopika & Desai 2020). In the midst of this work, we have compiled the total as well as baryonic masses at various radii, which we use here.

However, since the total Newtonian dynamical masses for these clusters (obtained by solving the equation of hydrostatic equilibrium) along with the errors, have been calculated in Gupta & Desai (2020) at R_{500}, we directly use these values to test Alzain's model. We therefore use the estimated M_{500} and R_{500} values along with errors in Equation (13) to estimate $M_{\rm dyn}$. Here we choose $a_0 = (1.2 \pm 0.02) \times 10^{-10}$ m s^{-2}, which is the fit parameter value obtained for SPARC data (McGaugh *et al.* 2016; Li *et al.* 2018). The resulting dynamical masses in Alzain's model for 12 clusters, along with the 1σ errors in the Vikhlinin sample can be found in Table 1. The error in $M_{\rm dyn}$ was obtained by standard error propagation from the errors in R_{500}, a_0 and M_N.

If this model is to successfully dispense with dark matter, the dynamical mass should agree with the baryonic mass (within errors). If the dynamical mass is much larger than the baryonic mass, it implies that Alzain's model also requires the presence of dark matter. If it is much less than the baryonic mass, it implies that the model cannot be physical. We compare the dynamical mass to the baryonic mass estimates for this cluster. This baryonic mass is the sum of hot diffuse gas in the intracluster medium and stars, and has been estimated for this sample at various radii in Gopika & Desai (2020). We use those estimates to compare with the dynamical mass from Alzain's model. We quantify the discrepancy (D) between the dynamical mass in Alzain's model and the baryonic mass in terms of number of standard deviations (σ) using

$$D = \frac{M_{\rm dyn} - M_{\rm bar}}{\sqrt{\sigma^2_{M_{\rm dyn}} + \sigma^2_{M_{\rm bar}}}}, \qquad (14)$$

where $\sigma_{M_{\rm dyn}}$ and $\sigma_{M_{\rm bar}}$ denote the uncertainty in the dynamical and baryonic mass. Such a metric is also widely used in cosmology, e.g., to quantify the Hubble constant tension (Vagnozzi 2020).

An acid test for any model to be the correct description of nature, is that the absolute value of D should be $< 1\sigma$. From Table 1, we see that D is negative for most clusters, with deviation $> 2\sigma$, for almost all the cases, with the maximum deviation equal to about 10σ (for A262). For only three clusters (A133, A1413 and A2390), the discrepancy is $<1\sigma$. Another way to quantify the deviation using the full dataset is to calculate χ^2 between the baryonic mass and Alzain's dynamical mass. Here, χ^2 is constructed

Table 1. Calculation of dynamical mass in Alzain's theory from Equation (13) using the Newtonian dynamical mass compiled in Gupta & Desai (2020) ($M_{\rm Newt}$) at R_{500} and considering $a_0 = 1.2 \times 10^{-10}$ m s^{-2}, along with the 1σ error bars. For comparison, we also show the baryonic mass (M_b) compiled in Gopika & Desai (2020). The last column indicates the discrepancy between the two (Equation 14) in terms of the number of sigmas. A positive value of D implies that the Alzain dynamical mass is greater than the baryonic one and vice-versa.

Cluster	R_{500} (kpc)	$M_N(R_{500})$ $(10^{14} M_\odot)$	$M_b(R_{500})$ $(10^{13} M_\odot)$	$M_{\rm dyn}$ $(10^{14} M_\odot)$	$D(\sigma)$
A133	1007 ± 41	4.08 ± 2.35	3.11 ± 0.69	-2.07 ± 2.39	-1.0
A262	650 ± 21	0.826 ± 0.19	1.08 ± 0.31	-1.5 ± 0.16	-9.9
A383	944 ± 32	3.19 ± 1.00	4.13 ± 0.94	-2.2 ± 0.96	-2.7
A478	1337 ± 58	7.41 ± 1.89	9.88 ± 2.01	-3.43 ± 2.2	-2.0
A907	1096 ± 30	4.41 ± 1.3	6.09 ± 1.46	-2.85 ± 1.25	-2.8
A1413	1299 ± 33	9.94 ± 5.8	8.95 ± 1.93	0.42 ± 84.9	-0.1
A1795	1235 ± 36	5.75 ± 1.46	6.95 ± 1.31	-3.48 ± 1.46	-2.8
A1991	731.7 ± 33	1.38 ± 0.60	1.47 ± 0.37	-1.73 ± 0.45	-4.1
A2029	1362 ± 43	8.63 ± 2.8	10.5 ± 2.34	-2.53 ± 3.3	-1.1
A2390	1416 ± 48	12.84 ± 5.7	15.9 ± 3.69	2.06 ± 9.1	0.05
RXJ1159	700 ± 57	1.21 ± 0.43	0.75 ± 0.15	-1.61 ± 0.46	-3.6
MKW4	634 ± 28	0.75 ± 0.24	0.62 ± 0.14	-1.45 ± 0.19	-7.8

by taking into account errors in a_0, R_{500} and the total M_N using the prescription in Desai (2016).

$$\chi^2 = \sum_{i=1}^{N} \frac{(M_b^i - M_{\rm dyn}^i)^2}{\left[\sigma_{M_b}^2 + \left(\frac{\partial M_{\rm dyn}}{\partial a_0}\sigma_{a_0}\right)^2 + \left(\frac{\partial M_{\rm dyn}}{\partial M_N}\sigma_{M_N}\right)^2 + \left(\frac{\partial M_{\rm dyn}}{\partial R_{500}}\sigma_{R_{500}}\right)^2\right]}, \quad (15)$$

where σ_{a_0}, σ_{M_N} and $\sigma_{R_{500}}$ denote the errors in a_0, M_N and R_{500} respectively. We then obtain $\chi^2/\text{dof} = 302/12$, thus indicating a poor fit.

Therefore, Alzain's model having the same value of a_0 same as that obtained a fit to the SPARC sample (Li *et al.* 2018) cannot be a viable model, in order to obviate the need for dark matter in clusters.

3.1 *a_0 as a free parameter*

We now discuss if Alzain's model can be reconciled with cluster observations using a different value for a_0. Many previous tests of Milgrom's acceleration law for clusters with varying a_0 have found larger value for a_0 than that for galaxies (The & White 1988; Tian *et al.* 2020; Chan & Del Popolo 2020). We now vary a_0 to find an optimum value which is consistent with the Chandra cluster sample. To do this, we evaluate χ^2 in Equation (15) as a function of a_0, and determine the value of a_0, which minimizes χ^2. This best-fit value is equal to $a_0 = (4.69 \pm 0.6) \times 10^{-11}$ m s^{-2}, resulting in a χ^2/dof of 8.1/11. This value is smaller than that obtained from fits to the SPARC sample or earlier galaxy measurements (Famaey & McGaugh 2012). Therefore, the only way to make this model compatible with data is to posit a_0 to be about 2.2 times smaller than the value determined by galaxy observations. We note that recent tests of radial acceleration relation for clusters is indicative of a higher value for a_0.

4. Conclusion

We test a recently proposed model of MOND by Alzain (2017) based on modified inertia, wherein Galilean invariance is violated and the new Lorentz transformations are non-linear. For this purpose, we studied a sample of 12 relaxed galaxy clusters using deep Chandra observations, for which detailed gas density and temperature profiles are available (Vikhlinin *et al.* 2006). We then determine the dynamical mass and associated errors using the modified equation of hydrostatic equilibrium (after assuming that the Milgrom's acceleration constant a_0 is the same as that determined using SPARC sample (Li *et al.* 2018)), and compare it with the baryonic mass (consisting of hot diffuse gas and stars). We find that with this value for a_0, the MOND dynamical mass in Alzain's model for most of the clusters is much smaller than the baryonic mass by more than 2σ (Table 1). The resulting χ^2/dof is also >1, indicating that this model is not viable.

We then test a variant of Alzain's model, where a_0 is a free parameter, and find that for this model to be consistent with Chandra X-ray cluster sample, the best-fit value of a_0 is $(4.69 \pm 0.6) \times 10^{-11}$ m s^{-2}, about two times smaller than the value for the SPARC sample.

Therefore, we conclude that the original model proposed by Alzain (2017) is not a viable description of gravity in the low acceleration regime, and cannot obviate the need for dark matter in clusters. Although this model can explain the Chandra cluster observations with a lower value of a_0, a more definitive test must be done with a larger X-ray cluster sample. This may soon be possible with the recent launch of the e-ROSITA X-ray satellite, which should discover about 100,000 clusters. If that data is also compatible with the same value, then the variation of a_0 from clusters to galaxy must also be explained by a more fundamental theory.

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References

Ade P. *et al.* 2016 Astron. Astrophys., 594, A13

Aguirre A., Schaye J., Quataert E. 2001 Astrophys. J., 561, 550

Allen S. W., Evrard A. E., Mantz A. B. 2011 Ann. Rev. Astron. Astrophys., 49, 409

Alzain M. 2017 J. Astrophys. Astron., 38, 59

Angus G. W., Famaey B., Buote D. A. 2008 Mon. Not. R. Astron. Soc., 387, 1470

Bertone G., Hooper D., Silk J. 2005 Phys. Rep., 405, 279

Boran S., Desai S., Kahya E. O., Woodard R. P. 2018 Phys. Rev. D., 97, 041501

Canepa A. 2019 Rev. Phys., 4, 100033
Chan M. H., Del Popolo A. 2020 Mon. Not. R. Astron. Soc., 492, 5865
Desai S. 2016 Astrophys. Space Sci., 361, 138
Dodelson S. 2011 Int. J. Mod. Phys. D, 20, 2749
Edmonds D., Farrah D., Minic D., Ng Y. J., Takeuchi T. 2018 Int. J. Mod. Phys. D, 27, 1830001
Famaey B., McGaugh S. S. 2012 Living Rev. Relativ., 15, 10
Gopika K., Desai S. 2020 Phys. Dark Univ., 30, 100707
Gupta S., Desai S. 2019 Class. Quantum Gravity, 36, 105001
Gupta S., Desai S. 2020, Phys. Dark Univ., 28, 100499
Jungman G., Kamionkowski M., Griest K. 1996 Phys. Rep., 267, 195
Li P., Lelli F., McGaugh S., Schombert J. 2018 Astron. Astrophys., 615, A3
McGaugh S. 2020, Galaxies, 8, 35
McGaugh S. S., Lelli F., Schombert J. M. 2016 Phys. Rev. Lett., 117, 201101
Merritt D. 2017 Stud. History Philos. Mod. Phys., 57, 41
Milgrom M. 1983 Astrophys. J., 270, 365
Milgrom M. 1994 Ann. Phys., 229, 384
Natarajan P., Zhao H. 2008 Mon. Not. R. Astron. Soc., 389, 250
Pointecouteau E., Silk J. 2005 Mon. Not. R. Astron. Soc., 364, 654
Rahvar S., Mashhoon B. 2014 Phys. Rev. D, 89
Ratra B., Vogeley M. S. 2008 PASP, 120, 235
Sanders R. H. 2003 Mon. Not. R. Astron. Soc., 342, 901
The L. S., White S. D. M. 1988 Astron. J., 95, 1642
Tian Y., Umetsu K., Ko C.-M., Donahue M., Chiu I. N. 2020 Astrophys. J., 896, 70
Vagnozzi S. 2020 Phys. Rev. D., 102, 023518
Vikhlinin A., Kravtsov A., Forman W., *et al.* 2006 Astrophys. J., 640, 691
Vikhlinin A., Markevitch M., Murray S. S., *et al.* 2005 Astrophys. J., 628, 655